

## COMS W3261: Theoretical Computer Science.

Instructor: Tal Malkin

### Problem Set 9

Due: Thur, 04/09/09.

Late Homework may be submitted by Tue 4/14/09 without penalty, because of Passover. However, be aware that another homework is likely to be due on Thur 4/16/09.

Reading: Chapter 4

Note: from now on, unless otherwise noted, you can use a high-level description for any TM (algorithm) you describe (as we do in class). You still need to argue the correctness of your solutions.

1. Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language, and show that it is decidable.
2. Let  $L = \{\langle N \rangle \mid N \text{ is an NFA such that all strings accepted by } N \text{ have } 000 \text{ as a substring}\}$ .

Prove that  $L$  is decidable.

3. We proved in class that every set of strings (language) is countable. On the other hand, we proved that the set of real numbers is not countable. But, one may give the following argument to claim that this cannot be the case:

*Every real number can be described as a sequence of decimal numbers followed by a . and then followed by another sequence of decimal numbers. Thus, the real numbers can be viewed as the set of all strings over the alphabet  $\{0, 1, \dots, 9, .\}$  containing . exactly once. This is a set of strings (a language), and thus must be countable, as proved in class.*

What is wrong with the above argument?

4. (a) Let  $A$  be some infinite countable set. Prove that  $\mathcal{P}(A) = \{B \mid B \subseteq A\}$  is not countable, using the diagonalization method. (Hint: note that a subset  $B \subseteq A$  can be defined by specifying for each element of  $A$  whether it is in the subset or not.)  
(b) Prove that for any finite alphabet  $\Sigma$ , the set of all languages over  $\Sigma$  is not countable.<sup>1</sup>
5. Extra credit: Sipser 4.17 (A language  $C$  is TM-recognizable if and only if there exists a decidable language  $D$  such that  $C = \{x \mid \exists y \text{ s.t. } (x, y) \in D\}$ .)

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<sup>1</sup>As mentioned in class, this is another way to prove that there exist languages that are not TM-recognizable, since we proved that the set of all TM-recognizable languages is countable.