Due at 2:40pm Thursday, February 22, 2018.

1. Let $G$ be an $r$-regular graph on $2r$ vertices. Show that $G$ has a perfect matching.

2. (a) Let $e$ be an edge in a 2-connected graph $G \neq K_3$. Show that either $G - e$ or $G/e$ is again 2-connected.
   
   (b) Does every 2-connected graph $G \neq K_3$ have an edge $e$ such that $G/e$ is still 2-connected?\(^1\)

3. Derive the marriage theorem from König’s theorem.\(^2\)

4. Show that a graph $G$ contains $k$ independent edges if and only if $q(G - S) \leq S + |G| - 2k$ for all sets $S \subseteq V(G)$.\(^3\) [Hint: add $|G| - 2k$ new vertices and connect them to every other vertex, including to one another. What can you say about matchings in this new graph?]

5. Let $G$ be a graph, and $H := L(G)$ its line graph.\(^4\)
   
   (a) Show that $H$ is Hamiltonian if $G$ has a spanning Eulerian subgraph.
   
   (b) Deduce that $H$ is Hamiltonian if $G$ is 4-edge-connected. [Hint: make all the vertices have even degree by deleting edges, but be careful not to disconnect the graph.]

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\(^1\)Diestel, §3, #9
\(^2\)Diestel, §2, # 5
\(^3\)Diestel §2, # 20
\(^4\)Diestel, §10, # 5