1. Give regular expressions generating each of the following languages
   (a) \( \{ w \in \{0,1\}^* \mid w \text{ does not contain the substring 10} \} \)
   (b) \( \{ w \in \{0,1\}^* \mid w \text{ does not contain the substring 11} \} \)
   (c) \( \{ w \in \{0,1\}^* \mid w \text{ has both 11 and 101 as substrings} \} \)

2. Consider the language \( L_1 = \{ w \in \{0,1\}^* \mid w \text{ does not end in 10} \} \).
   (a) Come up with a regular expression generating \( L_1 \), and provide a short informal justification for why it works.
   (b) Construct a DFA/NFA for \( L_1 \), then apply the construction we saw in class to convert it to a regular expression. Show your work.

3. Prove that the following languages are not regular
   (a) \( L_2 = \{ x+y=z \mid x, y, z \text{ are binary representations of integers, and } z \text{ is the sum of } x \text{ and } y \} \) over the alphabet \( \Sigma = \{0, 1, +, =\} \)
      (for example, the strings \( 1+1=10 \) and \( 101+10=111 \) are in the language, while the strings \( 1+10=100 \) and \( ++1 \) are not in the language).
   (b) Define the following language over the alphabet \( \{a, b, c, d\} \):
       \[ L_3 = \{ cw \mid w \in \{a, b\}^*, w = w^R \} \cup \{ dw \mid w \in \{a, b\}^*, |w| > 2 \} \]
       where \( w^R \) is the reverse of \( w \). Thus, \( L_3 \) consists of all strings starting with the letter \( c \) followed by a palindrome over \( \{a, b\} \), or starting with the letter \( d \) followed by a string of length at least 3 over \( \{a, b\} \). For example, the strings \( cb, cabba, dabba, dbbbba \) are all in \( L_3 \), and the strings \( abba, db, dddddd \) are all not in \( L_3 \).

4. For the alphabet \( \{a, b, c\} \) define \( L_4 = \{ c^n w \mid n \geq 0, w \in \{a, b\}^*, \text{ and if } n \text{ is odd then } w \text{ has the same number of } a's \text{ and } b's \} \).
   (a) prove that \( L_4 \) is not regular
   (b) prove that \( L_4 \) satisfies the pumping lemma.

5. **Extra Credit:** For a language \( L \subseteq \Sigma^* \) define \( f(L) = \{ w \in \Sigma^* \mid w w^R \in L \} \)
   Is the class of regular languages closed under \( f \)? Prove your answer.