COMS W4203: Graph Theory - Midterm Review

Timothy Sun

Columbia University

Basic Notions

- Simple graphs, multigraphs, directed graphs
- Handshaking lemma
- Neighbors, degree (min, max, average)
- Paths (radius, diameter), cycles (circumference, girth)

Eulerian and Hamiltonian graphs

- Eulerian = even degree
- Sufficient conditions for Hamiltonicity (Dirac, Chvatal)
  - Degree sequences (Havel-Hakimi)

Problem

Find a Hamiltonian graph with degree sequence
$2, 2, 2, 2, 3, 3, 3, 3, 3, 3$.

Matchings

- Bipartite graphs (alternating/augmenting paths)
  - Konig's theorem
  - Hall's marriage theorem
- Tutte's theorem (Petersen's theorem)

Problem

Construct a cubic graph with no perfect matching. Is your graph bipartite?

Connectivity

- Menger's theorem(!)
- Ear decompositions, Robbins's theorem
- 3-connected graphs (Tutte's wheel theorem)
- tree packing, covering (Nash-Williams)

Problem

What is the connectivity of the octahedral graph $K_{2n} - 2K_n$*?*

Planarity

- Jordan Curve Theorem, Euler's formula
- Kuratowski, Mac Lane (linear algebra), Whitney (duality), Hanani-Tutte (crossings)

Problem

Characterize the Mobius ladders ML_n, $n \geq 2$ that are planar.

Problem

Show that a Venn diagram with 4 **circles** *is impossible.*

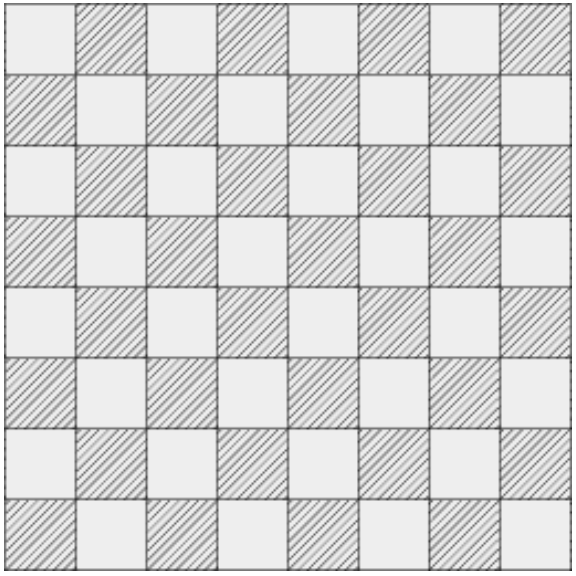
Coloring

- Cliques, independent sets
- 6, 5 colors suffice for planar graphs
- Greedy colorings, Brooks's theorem

Problem

What is the most number of nonattacking bishops that can be placed on a chessboard?

Coloring



Coloring