COMS W3261: Theoretical Computer Science.
Instructor: Tal Malkin

Problem Set 8

Due: Thur, 04/02/09.
Reading: Chapter 3

1. (a) Prove that the class of Turing-recognizable languages is closed under the concatenation operation.

   (b) Prove that the class of Turing-decidable languages is closed under the complementation operation.

   (c) Does your proof of part (a) work to prove the same claim for the class of Turing-decidable languages? Does your proof of part (b) work to prove the same claim for the class of Turing-recognizable languages? Explain your answers.

2. Let $NE = \{\langle M \rangle \,|\, M \text{ is a TM that accepts some string}\}$ (namely all $\langle M \rangle$ that recognize a non-empty language).

   Prove that NE is Turing-recognizable (show an algorithm to recognize it, and argue the correctness of your algorithm).

3. Prove that a language L is decidable if and only if there is some enumerator that enumerates the language in lexicographic order.

4. Recall that a non-deterministic TM (NTM) is called a decider, if it always halts (all computations, on all inputs). In this problem you will show that having some halting computation on every input, does not necessarily mean the NTM is a decider.

   Assume L is some language that is TM-recognizable but not TM-decidable. Construct a non-deterministic TM N recognizing L, such that N has the following properties: (1) For any input string w, there exists at least one computation of N on w that is halting; (2) N is not a decider.