COMS W4203: Graph Theory

Timothy Sun

Columbia University

Administrative Stuff

COMS W4203: Graph Theory

- General breadth requirement
- Foundations of CS track elective

Prerequisite: COMS W3203: Discrete Mathematics

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Course Website:
`http://www.cs.columbia.edu/~tim/teaching/cs4203/`

Instructor: Timothy Sun (tim@cs.columbia.edu)
Office Hours: 4-6pm F, 521 CSB

TA: Geelon So (geelon.so@columbia.edu)
Office Hours: 5:30-7pm W, TA room (1st floor Mudd)

TA: TBD

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Textbook: Diestel, Graph Theory, 5th Edition

`http://diestel-graph-theory.com/`

Free! (low quality scans)

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Grading for this class will be broken down into:

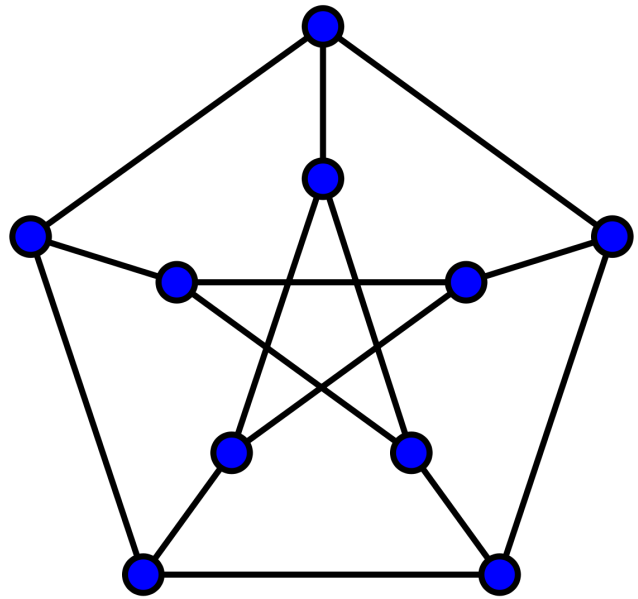
- Homework (5-6 problem sets): 30%.
- In-class midterm: 30%.
- Final exam: 40%.

10% off each day late on HW, up to 7 days.

Graph theory

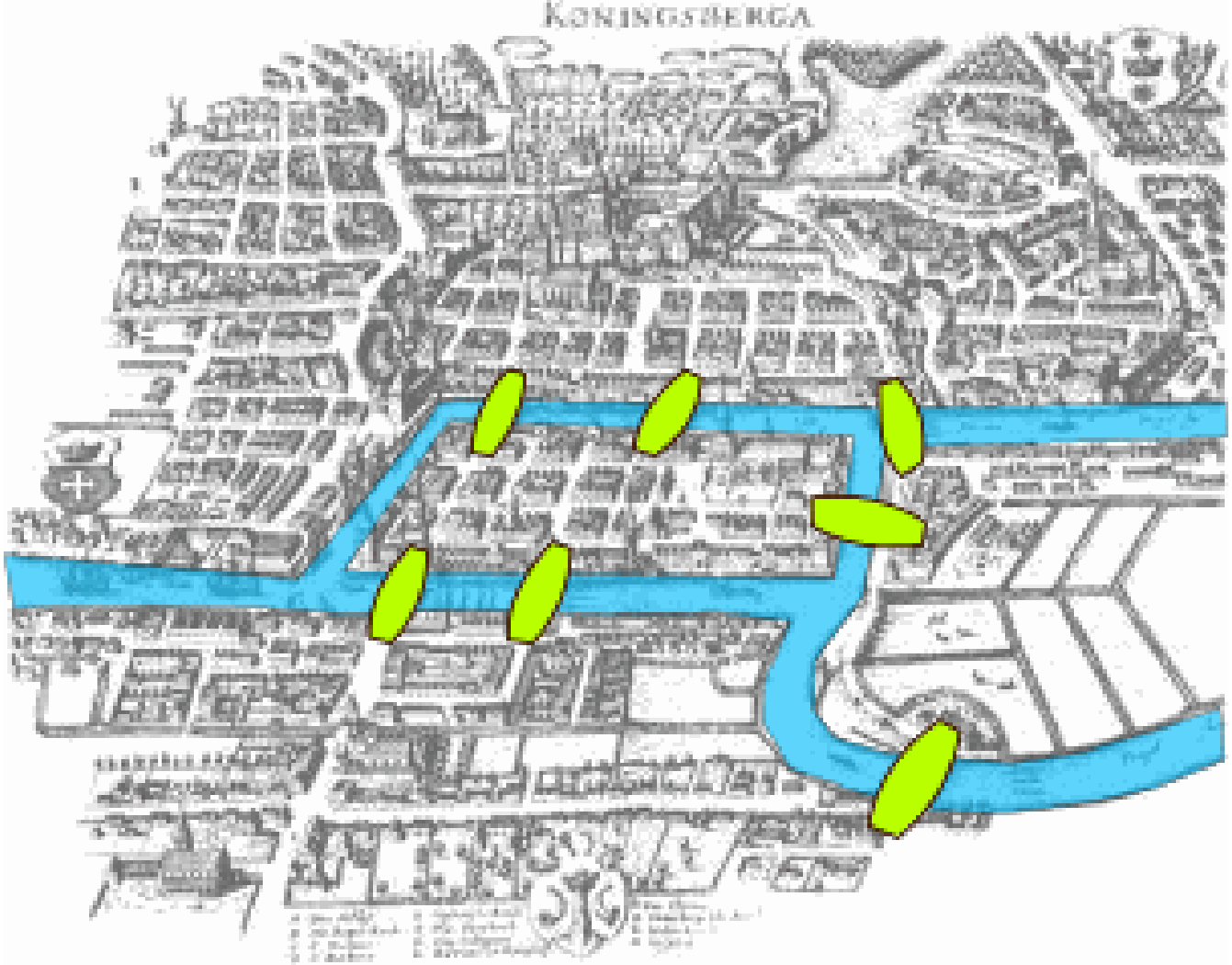
Graphs model "pairwise relationships" in real-life phenomena, e.g.

- social networks
- chemical bonds (e.g. hydrocarbons, fullerenes)
- circuitry



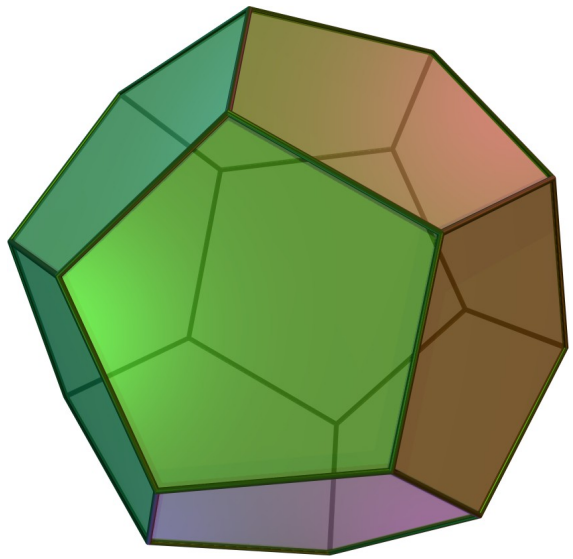
Eulerian circuits and Hamiltonian paths

Is there a way to cross each bridge exactly once?



Eulerian circuits and Hamiltonian paths (cont'd)

Is there a way to visit each vertex once?



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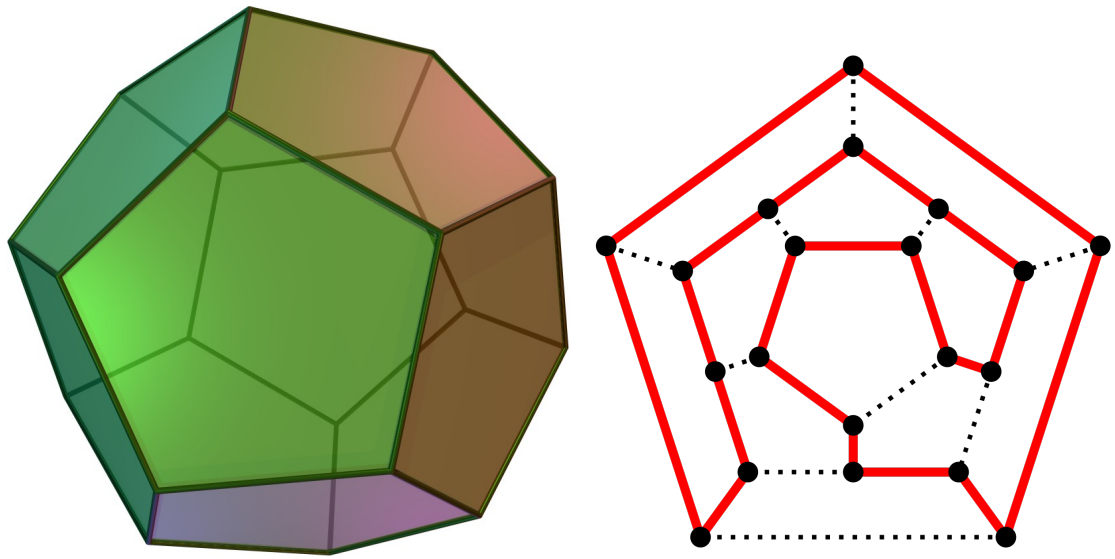

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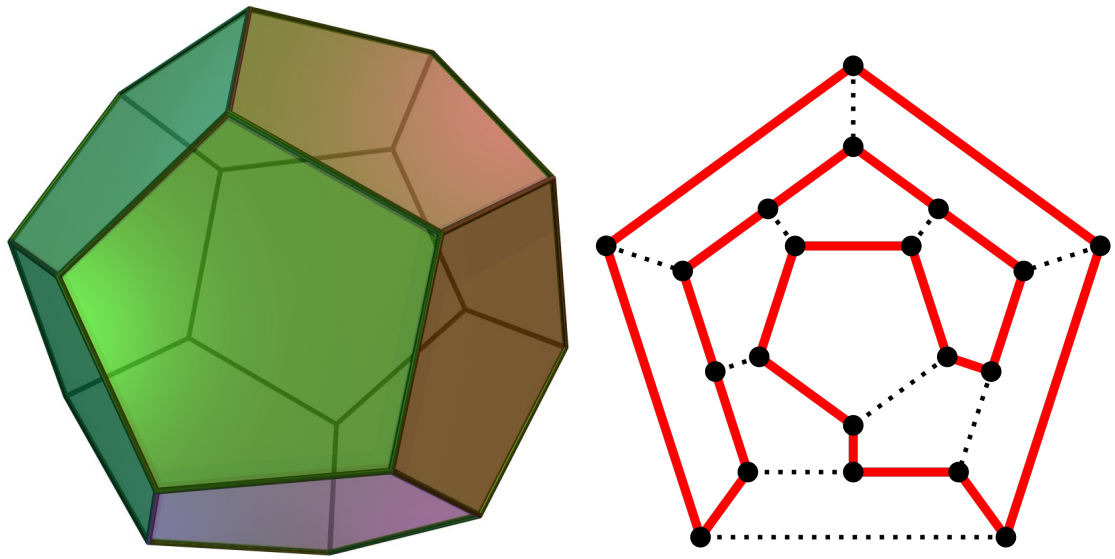


One of Karp's 21 NP-complete problems

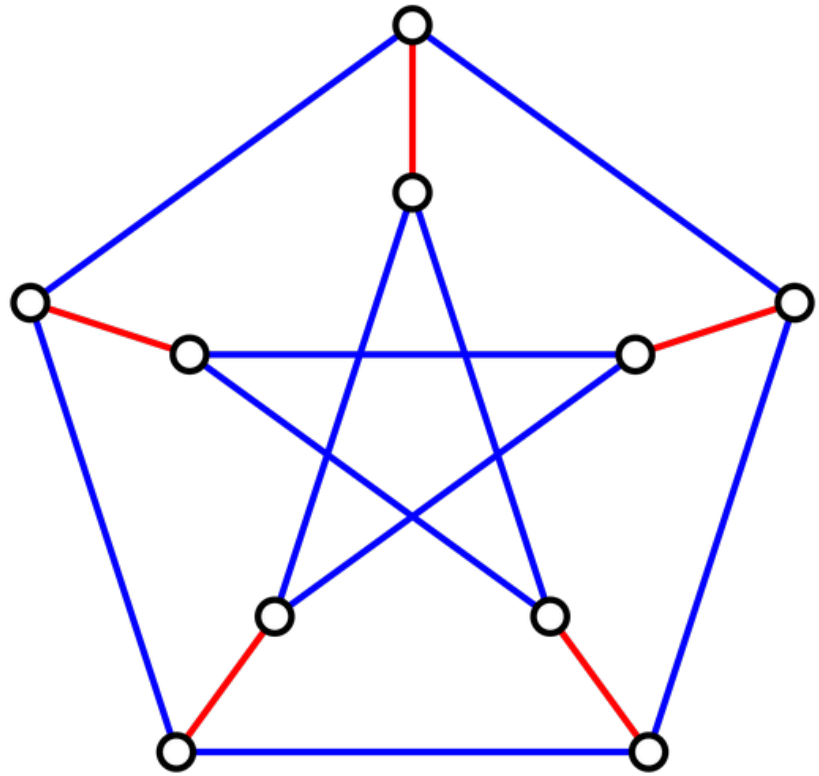
Connectivity

Higher orders of connectivity: can the streets of Manhattan be given all one-way directions such that you can drive from any building to any other building?

Matchings

A **matching** is a subset of edges, no two of which share a vertex.

- Characterizations of maximum matchings
- Algorithmic aspects (P vs NP)



Nobel Prize in CS?

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2012

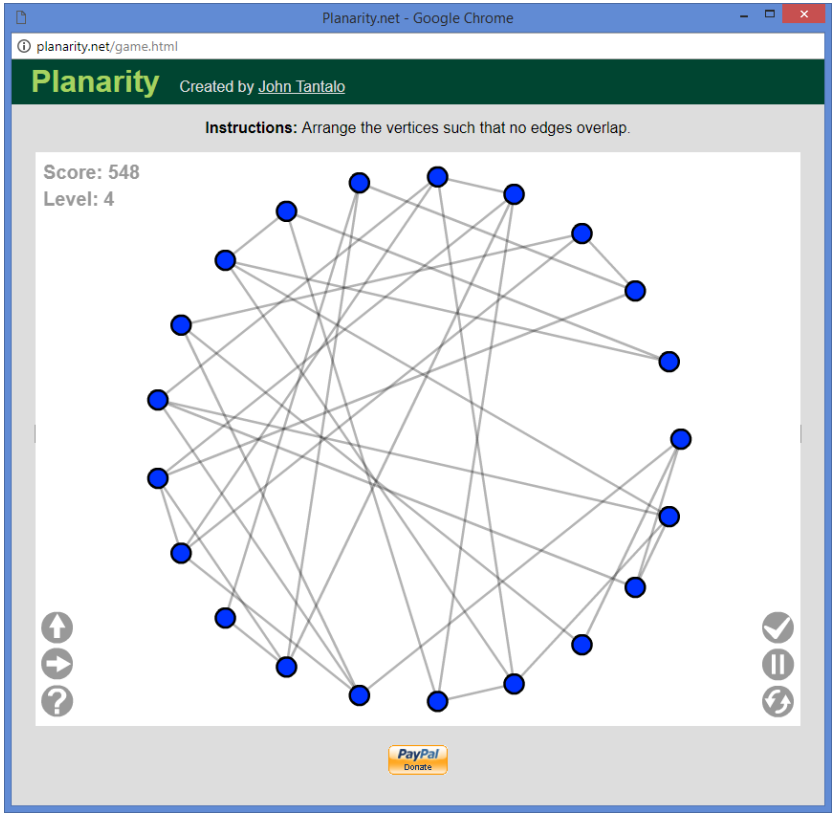
Photo: U. Montan
Alvin E. Roth
Prize share: 1/2

Photo: U. Montan
Lloyd S. Shapley
Prize share: 1/2

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2012 was awarded jointly to Alvin E. Roth and Lloyd S. Shapley *"for the theory of stable allocations and the practice of market design"*

Planarity

Can you untangle this graph so that no edges cross?



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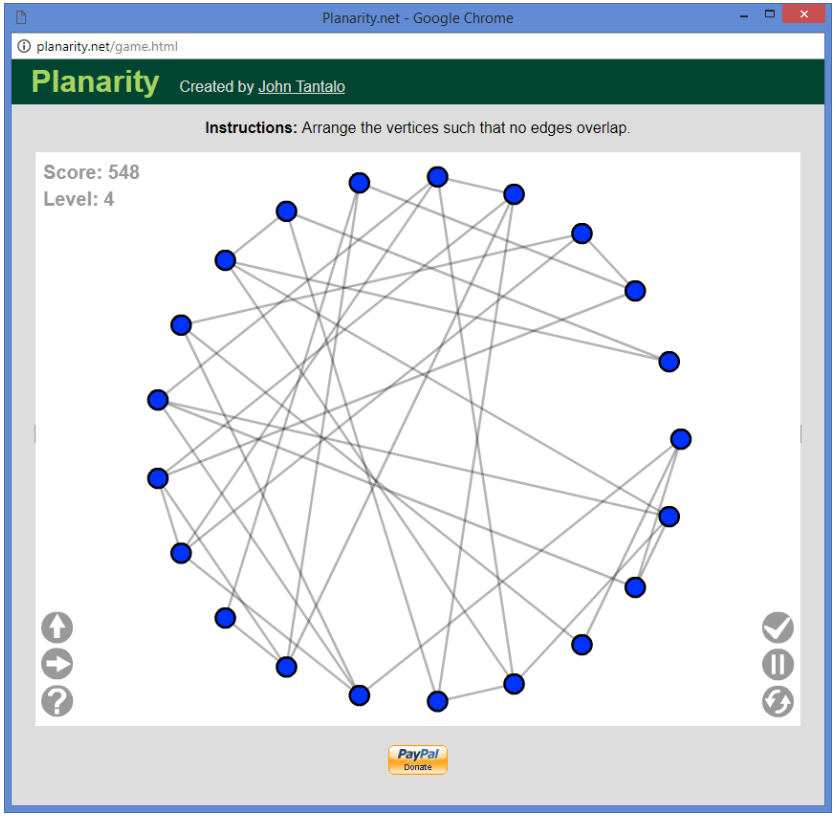

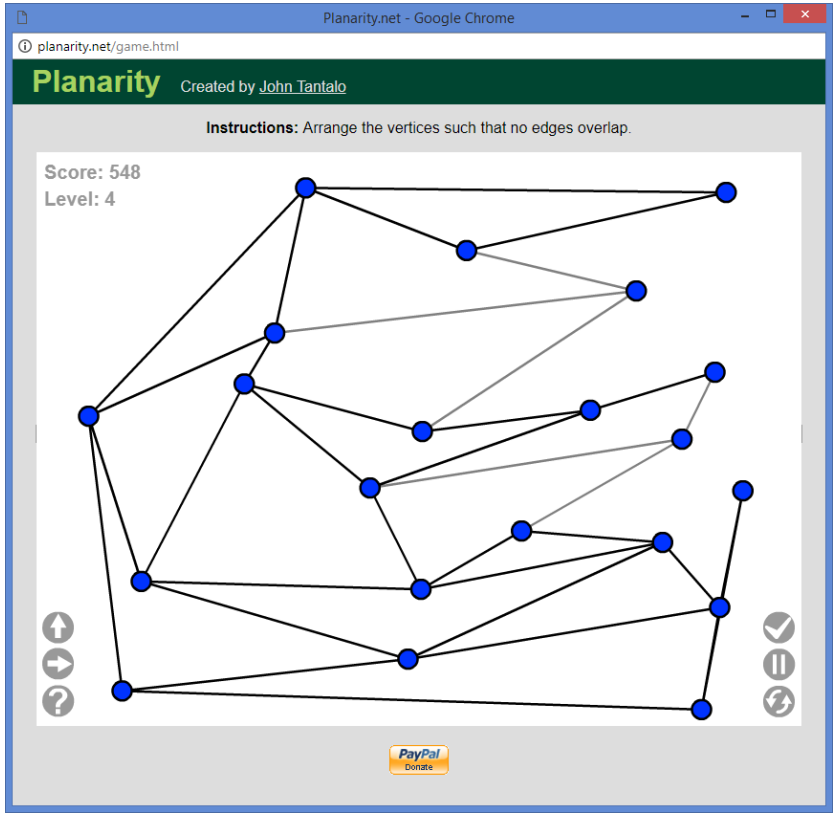

Planarity

Some graphs can't be untangled.

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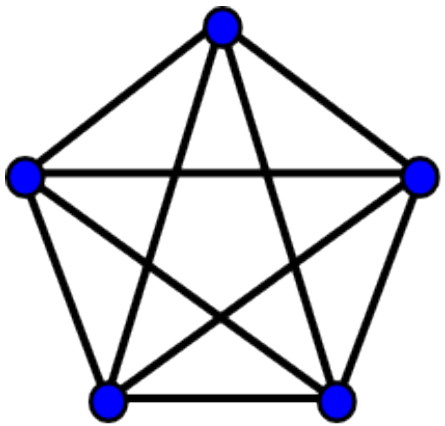

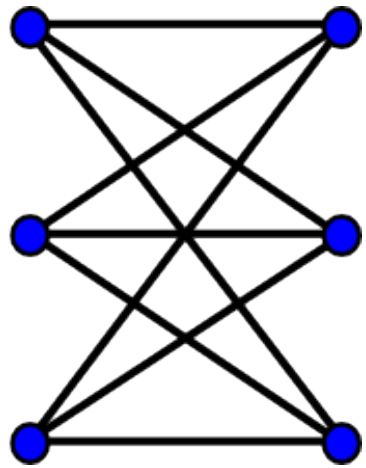

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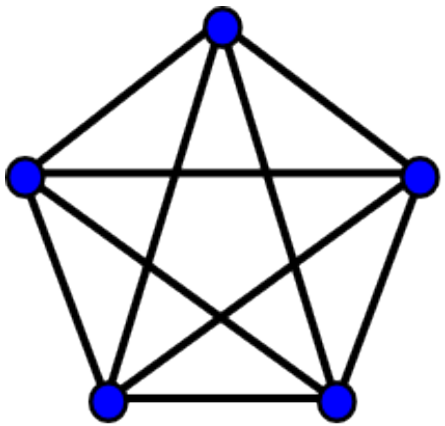
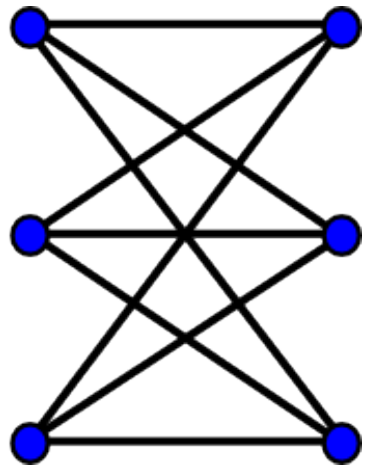


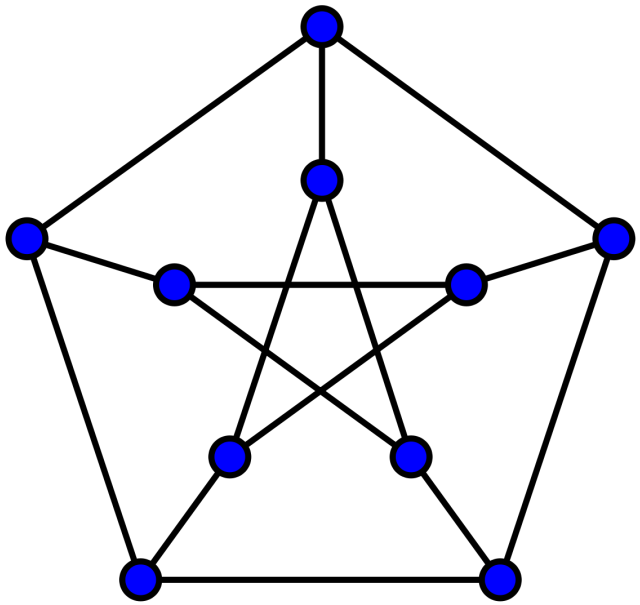


How can you check if such a drawing exists?

Generalizations of Planarity

Parameters of graphs where planarity is the simplest case:

- Crossing number



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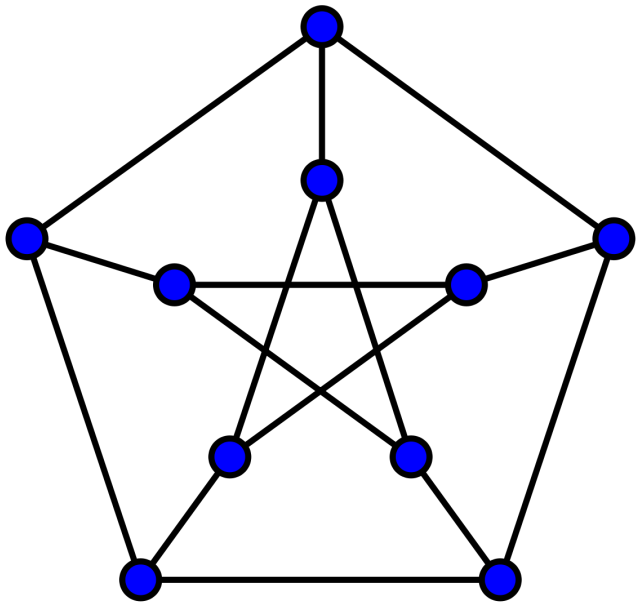
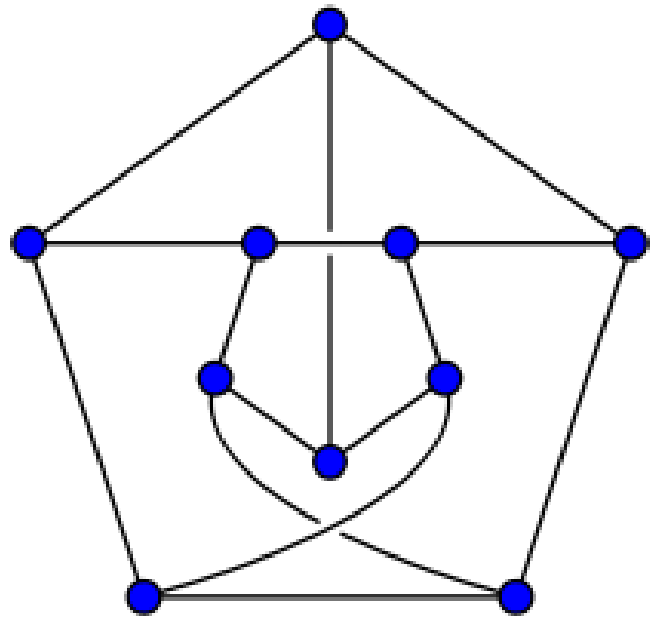
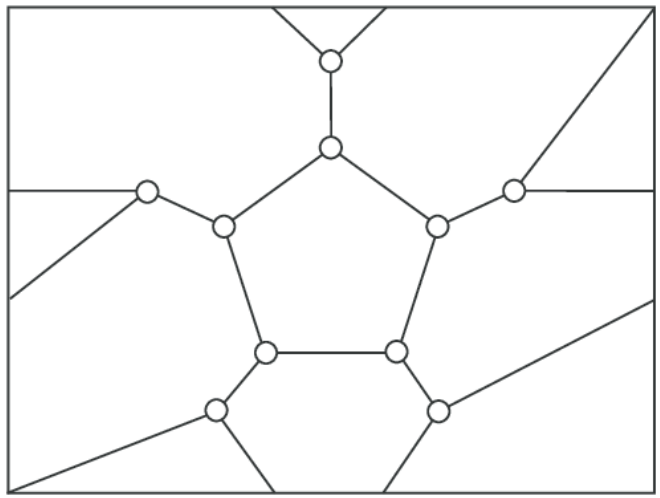




Generalizations of Planarity

- Genus



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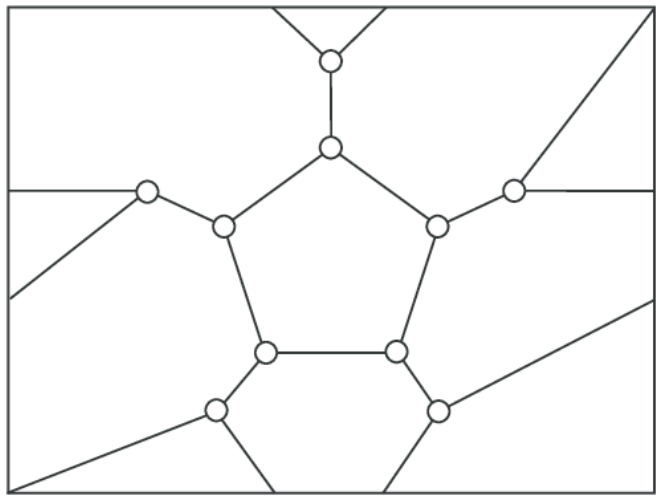
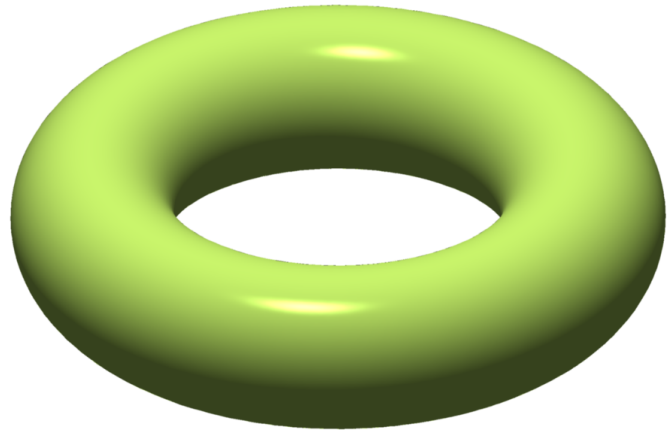
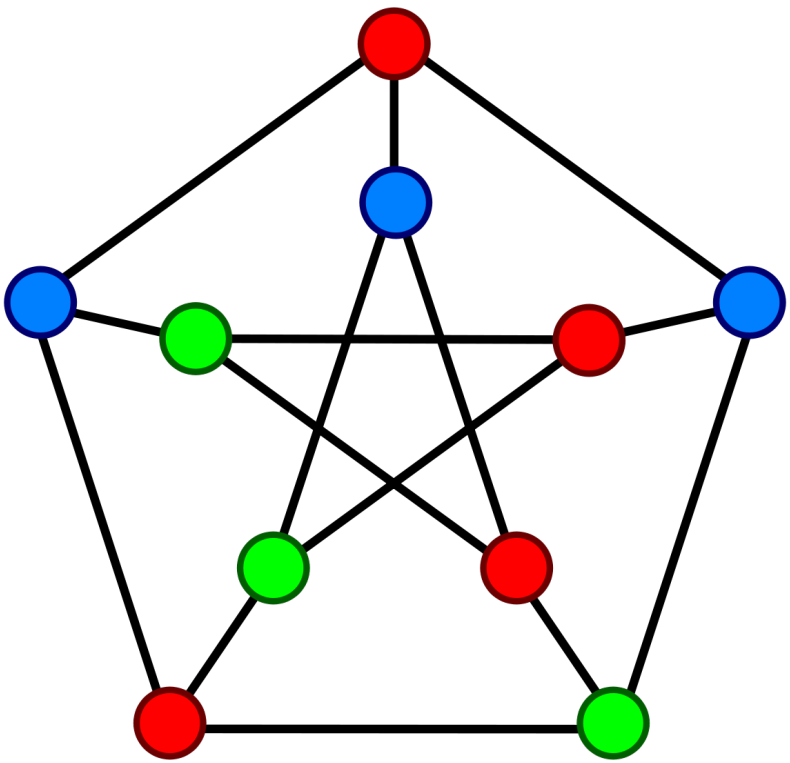
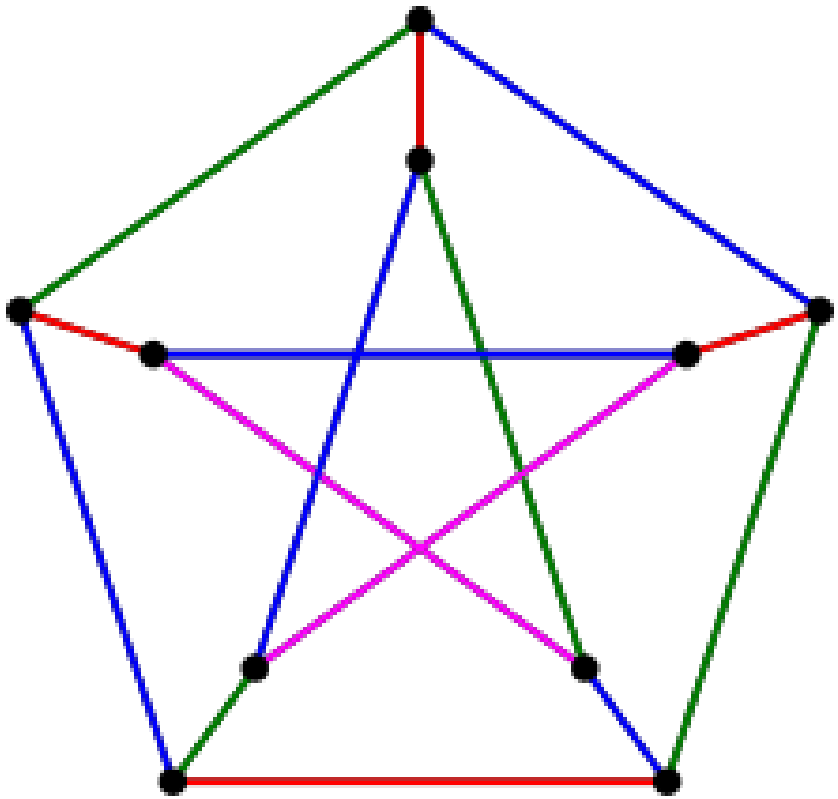




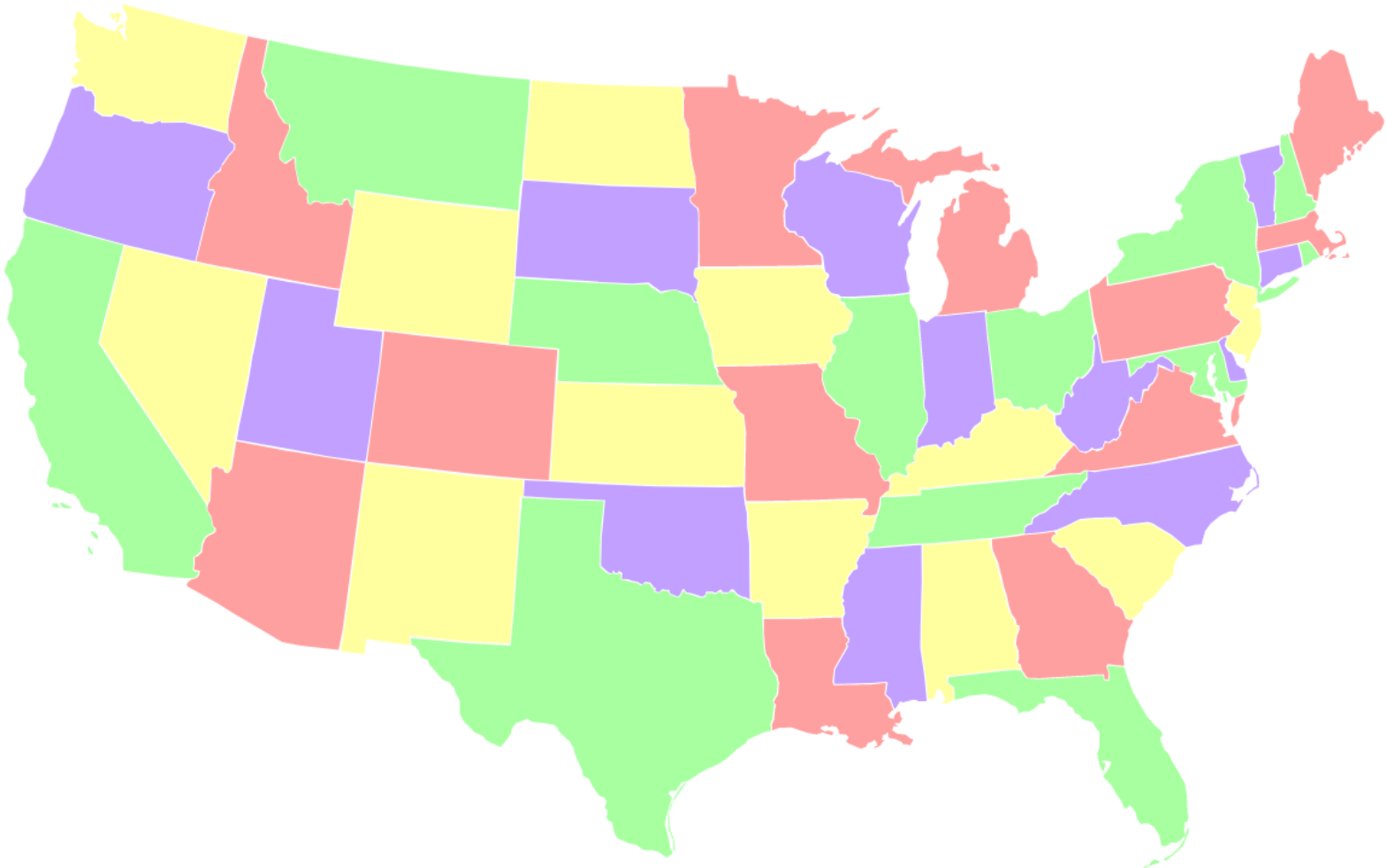
Coloring

Color vertices/edges such that "neighboring" elements are given different colors.





Four Color Theorem

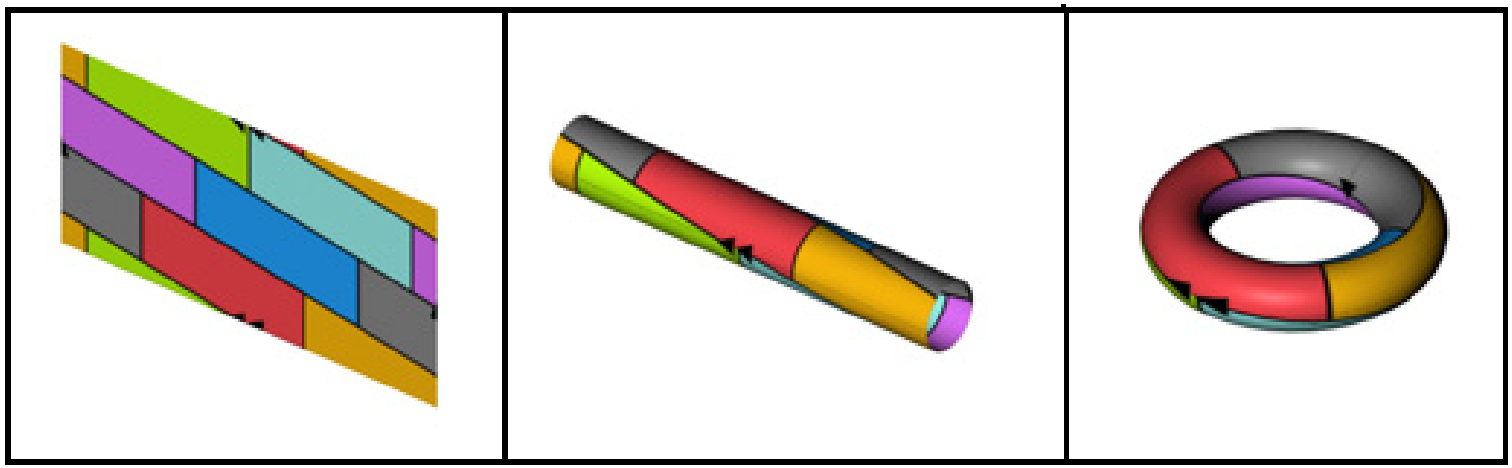


Four Color Theorem (cont'd)

"This leaves the reader to face 50 pages containing text and diagrams, 85 pages filled with almost 2500 additional diagrams, and 400 microfiche pages that contain further diagrams and thousands of individual verifications of claims made in the 24 lemmas in the main sections of text. In addition, the reader is told that certain facts have been verified with the use of about twelve hundred hours of computer time and would be extremely time-consuming to verify by hand. The papers are somewhat intimidating due to their style and length and few mathematicians have read them in any detail."

Map Color Theorem

"Generalization" of Four Color Theorem to higher-genus surfaces



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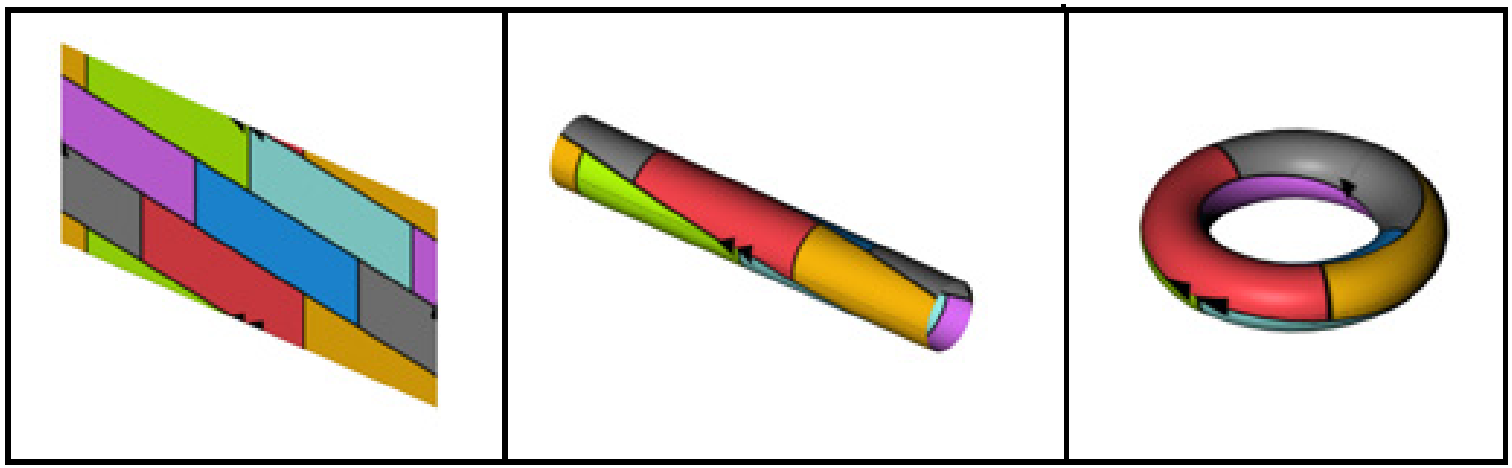

Every map on the torus can be colored with 7 colors.

Extremal Graph Theory/Ramsey Theory

Global properties $\Rightarrow$ local substructure.

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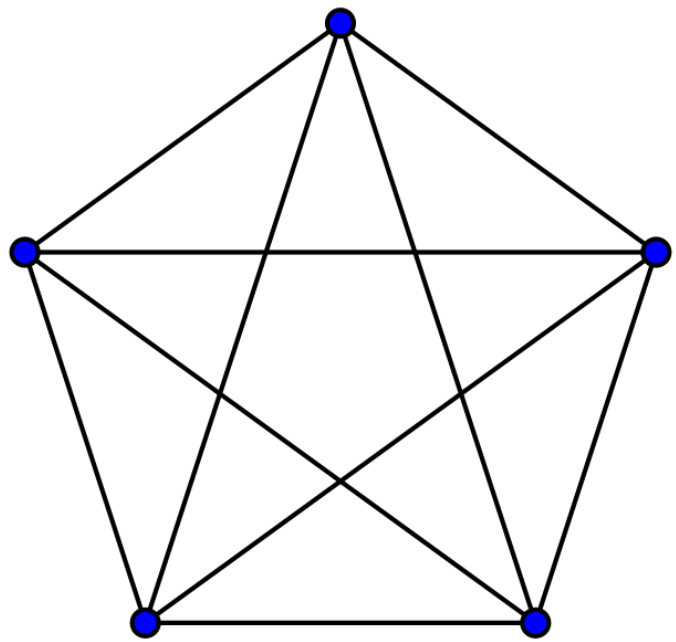
Global properties $\Rightarrow$ local substructure.

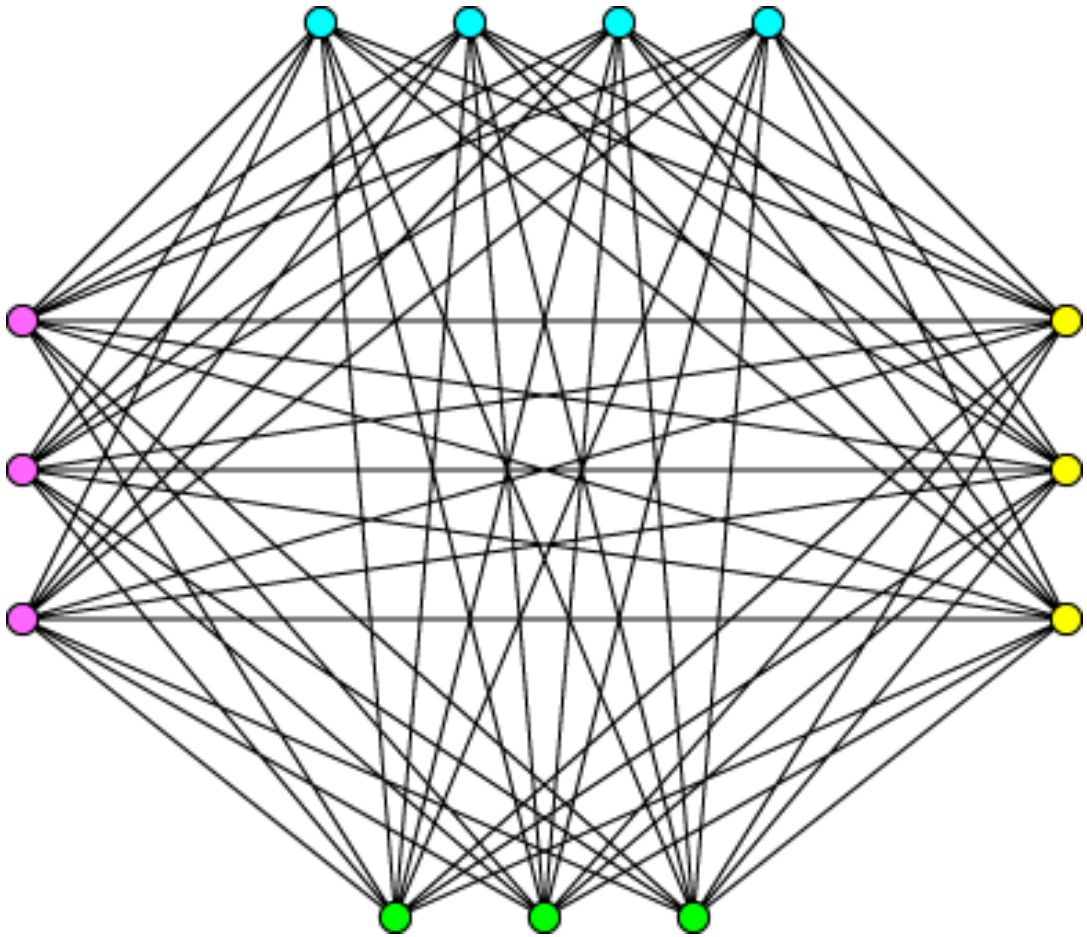
E.g., how many edges do we need to guarantee



as a subgraph?

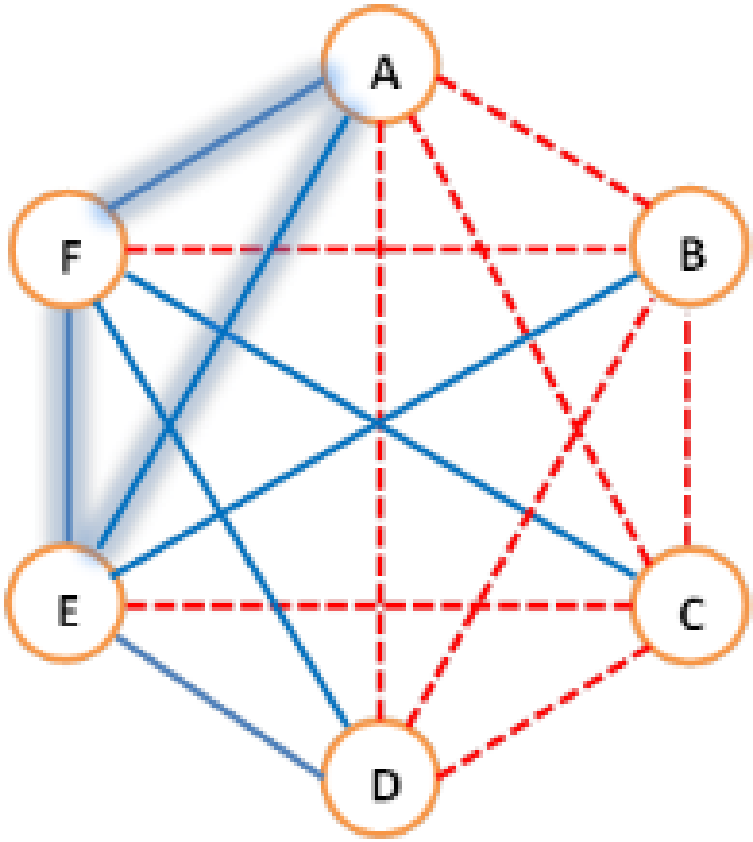
Extremal Graph Theory/Ramsey Theory

For 13 vertices, this is the most edges without that subgraph.



Theorem on friends and strangers

In a group of six people, three people are either all friends or all strangers.



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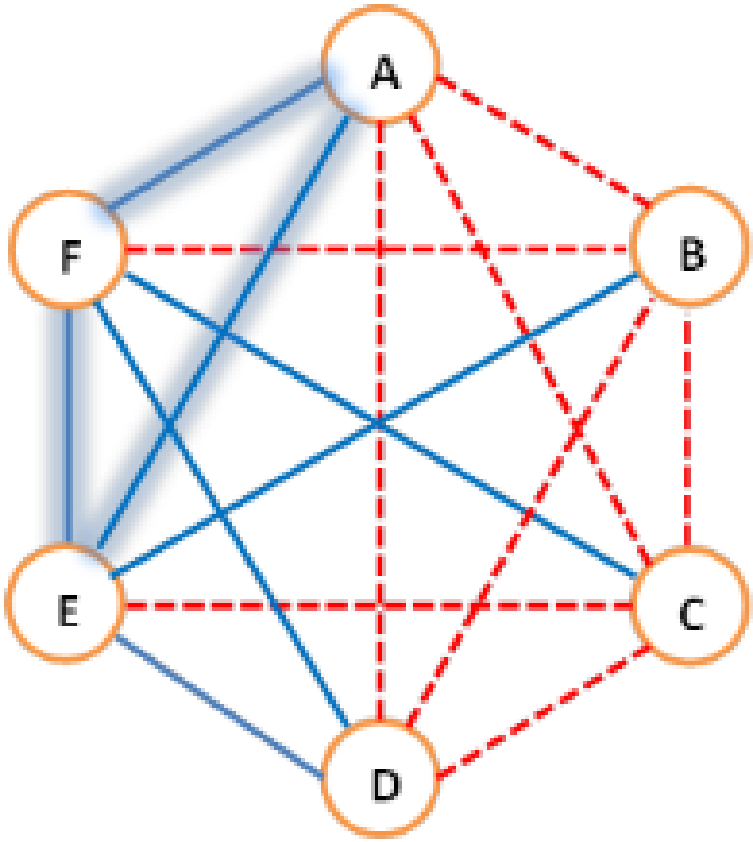

Lower bound the starting point of *random graphs* and the *probabilistic method*.

Questions?