Due at 2:40pm Thursday, March 8, 2018.

1. Show that every planar graph is a union of three forests.[1]

2. Let F, F' be forests on the same set of vertices, with $|E(F)| < |E(F')|$. Show that F' has an edge $e \notin E(F)$ such that $F + e$ is again a forest.[2]

3. (a) A *fullerene* is a molecule that is made up entirely of carbon atoms forming a cubic plane graph all whose faces are pentagons or hexagons. Show that, since carbon atoms can form double bonds, every such graph can be realized in principle by (4-valent) carbon atoms. [i.e. starting with such a plane graph, show that you can double some edges to make a multigraph that is 4-regular][3]

   (b) A football is made of pentagons and hexagons, not necessarily of regular shape. They are sewn together so that their seams form a cubic planar graph. How many pentagons does the football have?[4]

   (c) Fullerenes are less stable if they contain adjacent pentagons. Show that stable fullerenes have at least 60 carbon atoms.[5]

4. For every $n > 1$, find a bipartite graph on $2n$ vertices, ordered in such a way that the greedy algorithm uses n rather than 2 colors.[6]

5. Consider the following approach to vertex coloring. First, find a maximal independent set of vertices and color these with color 1; then find a maximal independent set of vertices in the remaining graph and color those 2, and so on. Compare this algorithm with the greedy algorithm: which is better?[7]

[1] Diestel, §4#4
[2] Diestel, §1#22
[3] Diestel, §4#6
[4] Diestel, §4#7
[5] Diestel, §4#8
[6] Diestel, §5#5
[7] Diestel, §5#6