3261 Midterm
March 7, 2007
Prof. Tal Malkin

Closed book exam, one "cheat-sheet" allowed.

Write all answers in the space provided. If you run out space use the back of the page. If you need scratch paper, ask the TA or the professor. Exam starts at 4:10pm and ends at 5:25pm. Total no. of points is 100. Write your name and email address on the top of any *loose* page.

To show that a language is regular (or context free) you can show a DFA/NFA/regular expression (or a CFG/PDA) and argue its correctness informally, but unless otherwise specified, you do not need to provide a formal proof that your construction is correct.

If you use theorems proved in class, you should explicitly state what you are using.

It is highly recommended that you read the entire exam before proceeding as questions are not necessarily in order of difficulty.

Name: _____

Email: _____

1a(4)	1b(10)	2(14)	3a(10)	3b(10)	3c(12)	4(10)	5a(6)	5b(6)	5c(6)	5d(6)	5e(6)	Total(100)

1. Let $N = (\{q, r\}, \{a, b, c\}, \delta, q, \{r\})$ be an NFA, where δ is defined as follows:

$\delta(q, \epsilon) = \{r\}$
$\delta(q, a) = \{q\}$
$\delta(r, b) = \{r\}$
$\delta(r, c) = \{q\}$.

(a) (4 points) Draw the state diagram for N.

(b) (10 points) Use the construction for converting an NFA to a DFA, to convert N to an equivalent DFA (it is enough to show the state diagram for the DFA, labeling the states in a way that shows you understood the construction).

2. (14 points) Prove that the class of regular languages is closed under intersection.

3. (a) (10 points) Let L_1 be the set of strings over $\{0, 1\}$ that can be written in the form $1^k y$ where y contains at least k 1s, for some $k \geq 1$. Show that L_1 is a regular language.

(Note that the same string could fit the definition for more than one value of k. For example, 11010100 can be seen as 1 followed by the string $y = 1010100$, which contains at least one 1, or as 11 followed by 010100, which contains at least two 1s. On the other hand, strings such as 100 or 000110110 are not in L_1 because there is no value of k for which the definition applies.)

(b) (10 points) Let L_2 be the set of strings over $\{0, 1\}$ that can be written in the form $1^k 0 y$ where y contains at least k 1s, for some $k \geq 1$. Show that L_2 is not a regular language.

(c) (12 points) Show that L_1 and L_2 are both context free languages.

4. (10 points) Show a PDA for the language $\{a^m b^n | m \neq n\}$ over {a,b}

5. Please provide a short answer and a short (and clear) justification for each of the following.

   (a) (6 points) Give a regular expression for the language over $\{0, 1\}$ consisting of all strings where the third to last symbol is 0.

   (b) (6 points) True or False: $\{www | w \in \{a, b\}^{3261}\}$ is regular.

(c) (6 points) True or False: If a DFA M has n states, then $L(M)$ has a string of length at least n.

(d) (6 points) True or False: If a DFA M has n states, and $L(M)$ has a string of length at least n, then $L(M)$ has a string of length at least $100n$.

(e) (6 points) Let $L = \{a^n b^n c^n | n \geq 0\} \cup \{a, b, c\}$. True or False: L^* is context free.