

Problem Set 6

Due: Tue, 10/16/07.

Reading: Chapter 2.1, 2.2

1. Consider the CFG G with start variable S and the following derivation rules R :

$$S \longrightarrow A1A1A$$

$$A \longrightarrow \epsilon \mid 0A \mid 1A$$

- (a) (5 points) What are the variables and terminals of G ? Give a string of length 7 which is in $L(G)$ and a string of length 5 which is not in $L(G)$.
 - (b) (10 points) The language generated by G happens to be a regular language. Give the English description and the regular expression for it.
 - (c) (10 points) Prove that G is ambiguous.
 - (d) (Extra Credit:) Show a non-ambiguous grammar for the same language.
2. Consider the language $L = \{w \mid w \text{ has an even number of 0's}\}$ over the alphabet $\{0, 1\}$.
 - (a) (10 points) Give a context free grammar for L .
 - (b) (5 points) Give the derivation and derivation tree for 101011
 3.
 - (a) (5 points) What needs to be proved in order to show that the class of context free languages is closed under the regular operations?
 - (b) (10 points) Now you are asked to do it: Prove that the class of context free languages is closed under the regular operations. (Hint: Use context-free grammars, and generate a CFG for the resulting language, using a new start symbol.)
 - (c) (10 points) Use the above to prove that regular languages are context free, by showing that every regular expression has an equivalent CFG. (Note that this is a third alternative to the two proofs seen in class).
 4. Give an informal description and a state diagram of pushdown automata for the following language over the alphabet $\Sigma = \{1, +, =\}$:

$$L_2 = \{1^n + 1^m = 1^{n+m} : n \geq 1, m \geq 1\}$$

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