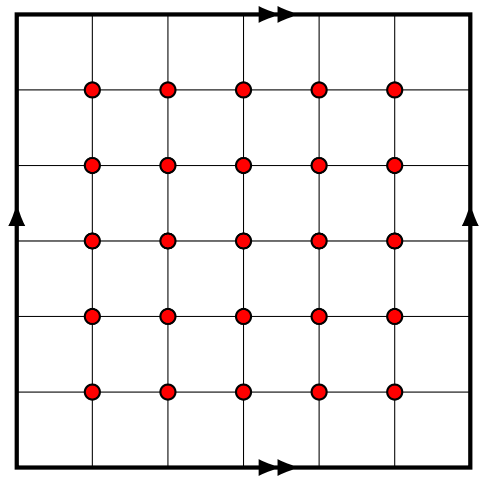
1. Let G be any simple graph on 5 vertices and 7 edges. Show that the diameter of G is 2.

2. The graph $C_5 \times C_5$ has 25 vertices and is shown here embedded in the torus:



(a) Compute the chromatic number of $C_5 \times C_5$.

(b) Compute the independence number of $C_5 \times C_5$.

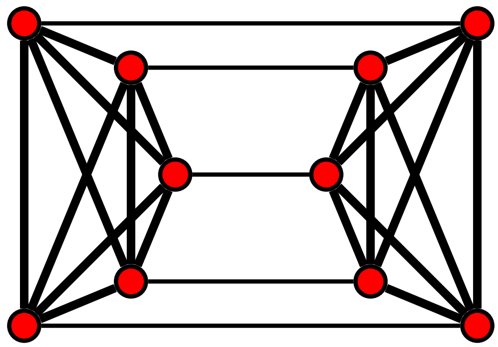
3. Let $n = R(3,4)$ be the smallest integer such that for every graph on n vertices, either it has a K_3 subgraph or its edge-complement has a K_4 subgraph.

   (a) Give an 8-vertex graph that has neither a K_3 or $\overline{K_4}$ subgraph, showing that $R(3,4) > 8$. (Hint: consider a Hamiltonian graph)

   (b) Prove that there is no 9-vertex cubic graph.

   (c) Show that $R(3,4) \leq 9$. (Hint: sharpen the proof of Ramsey's theorem using part (b))

4. Let H be the "5-prism" shown below, formed by taking two copies of K_5 and adding five independent edges between them:



(a) What is the minimum genus of G? (Hint: find a toroidal embedding of K_5 with a face incident with every vertex)

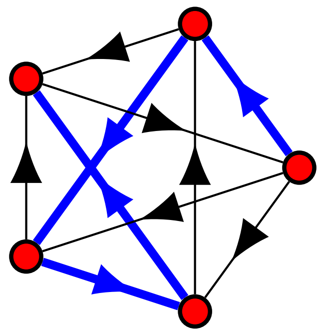
(b) What is the maximum genus of G?

5. Without using Seymour's 6-flow theorem, prove the following:

   (a) Suppose G has 3 spanning trees such that no edge appears in all 3 trees. Show that G has an 8-flow. (Hint: express G as the union of 3 even graphs, as we did for the 4-edge-connected case)

   (b) By doubling every edge (i.e. replacing each edge with a pair of parallel edges), show that every 3-edge-connected graph has an 8-flow.

6. Recall that a *tournament* on n vertices is the complete graph K_n with directions on each of the edges (i.e. imagine a round-robin tournament). A *directed* path is a path where the edges of that path are pointing in the same direction. An example of this is shown below, where the thick path in blue is a directed Hamiltonian path.



Show that for each n, there exists a tournament with $n!/2^{n-1}$ directed Hamiltonian paths.