Homework 2

Due: Wednesday, 2/7/18 at 11:59pm

1. Let $L_1 \subseteq \{a, b\}^*$ be defined by $L_1 = \{xy \mid \text{there are twice as many } a's \text{ in } x \text{ as in } y\}$. Show that $L_1$ is regular by giving a state diagram of either a DFA or an NFA that recognizes it.

2. Define $L_2 = \{w \in \{0, 1, \ldots, 9\}^* \mid w \neq \epsilon \text{ and the last (rightmost) digit in } w \text{ has not appeared in the string before}\}$. For example, the strings 3261, 5778, 0, are all in $L_2$, but the strings 101, 1989, 00 are all not in $L_2$. Show that $L_2$ is regular by giving a state diagram of either a DFA or an NFA that recognizes it.

3. Let $L_{\text{odd}} = \{w \in \{0, 1\}^* \mid w \text{ contains an odd number of } 1s\}$.
   (a) What is $L_{\text{odd}}^*$? (arrive at a direct description of this language that does not refer to $L_{\text{odd}}$)
   (b) Start with a DFA recognizing $L_{\text{odd}}$, and use the construction we saw in class to obtain an NFA recognizing $L_{\text{odd}}^*$ (Do not attempt to optimize the NFA or build your own DFA/NFA for the language – just apply the construction we saw in class when we proved regular languages are closed under the star operation).
   (c) For the NFA you constructed above, show the computation trees for the strings 101 and 00 (you may omit branches that die before they reach the end of the input). Determine whether each of these strings is in $L_{\text{odd}}^*$ or not.

4. Let $a \in \Sigma$ be an arbitrary fixed symbol. For a language $L \subseteq \Sigma^*$ define
   
   $$\text{drop}_a(L) = \{w \in \Sigma^* \mid aw \in L\}$$

   Prove that the class of regular languages is closed under the $\text{drop}_a$ operation.

5. Extra Credit: Read the Myhill-Nerode notes on the class webpage.
   Use the Myhill-Nerode theorem to find the number of states in the minimal DFA recognizing each of the following languages (both defined over $\Sigma = \{0, 1, \ldots, 9\}$).
   (a) $L_7 = \{x \in \Sigma^* \mid x \text{ is a decimal representation of a number divisible by 7}\}$.
   (b) $L_6 = \{x \in \Sigma^* \mid x \text{ is a decimal representation of a number divisible by 6}\}$.

   Note: This problem is not fully self-contained, even if you read the MN notes, because it requires some basic knowledge of number theory/ modular arithmetic.