1. (50 points) For each of the following languages, either use Rice’s theorem to prove that the language is not decidable, or argue that Rice’s theorem cannot be used, and find another way to determine (and prove) whether or not the language is decidable.

   (a) \( L_1 = \{ \langle M \rangle \mid M \text{ is a TM and } L(M) \text{ is uncountable} \} \).

   (b) \( L_2 = \{ \langle M \rangle \mid M \text{ is a TM and every string that } M \text{ accepts starts with 0} \} \).

   (c) \( L_3 = \{ \langle M, w \rangle \mid M \text{ is a TM and } M \text{ accepts the string } \epsilon \text{ or the string } w \text{ (or both)} \} \).

2. (25 points) Exactly one of the above three languages is not recognizable. Identify which one it is, and prove that it is not recongizable by showing a mapping reduction from a non-recongizable language.

3. (25 points) Let \( L_4 = \{ \langle M \rangle \mid M \text{ is a TM and } M \text{ accepts at most two different strings} \} \). Prove that \( L_4 \) is not recognizable.

4. Extra Credit: Formulate and prove a version of Rice’s theorem that works for properties of pairs of recognizable languages, showing that a language of the form \( \{ \langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are TMs and } \ldots \} \) is not decidable (fill in the “...” and the rest of the theorem appropriately, and prove it).

   For example, your theorem should apply to solve problem 3 from the last homework.