COMS W3261: Theoretical Computer Science.

Instructor: Tal Malkin

## Problem Set 9

Due: Tue, 11/20/07

Note: This homework is shorter than usual, and will only count as half a homeork

Reading: Chapter 4

1. Let  $L = \{\langle N \rangle | N \text{ is an NFA such that all strings accepted by } N \text{ have } 000 \text{ as a substring} \}.$ 

Prove that L is decidable.<sup>1</sup>

2. Let  $NE = \{\langle M \rangle | M \text{ is a TM that accepts some string} \}$  (namely all  $\langle M \rangle$  that recognize a non-empty language).

Prove that NE is Turing-recognizable. (Hint: Construct a TM/algorithm that recognizes NE. Make sure that any  $\langle M \rangle \in NE$  will indeed be accepted by your TM after a finite number of steps).

3. Let A be a countable set. Prove that  $\{B|B\subseteq A\}$  is not countable, using the diagonalization method.

<sup>&</sup>lt;sup>1</sup>To see some examples, check the solved problems in the textbook Chapter 4.