Due at 2:40pm Thursday, February 8, 2018.

1. Prove that if a tree has an even number of edges, then it has at least one vertex of even degree.

2. How many 2-regular simple graphs (up to isomorphism) are there with 10 vertices?

3. Show that any two longest paths in a connected graph must have at least one vertex in common.

4. Let $d \in \mathbb{N}$ and $V := \{0, 1\}^d$; thus, V is the set of all $0 - 1$ sequences of length d. The graph on V in which two such sequences form an edge if and only if they differ in exactly one position is called the *d-dimensional cube*. Determine the average degree, number of edges, diameter, girth, and circumference of this graph. (Hint for the circumference: induction on d.)[1]

   [This graph is usually given the notation Q_d.]

5. Show that $\mathrm{rad}(G) \le \mathrm{diam}(G) \le 2\,\mathrm{rad}(G)$.[2] Give an example of a graph where these inequalities are all strict (i.e., "$<$" instead of "$\le$").

6. Show that every automorphism [an isomorphism from a graph to itself] of a tree fixes a vertex or an edge.[3]

7. An oriented complete graph [i.e., giving a direction to each edge of the complete graph, forming a directed graph] is called a *tournament*. Show that every tournament contains a (directed) Hamilton path.[4]

8. Find a connected graph G whose square G^2 has no Hamilton cycle.[5]

[1] Diestel §1, #2
[2] Diestel §1, #6
[3] Diestel §1, #28
[4] Diestel §10, #1
[5] Diestel §10, #13