

Problem Set 10

Due: Tue, 12/04/07

Reading: Chapter 4, 5.1, 5.3, 6.3

1. Recall that we defined $X_{TM} = \{\langle M \rangle : M \text{ is a TM and } M \text{ does not accept } \langle M \rangle\}$ and proved that it is not decidable. Also recall that we proved that a language is decidable if and only if it is both recognizable and co-recognizable. Define the language

$$NE_{TM} = \{\langle M \rangle : M \text{ is a TM and } L(M) \text{ is not empty}\}.$$

- (a) (10 points) Prove that X_{TM} is not recognizable.
 - (b) (10 points) Prove that NE_{TM} is undecidable, by showing that $X_{TM} \leq_T NE_{TM}$. Specifically, let H be a decider for NE_{TM} , show a decider D for X_{TM} .
 - (c) (5 points) Why can't the same proof be used to prove that NE_{TM} is not recognizable? Specifically, if H is a recognizer for NE_{TM} , why wouldn't D (that you built in part (b)) be a recognizer for X_{TM} ?
 - (d) (10 points) Prove that NE_{TM} is recognizable.
2. (10 points) Let $L_{TM} = \{\langle M \rangle \mid M \text{ is TM such that all strings accepted by } M \text{ have } 000 \text{ as a substring}\}$.

Prove that L_{TM} is undecidable.

(Hint: one way to do it is by showing that $A_{TM} \leq_T L_{TM}$. Another (similar) way is by showing that $\overline{A_{TM}} \leq_T L_{TM}$.)

3. (10 points) Show that it is impossible to write a Java program which, given as input an arbitrary Java program, can determine whether or not there is any input on which the given program prints out "AN ERROR HAS OCCURED". (You may assume Java programs are equivalent to Turing Machines.)
4. (10 points) Show that a language A is decidable if and only if $A \leq_m 0^*1^*$.
5. (15 points) 5.24 in text (proving a certain language is neither recognizable nor co-recognizable).