Due at 2:40pm Thursday, April 26, 2018.

- 1. Show that every bipartite graph on n vertices is  $(\lceil \log_2 n \rceil + 1)$ -choosable. (Hint: apply randomization on the *colors*, not vertices or edges.)
- 2. Find an embedding of the following graph in the torus:



3. Without using Kuratowski's theorem, show that the following graph is nonplanar (Hint: count the number of short cycles):



4. Recall that  $K_n - C_n$  is the graph formed by taking the complete graph  $K_n$  and deleting a Hamiltonian cycle. Using the Map Color Theorem, show that for sufficiently large n, the genus of  $K_n - C_n$  is strictly increasing.

Let  $O_n$  denote the "generalized" octahedral graph<sup>1</sup> formed by taking  $K_n$  and deleting a maximum matching of  $\lfloor n/2 \rfloor$  edges. Using the Map Color Theorem [i.e., the genus of the complete graphs] show that for sufficiently large n, the genus of  $O_n$  is **strictly** increasing.

<sup>&</sup>lt;sup>1</sup>Previously, we've seen it defined for even n.