1. Prove that if a tree has an even number of edges, then it has at least one vertex of even degree.

2. How many 2-regular simple graphs (up to isomorphism) are there with 10 vertices?

3. Show that any two longest paths in a connected graph must have at least one vertex in common.

4. Let \( d \in \mathbb{N} \) and \( V := \{0, 1\}^d \); thus, \( V \) is the set of all \( 0-1 \) sequences of length \( d \). The graph on \( V \) in which two such sequences form an edge if and only if they differ in exactly one position is called the \( d \)-dimensional cube. Determine the average degree, number of edges, diameter, girth, and circumference of this graph. (Hint for the circumference: induction on \( d \).)

[This graph is usually given the notation \( Q_d \).]

5. Show that \( \text{rad}(G) \leq \text{diam}(G) \leq 2 \text{rad}(G) \). Give an example of a graph where these inequalities are all strict (i.e., “<” instead of “≤”).

6. Show that every automorphism [an isomorphism from a graph to itself] of a tree fixes a vertex or an edge.

7. An oriented complete graph [i.e., giving a direction to each edge of the complete graph, forming a directed graph] is called a tournament. Show that every tournament contains a (directed) Hamilton path.

8. Find a connected graph \( G \) whose square \( G^2 \) has no Hamilton cycle.

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1Diestel §1, #2
2Diestel §1, #6
3Diestel §1, #28
4Diestel §10, #1
5Diestel §10, #13