Due at 2:40pm Tuesday, April 17, 2018.

1. (a) Give a direct proof that a graph has a 2-flow if and only if every vertex has even degree, i.e., do not use any results on $\mathbb{Z}_2$-flows.
   (b) Show that every graph with a Hamiltonian cycle has a 4-flow.[1]

2. (a) Determine the flow number of $C_5 * K_1$, the wheel with 5 spokes.[2]
   (b) Determine the flow number of the Petersen graph.

3. (a) Show that every graph on n vertices and $(k-1)n+1$ edges, where $n > k$, contains every tree on k edges.
   (b) Show that the Erdős-Sós conjecture [see Diestel, p.179] is best possible in the sense that, for every k and infinitely many n, there is a graph on n vertices and with $\frac{1}{2}(k-1)n$ edges that contains no tree with k edges.[3]

4. Let $m, n \in \mathbb{N}$, and assume that $m-1$ divides $n-1$. Show that every tree T of order m satisfies $R(T, K_{1,n}) = m + n - 1$.[4]

5. Show that, for constant $p \in (0, 1)$, almost every graph in $\mathcal{G}(n, p)$ has diameter 2.[5]

[1] Diestel, §6#16
[2] Diestel, §6#18
[3] Diestel, §7#15
[4] Diestel, §9#13
[5] Diestel, §11#7