1. Let \( A = \{10^i1 \mid i \geq 0\} = \{11, 101, 1001, 10001, \ldots\} \)
   
   (a) Prove that \( A \) has uncountably many subsets.
   (b) Conclude that there exists a language \( L \subset A \) that is not recognizable.

2. Consider the following transformation that, given a string \( \langle M, w \rangle \) where \( M \) is a TM, outputs the string \( \langle M' \rangle \) where \( M' \) is the following TM:

   \( M' \), on an input string \( x \):
   
   • If \( x = 1^i \) for some \( i \geq 0 \), accept.
   • Else, run \( M \) on \( w \) and output the same.

   (a) If \( M \) accepts \( w \), what is \( L(M') \)?
   (b) If \( M \) does not accept \( w \), what is \( L(M') \)?
   (c) Let \( L_1 = \{\langle M \rangle \mid M \) is a TM and \( L(M) = L(1^*)\} \). Prove that \( L_1 \) is not decidable.

3. Let \( L_2 = \{\langle M_1, M_2 \rangle \mid M_1, M_2 \) are TMs and \( L(M_1) \cap L(M_2) \neq \emptyset\} \).

   Prove that \( L_2 \) is not decidable.

4. Prove that for every recognizable language \( L \), it holds that \( L \preceq_T A_{TM} \)

5. **Extra Credit:** Prove that there is no language \( B \) such that \( A \preceq_T B \) for all \( A \).