COMS W3261: Theoretical Computer Science.
Instructor: Tal Malkin

Problem Set 3

Due: Thur, 02/12/09.
Reading: Chapter 1.2.

1. Let $L = \{0^i 10^j | i \geq 0, j \geq 1\}$.
   (a) Give a state diagram of an NFA for L.
   (b) Give a short informal description of L^*.
   (c) Use the construction given in class (found in the proof of theorem 1.49) to give the state diagram of the NFA recognizing L^*.

2. (a) Problem 1.16(b) in text (NFA to DFA conversion).
   (b) Problem 1.8(a) in text (using union construction).
   (c) Problem 1.9(b) in text (using concatenation construction).

3. In class we showed that given a DFA that recognizes a language L, swapping the accept and nonaccept states yields a new DFA that recognizes the complement of L. We also mentioned that this is not necessarily true for NFAs. In this problem you will prove the latter,[1] and show a construction that does work for NFAs.

   Let $N_1 = (Q, \Sigma, \delta, q_0, F)$ be an NFA recognizing the language $L_1 = L(N_1)$. Consider the NFA $N_2 = (Q, \Sigma, \delta, q_0, Q \setminus F)$ obtained by swapping the accept and nonaccept states in N_1. Let $L_2 = L(N_2)$. For parts (a) and (b) below you should provide state diagrams, while for part (c) you should use the formal notation.

   (a) Give an example of an NFA N_1 and a string such that the string is accepted by both N_1 and N_2.
   (b) Give an example of an NFA N_1 and a string such that the string is rejected by both N_1 and N_2.
   (c) Given the NFA N_1 for the language L_1, show how to construct an NFA (or a DFA) that recognizes $\bar{L_1}$ (the complement of L_1).

4. Problem 1.31 in text (showing that the class of regular languages is closed under the *reverse* operation).

[1] part (a) proves that $L_2 \not\subseteq \bar{L_1}$, and part (b) proves that $\bar{L_1} \not\subseteq L_2$, so either one of them suffices to prove that $L_2 \neq \bar{L_1}$.