Due at 2:40pm Thursday, February 22, 2018.

1. Let G be an r-regular graph on $2r$ vertices. Show that G has a perfect matching.

2. (a) Let e be an edge in a 2-connected graph $G \neq K_3$. Show that either $G - e$ or G/e is again 2-connected.

   (b) Does every 2-connected graph $G \neq K_3$ have an edge e such that G/e is still 2-connected?[1]

3. Derive the marriage theorem from König's theorem.[2]

4. Show that a graph G contains k independent edges if and only if $q(G-S) \leq S+|G|-2k$ for all sets $S \subseteq V(G)$.[3] [Hint: add $|G| - 2k$ new vertices and connect them to every other vertex, including to one another. What can you say about matchings in this new graph?]

5. Let G be a graph, and $H := L(G)$ its line graph.[4]

   (a) Show that H is Hamiltonian if G has a spanning Eulerian subgraph.

   (b) Deduce that H is Hamiltonian if G is 4-edge-connected. [Hint: make all the vertices have even degree by deleting edges, but be careful not to disconnect the graph.]

[1] Diestel, §3, #9
[2] Diestel, §2, # 5
[3] Diestel §2, # 20
[4] Diestel, §10, # 5