1. For each of the following languages, give a state diagram of a DFA that recognizes the language. For the first language $L_1$, also give the formal description of the DFA you construct.

(a) $L_1 = \{w \in \{a, b\}^* | \text{no two consecutive characters in } w \text{ are the same}\}$

(b) $L_2 = \{w \in \{a, b, c\}^* | w = a^i b^j c^k, \text{ where } i \geq 0, j \geq 1, k \geq 1, \text{ and } i + j + k \text{ is odd}\}$

(Recall that $x^i$ stands for the concatenation of $x$ with itself $i$ times.)

2. (a) Construct a DFA recognizing the following language:

$L_3 = \{w \in \{0, 1, \ldots, 9\}^* | w \text{ is a decimal representation of a natural number divisible by 7}\}$

(b) How would you change your DFA if you wanted to check whether the input number, when divided by 7, gives remainder 1 (or any other specific remainder)?

**Extra Credit:** Consider the same problem, but where the input is given in reverse, written from least significant digit to most significant digit. Specifically, construct a DFA (with as few states as you can) recognizing the following language:

$L_4 = \{w \in \{0, 1, \ldots, 9\}^* | w \text{ is a reversed decimal representation of a natural number divisible by 7}\}$

(example strings in $L_4$ include 12, 94, and 7041).

3. Let $\Sigma = \{a, b, c, d, e\}$. Define $L_5 = \{w \in \Sigma^* | \text{there is at least one symbol } \in \Sigma \text{ that does not appear in } w\}$.

Show that $L_5$ is regular by giving a state diagram of either a DFA or an NFA that recognizes the language.

4. Consider the NFA $N = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_1, q_3\})$, where $\delta$ is given by the following table.

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_0$</td>
<td>${q_0, q_1, q_2}$</td>
<td>${q_0}$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$q_1$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$q_2$</td>
<td>${q_3}$</td>
<td>${q_3}$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$q_3$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
</tbody>
</table>

(a) Give the state diagram of this machine.
(b) For each of the strings $abab$, $abbb$, $aabb$, determine whether or not the string is in $L(N)$. If so, show a computation (i.e., a sequence of states) that accepts the string. If not, construct the computation tree of $N$ on the string.

(c) What is the language recognized by this NFA? (describe the language directly, not in terms of “all strings accepted by $N$”)

(d) Apply the subset construction to convert the NFA $N$ to a DFA for the same language. Do not attempt to optimize the resulting DFA, simply follow the subset construction. You may give only the portion of the DFA that is reachable from the start state.