

COMS W3261: Theoretical Computer Science.

Instructor: Tal Malkin

Problem Set 2

Due: Tue, 9/18/07.

Reading: Chapters 1.1, 1.2.

1. For each of the following languages, give a state diagram of a DFA that recognizes the language. For the first language L_1 , also give the formal description of the DFA you construct. All languages are over the alphabet $\Sigma = \{0, 1\}$.
 - (a) $L_1 = \{w \mid |w| \text{ is an integer multiple of } 4\}$ (recall that $|w|$ is the length of the string).
 - (b) $L_2 = \{w \mid w \text{ is a representation of a binary number that is an integer multiple of } 4\}$.
 - (c) (extra credit): $L_3 = \{w \mid w \text{ is a representation of a binary number that is an integer multiple of } 3\}$.
 - (d) $L_4 = \{w \mid w \text{ does not contain the substring } 110\}$
2. Let $L_5 = \{w \mid \text{the two symbols before the last one in } w \text{ are } 01\}$ (over the alphabet $\Sigma = \{0, 1\}$). For example, 11011 is in L_5 but 110111 is not in L_5 .
 - (a) Construct a DFA with 5 states for L_5 .
 - (b) Construct an NFA with 4 states for L_5 .
 - (c) Give the computation tree of your NFA from (b) on the word 01010.
3. For each of the following languages, give a state diagram of an NFA that recognizes the language. For the third language L_8 , also give the formal description of the NFA you construct. All languages are over the alphabet $\Sigma = \{0, 1\}$.
 - (a) $L_6 = \{0, 1\}$
 - (b) $L_7 = \emptyset$
 - (c) $L_8 = \{w \mid w \text{ contains an even number of 0s or exactly two 1s}\}$
 - (d) $L_9 = \{w \mid w \text{ contains two 0s separated by a string } w' \text{ in } L_1\}$ where L_1 is the language defined in problem 1a above.