

Due at 2:40pm Tuesday, April 3, 2018.

1. Show that adding a new edge to a maximal planar graph of order at least 6 always produces both a TK_5 and a $TK_{3,3}$ subgraph.¹
2. Show that a 2-connected plane graph is bipartite if and only if every face is bounded by an even cycle.²
3. A k -chromatic graph G is called *critical* if $\chi(G - v) < k$ for every vertex $v \in G$. Determine the critical 3-chromatic graphs.³
4. For every integer k , find a 2-chromatic graph whose choice number is at least k .⁴
5. A family of (not necessarily distinct) subgraphs of a graph G is called a *double cover* of G if every edge of G lies on exactly two of these subgraphs. The *cycle double cover conjecture* asserts that every bridgeless multigraph admits a double cover by cycles. Prove the conjecture for graphs with a 4-flow.⁵
6. Prove that a plane triangulation is 3-colorable if and only if all its vertices have even degree.⁶

¹Diestel, §4#21

²Diestel, §4#24

³Diestel, §5#8

⁴Diestel, §5#28

⁵Diestel, §6#17

⁶Diestel, §6#21