COMS W3261: Theoretical Computer Science.
Instructor: Tal Malkin

Problem Set 11

Due: Thur, 04/21/09.

Reading: 4.2, 5.1, 5.3, 6.3

1. Let A be some decidable language, and let B some undecidable language. Your answers should apply to any such choices of A, B, but if it helps you to think of concrete examples, you may take $A = \{\langle M\rangle | M \text{ is a TM with at most 10 states}\}$ and $B = \text{Halt}_{TM} = \{\langle M, w\rangle : M \text{ halts on } w\}$. (If your answer applies only to these examples and not in general, you will get partial credit).

   (a) Prove that A is mapping reducible to B ($A \leq_m B$).

   (b) Prove that B is not mapping reducible to A ($B \not\leq_m A$).

2. (a) Prove that every language A is Turing-reducible to its complement $\overline{A}$.

   (b) Prove that if A is a recognizable language, and A is mapping-reducible to $\overline{A}$, then A is decidable.

3. Let ALL_{TM} be the language of Turing machines that always accept, namely:

$$\text{ALL}_{TM} = \{\langle M\rangle : M \text{ is a TM and } L(M) = \Sigma^*\}$$

   In this problem we show that ALL_{TM} is neither Turing-recognizable nor co-Turing recognizable. This is equivalent to saying that neither ALL_{TM} nor $\overline{\text{ALL}_{TM}}$ is recognizable.

   (a) Show that $\overline{\text{ALL}_{TM}}$ is not Turing-recognizable by showing $\overline{A_{TM}} \leq_m \overline{\text{ALL}_{TM}}$. (Hint: this might be easier to think about if you use the fact that for all languages A and B, we have $A \leq_m B$ iff $\overline{A} \leq_m \overline{B}$.)

   (b) **(Extra Credit)** Now show that ALL_{TM} is not Turing-recognizable by showing $\overline{A_{TM}} \leq_m \text{ALL}_{TM}$. (See hint from previous part.)