

A directed graph is said to be *strongly connected* if for every pair of vertices  $x, y$ , there is a directed path from  $x$  to  $y$ . An undirected graph is *strongly orientable* if there exists an assignment of directions to its edges to get a strongly connected graph.

**Theorem 1** (Robbins). *A graph is strongly orientable if and only if it is 2-edge-connected.*

*Proof.* ( $\Rightarrow$ ) Suppose  $G$  has a bridge  $e$ , and let  $C_1$  and  $C_2$  be the components that result from deleting  $e$ . Assume without loss of generality that  $e = v_1v_2$  is oriented from  $v_1 \in C_1$  to  $v_2 \in C_2$ . Then there cannot be a directed path from, say,  $v_2$  to  $v_1$ .

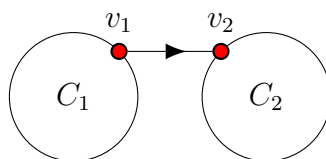


Figure 1: No strong orientations when there's a bridge.

( $\Leftarrow$ ) Every 2-edge-connected graph has a “closed” ear decomposition (i.e. one where the endpoints of an ear can coincide), which we use to construct a strongly connected graph. We induct on the number of ears. For the base case, we have a cycle, which we can make strongly connected by having all the edges point in the same direction. Adding an additional ear as a directed path or cycle keeps the graph strongly connected.

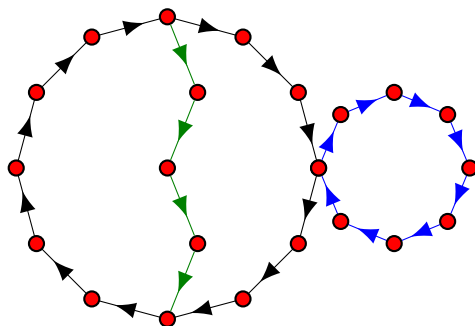


Figure 2: Add each ear as a directed path.

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