Problem Set 8

Due: Thur, 04/02/09.
Reading: Chapter 3

1. (a) Prove that the class of Turing-recognizable languages is closed under the concatenation operation.
   (b) Prove that the class of Turing-decidable languages is closed under the complementation operation.
   (c) Does your proof of part (a) work to prove the same claim for the class of Turing-decidable languages? Does your proof of part (b) work to prove the same claim for the class of Turing-recognizable languages? Explain your answers.

2. Let \( NE = \{\langle M \rangle | M \text{ is a TM that accepts some string} \} \) (namely all \( \langle M \rangle \) that recognize a non-empty language).
   Prove that \( NE \) is Turing-recognizable (show an algorithm to recognize it, and argue the correctness of your algorithm).

3. Prove that a language \( L \) is decidable if and only if there is some enumerator that enumerates the language in lexicographic order.

4. Recall that a non-deterministic TM (NTM) is called a decider, if it always halts (all computations, on all inputs). In this problem you will show that having some halting computation on every input, does not necessarily mean the NTM is a decider.
   Assume \( L \) is some language that is TM-recognizable but not TM-decidable. Construct a non-deterministic TM \( N \) recognizing \( L \), such that \( N \) has the following properties:
   (1) For any input string \( w \), there exists at least one computation of \( N \) on \( w \) that is halting; (2) \( N \) is not a decider.