Due at 2:40pm Thursday, April 26, 2018.

1. Show that every bipartite graph on $n$ vertices is $(\lceil \log_2 n \rceil + 1)$-choosable. (Hint: apply randomization on the colors, not vertices or edges.)

2. Find an embedding of the following graph in the torus:

![Graph](image)

3. Without using Kuratowski’s theorem, show that the following graph is nonplanar (Hint: count the number of short cycles):

![Graph](image)

4. Recall that $K_n - C_n$ is the graph formed by taking the complete graph $K_n$ and deleting a Hamiltonian cycle. Using the Map Color Theorem, show that for sufficiently large $n$, the genus of $K_n - C_n$ is strictly increasing.

Let $O_n$ denote the “generalized” octahedral graph\(^1\) formed by taking $K_n$ and deleting a maximum matching of $\lfloor n/2 \rfloor$ edges. Using the Map Color Theorem [i.e., the genus of the complete graphs] show that for sufficiently large $n$, the genus of $O_n$ is strictly increasing.

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\(^1\)Previously, we’ve seen it defined for even $n$. 