Let $\text{Subset-Sum} = \{\langle x_1, \ldots, x_n, t \rangle \mid \exists \text{ a non-empty subset } S \subseteq \{1, \ldots, n\} \text{ s.t. } \sum_{i \in S} x_i = t\}$. You may use the fact that this problem is NP-complete (we will define what this means in class on Monday).

1. Let $\text{HALF-Sum} = \{\langle x_1, \ldots, x_n, t \rangle \mid \exists \text{ a subset } S \subseteq \{1, \ldots, n\} \text{ of size } |S| = \lceil n/2 \rceil, \text{ such that } \sum_{i \in S} x_i = t\}$.
   Prove that $\text{HALF-Sum}$ is NP-complete.

2. Suppose that you are given an algorithm $M$ that decides $\text{Subset-Sum}$ in polynomial time. Show that in this case there is a polynomial time algorithm that, on input $\langle x_1, \ldots, x_n, t \rangle$, outputs a non-empty subset $S \subseteq \{1, \ldots, n\}$ such that $\sum_{i \in S} x_i = t$, or if no such subset exists, outputs “no solution”.
   In other words, you are proving that if there is polynomial time algorithm that decides whether or not there exists a subset, then there is a polynomial time algorithm that actually finds a subset if one exists.

3. Let $a \in \Sigma$ be an arbitrary fixed symbol. Recall that for a language $L \subseteq \Sigma^*$, we define $\text{drop}_a(L) = \{w \in \Sigma^* \mid aw \in L\}$.
   Prove that NP is closed under the $\text{drop}_a$ operation.