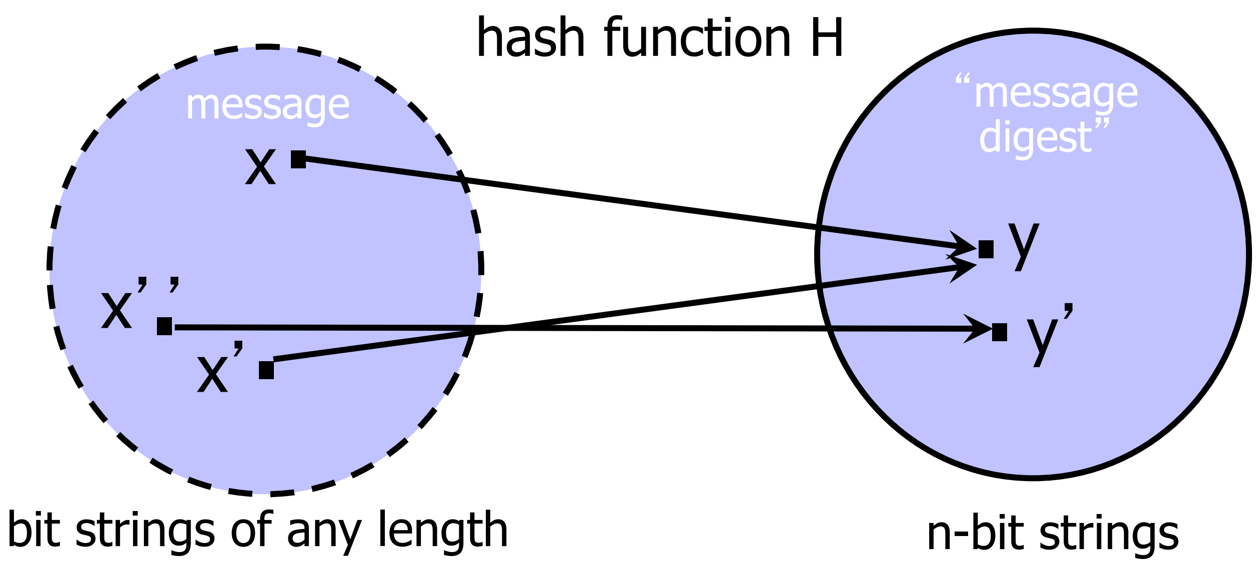
Cryptographic Hash Functions

*Slides borrowed from Vitaly Shmatikov

Hash Functions: Main Idea



- Hash function H is a lossy compression function
  - Collision: H(x)=H(x') for some inputs x≠x'
- H(x) should look "random"
  - Every bit (almost) equally likely to be 0 or 1
- A <u>cryptographic</u> hash function must have certain properties

One-Way

- Intuition: hash should be hard to invert
  - "Preimage resistance"
  - Given a random, it should be hard to find any x such that h(x)=y
    - y is an n-bit string randomly chosen from the output space of the hash function, ie, y=h(x') for some x'
- How hard?
  - Brute-force: try every possible x, see if h(x)=y
  - SHA-1 (a common hash function) has 160-bit output
    - Suppose we have hardware that can do 2^{30} trials a pop
    - Assuming 2^{34} trials per second, can do 2^{89} trials per year
    - Will take 2^{71} years to invert SHA-1 on a random image

Birthday Paradox

- T people
- Suppose each birthday is a random number taken from K days (K=365) – how many possibilities?
  - K^T - samples with replacement
- How many possibilities that are all different?
  - $(K)_T = K(K-1)\ldots(K-T+1)$ - samples without replacement
- Probability of no repetition?
  - $(K)_T/K^T \approx 1 - T(T-1)/2K$
- Probability of repetition?
  - $O(T^2)$

Collision Resistance

- Should be hard to find $x \neq x'$ such that $h(x)=h(x')$
- Birthday paradox
  - Let T be the number of values x, x', x'' … we need to look at before finding the first pair $x \neq x'$ s.t. $h(x)=h(x')$
  - Assuming h is random, what is the probability that we find a repetition after looking at T values? $O(T^2)$
  - Total number of pairs? $O(2^n)$
    - n = number of bits in the output of hash function
  - Conclusion: $T \approx O(2^{n/2})$
- Brute-force collision search is $O(2^{n/2})$, not $O(2^n)$
  - For SHA-1, this means $O(2^{80})$ vs. $O(2^{160})$

One-Way vs. Collision Resistance

- One-wayness does not imply collision resistance
  - Suppose g() is one-way
  - Define h(x) as g(x') where x' is x except the last bit
    - h is one-way (cannot invert h without inverting g)
    - Collisions for h are easy to find: for any x, h(x0)=h(x1)
- Collision resistance does not imply one-wayness
  - Suppose g() is collision-resistant
  - Define h(x) to be 0x if x is (n-1)-bit long, else 1g(x)
    - Collisions for h are hard to find: if y starts with 0, then there are no collisions; if y starts with 1, then must find collisions in g
    - h is not one way: half of all y's (those whose first bit is 0) are easy to invert (how?), thus random y is invertible with prob. 1/2

Weak Collision Resistance

- Given a randomly chosen x, hard to find x’ such that h(x)=h(x’)
  - Attacker must find collision for a specific x… by contrast, to break collision resistance, enough to find any collision
  - Brute-force attack requires $O(2^n)$ time
- Weak collision resistance does not imply collision resistance (why?)

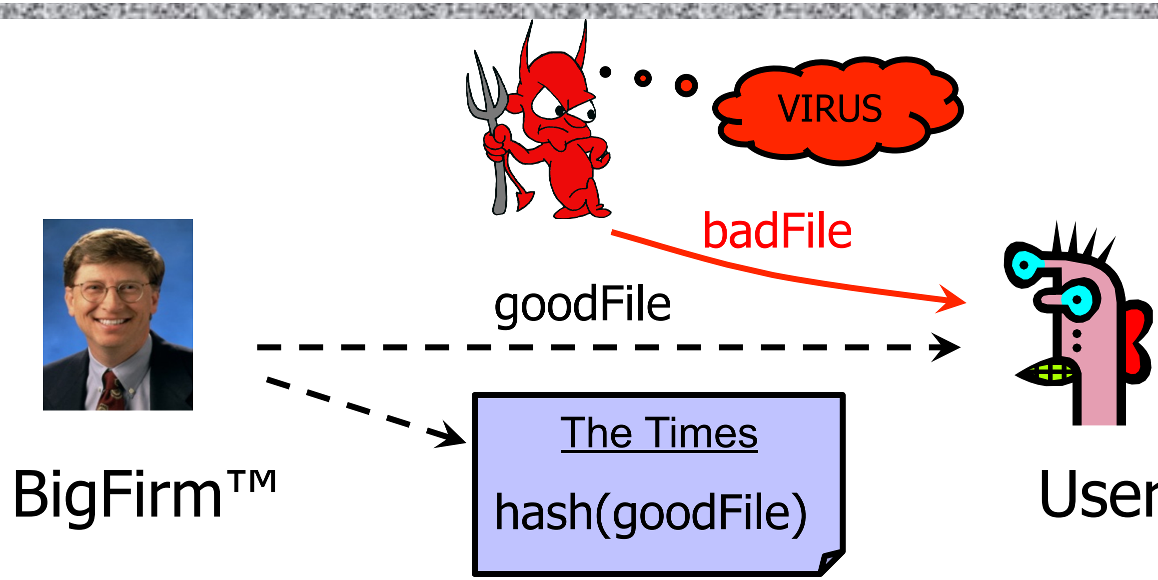
Hashing vs. Encryption

- Hashing is one-way. There is no "uh-hashing"!
  - A ciphertext can be decrypted with a decryption key… hashes have no equivalent of "decryption"
- Hash(x) looks "random", but can be compared for equality with Hash(x')
  - Hash the same input twice → same hash value
  - Encrypt the same input twice → different ciphertexts
- Cryptographic hashes are also known as "cryptographic checksums" or "message digests"

Application: Password Hashing

- Instead of user password, store hash(password)
- When user enters a password, compute its hash and compare with the entry in the password file
  - System does not store actual passwords!
  - Cannot go from hash to password!
- Why is hashing better than encryption here?
- Does hashing protect weak, easily guessable passwords?

Application: Software Integrity



Software manufacturer wants to ensure that the executable file is received by users without modification…

Sends out the file to users and publishes its hash in the NY Times

The goal is <u>integrity</u>, not secrecy

Idea: given goodFile and hash(goodFile), very hard to find badFile such that hash(goodFile)=hash(badFile)

Which Property Is Needed?

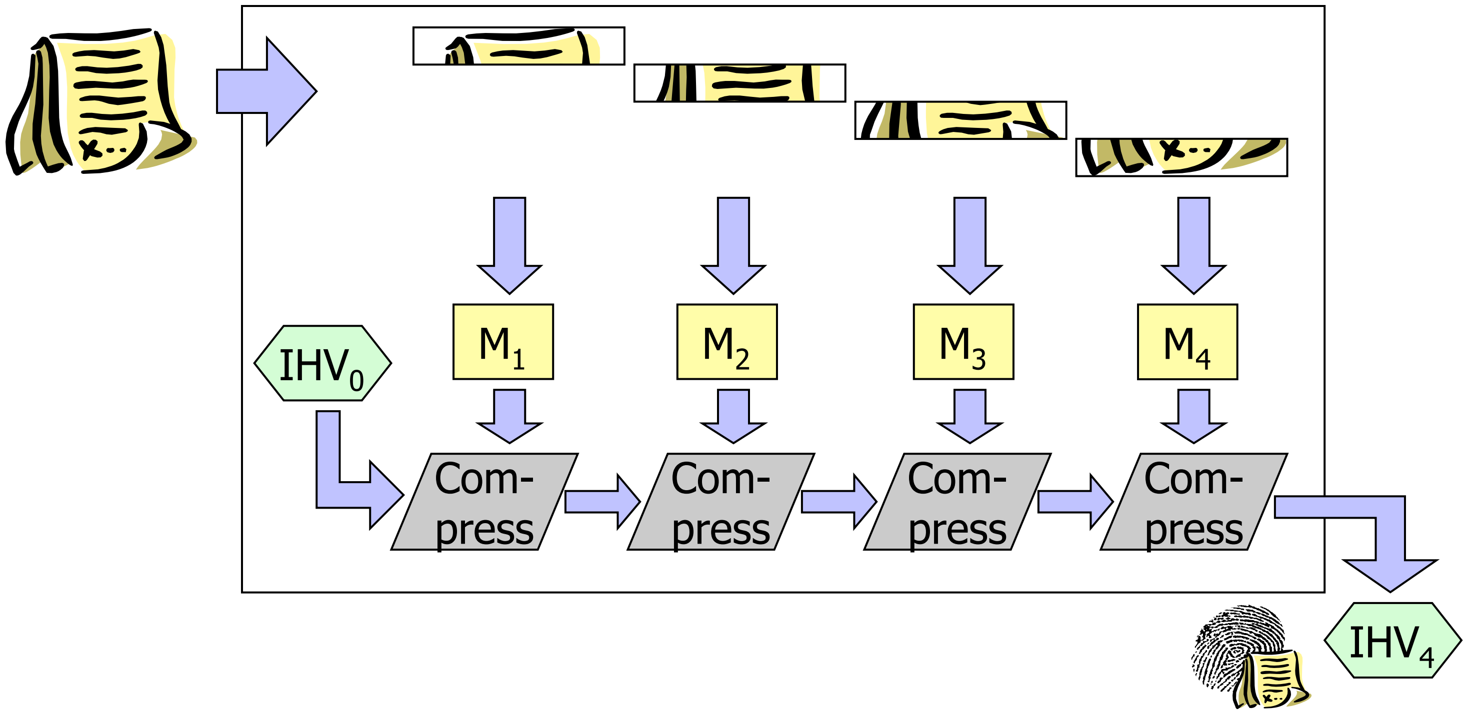
- Passwords stored as hash(password)
  - One-wayness: hard to recover entire password
  - Passwords are not random and thus guessable
- Integrity of software distribution
  - Weak collision resistance?
  - But software images are not random… maybe need full collision resistance
- Auctions: to bid B, send H(B), later reveal B
  - One-wayness… but does not protect B from guessing
  - Collision resistance: bidder should not be able to find two bids B and B' such that H(B)=H(B')

Common Hash Functions

- MD5
  - Completely broken by now
- RIPEMD-160
  - 160-bit variant of MD-5
- SHA-1 (Secure Hash Algorithm)
  - Widely used
  - US government (NIST) standard as of 1993-95
    - Also the hash algorithm for Digital Signature Standard (DSS)

Overview of MD5

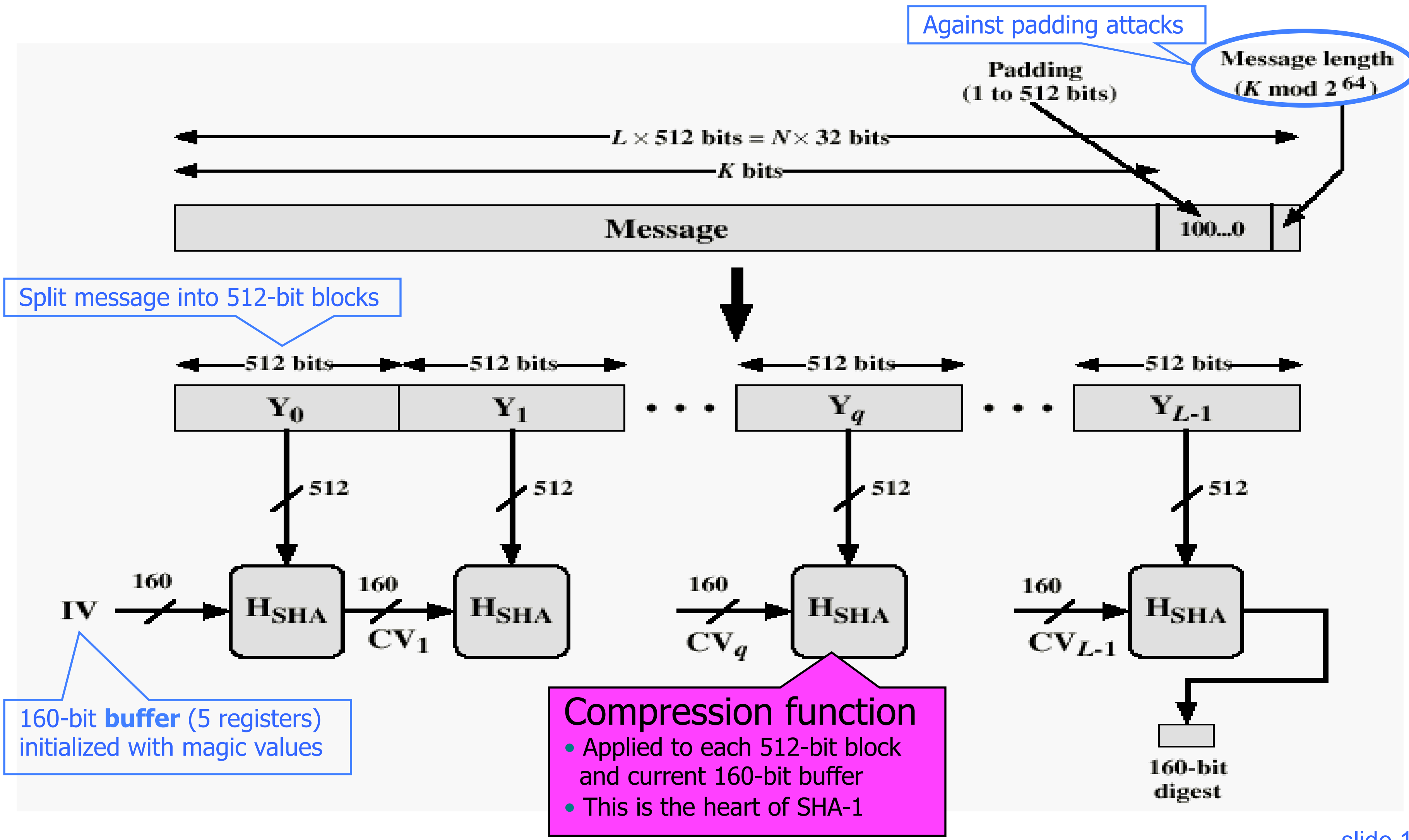
- Designed in 1991 by Ron Rivest
- Iterative design using compression function



History of MD5 Collisions

- 2004: first collision attack
  - The only difference between colliding messages is 128 random-looking bytes
- 2007: chosen-prefix collisions
  - For any prefix, can find colliding messages that have this prefix and differ up to 716 random-looking bytes
- 2008: rogue SSL certificates
  - Talk about this in more detail when discussing PKI
- 2012: MD5 collisions used in cyberwarfare
  - Flame malware uses an MD5 prefix collision to fake a Microsoft digital code signature

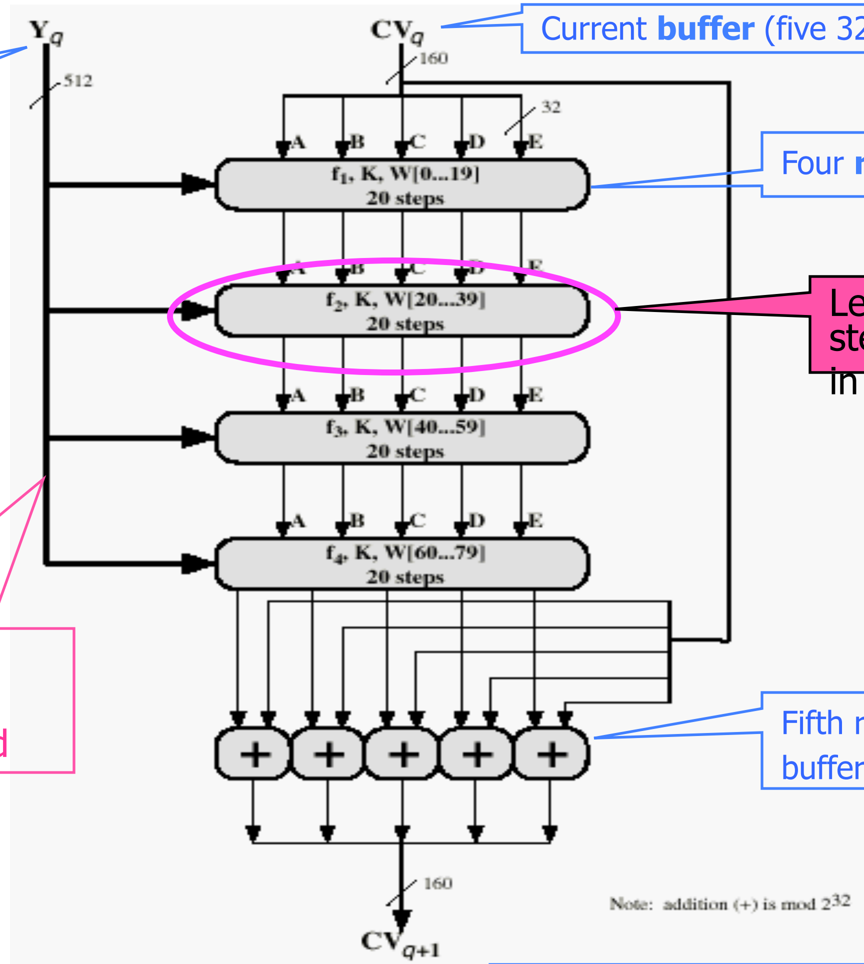
Basic Structure of SHA-1



SHA-1 Compression Function

Current **message block**

Current **buffer** (five 32-bit registers A,B,C,D,E)



Four **rounds**, 20 steps in each

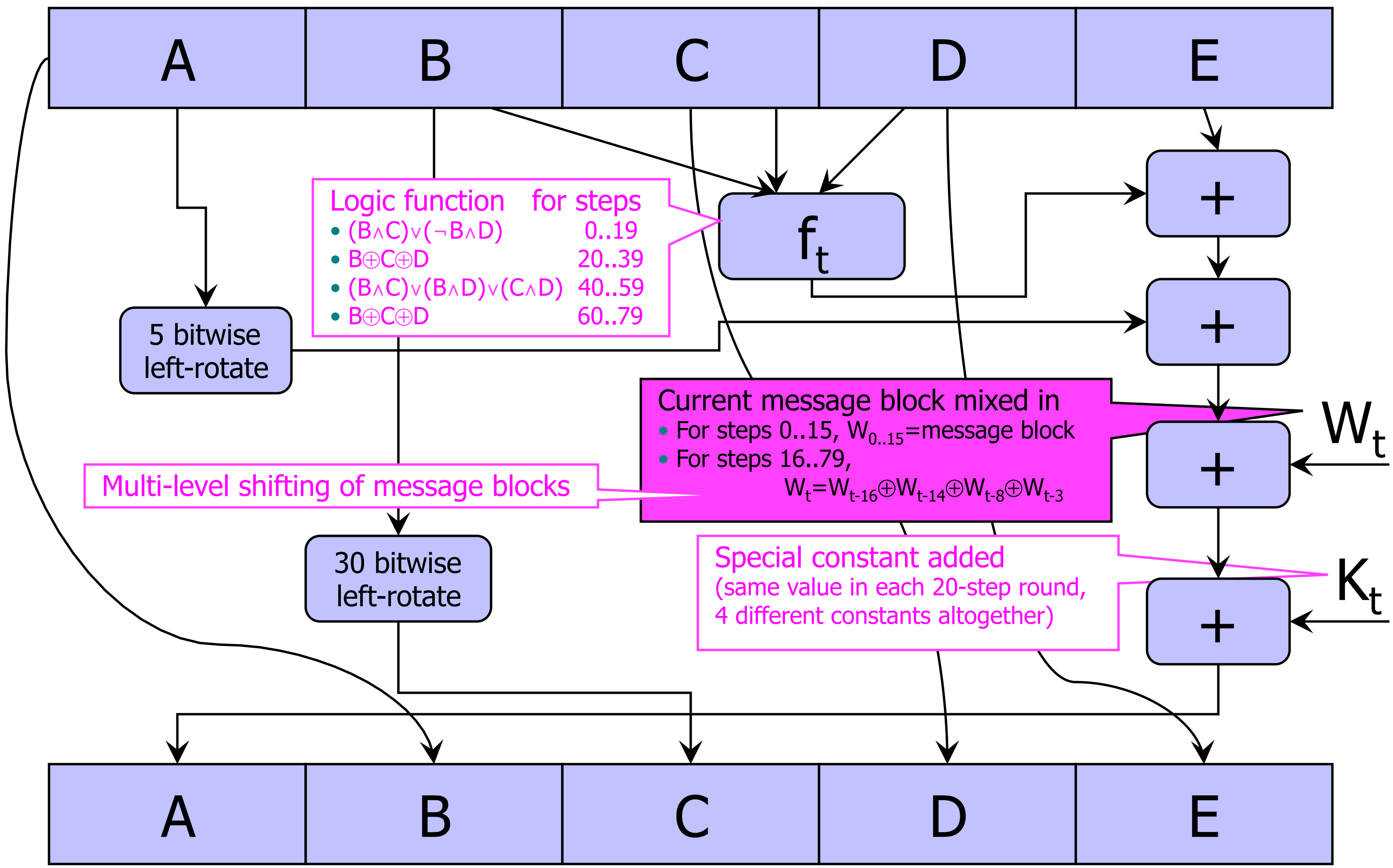
Let' s look at each step
in more detail…

Similar to a block cipher, with message itself used as the key for each round

Fifth round adds the original buffer to the result of 4 rounds

Buffer contains final hash value

One Step of SHA-1 (80 steps total)



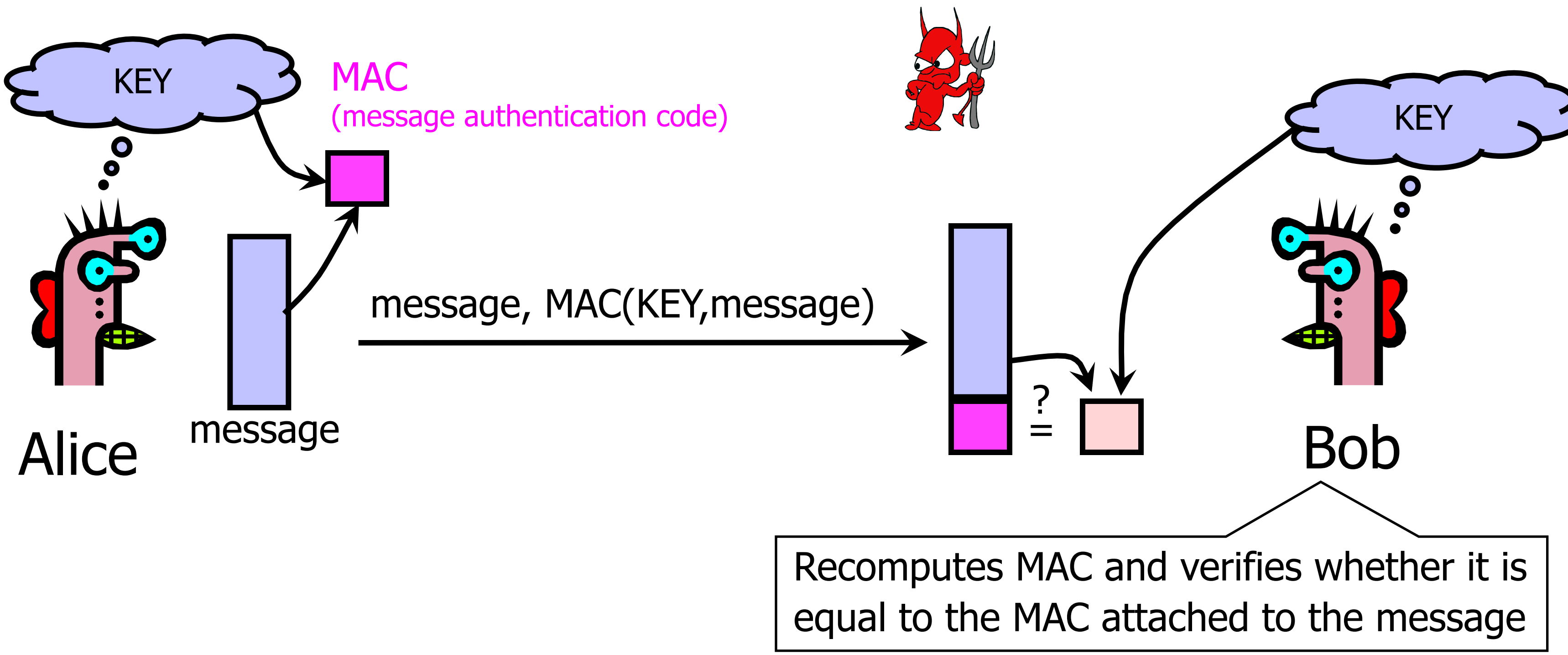
How Strong Is SHA-1?

- Every bit of output depends on every bit of input
  - Very important property for collision-resistance
- Brute-force inversion requires 2^{160} ops, birthday attack on collision resistance requires 2^{80} ops
- Some weaknesses discovered in 2005
  - Collisions can be found in 2^{63} ops

NIST Competition

- A public competition to develop a new cryptographic hash algorithm
  - Organized by NIST (read: NSA)
- 64 entries into the competition (Oct 2008)
- 5 finalists in 3rd round (Dec 2010)
- Winner: Keccak (Oct 2012)
  - standardized as SHA-3

Integrity and Authentication

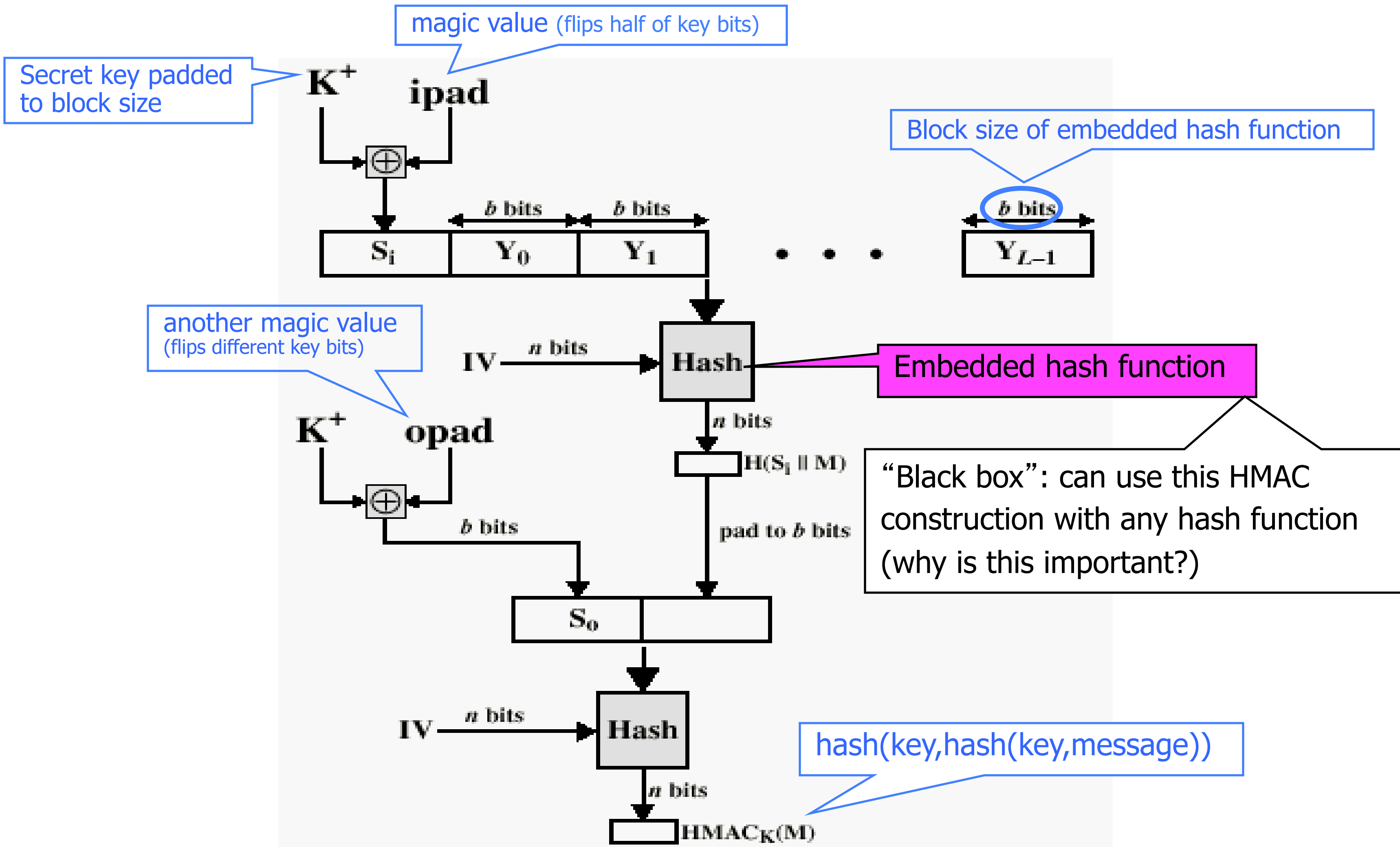


Integrity and authentication: only someone who knows KEY can compute correct MAC for a given message

HMAC

- Construct MAC from a cryptographic hash function
  - Invented by Bellare, Canetti, and Krawczyk (1996)
  - Used in SSL/TLS, mandatory for IPsec
- Why not encryption?
  - Hashing is faster than encryption
  - Library code for hash functions widely available
  - Can easily replace one hash function with another
  - There used to be US export restrictions on encryption

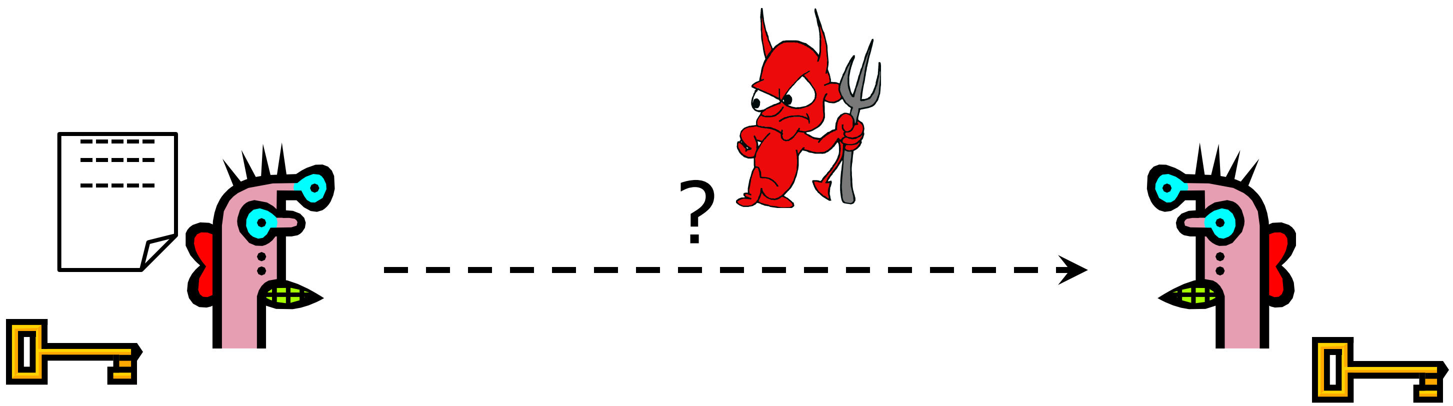
Structure of HMAC



"Black box": can use this HMAC construction with any hash function (why is this important?)

Overview of Symmetric Encryption

Basic Problem



Given: both parties already know the same secret

Goal: send a message confidentially

How is this achieved in practice?

Any communication system that aims to guarantee confidentiality must solve this problem

Kerckhoffs's Principle

- An encryption scheme should be secure even if enemy knows everything about it except the key
  - Attacker knows all algorithms
  - Attacker does not know random numbers
- Do not rely on secrecy of the algorithms ("security by obscurity")

Easy lesson:
use a good random number generator!

Full name:

Jean-Guillaume-Hubert-Victor-François-Alexandre-Auguste Kerckhoffs von Nieuwenhof

Randomness Matters!

The New York Times

Business Day
Technology

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Flaw Found in an Online Encryption Method

By JOHN MARKOFF
Published: February 14, 2012

SAN FRANCISCO — A team of European and American mathematicians and cryptographers have discovered an unexpected weakness in the encryption system widely used worldwide for online shopping, banking, e-mail and other Internet services intended to remain private and secure.

The flaw — which involves a small but measurable number of cases — has to do with the way the system generates random numbers, which are used to

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Internet users, there is nothing a
sites will need to make changes t
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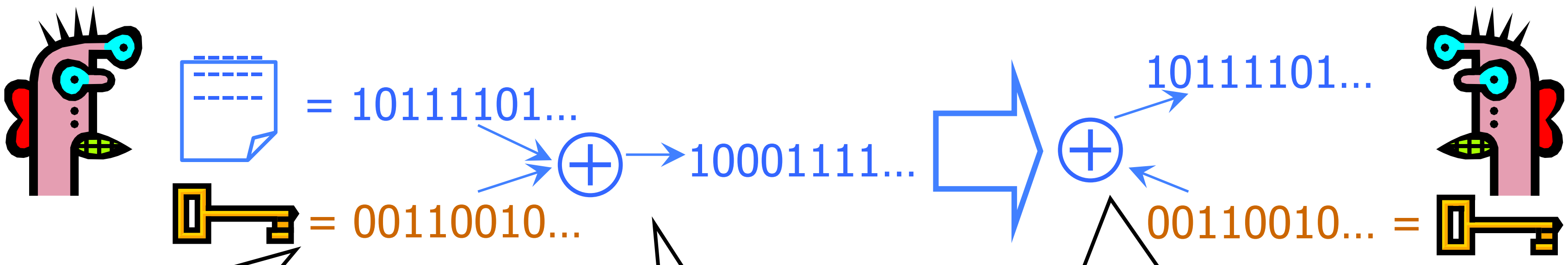
Uptime | IT and business computing insights

Crypto shocker: four of every 1,000 public keys provide no security (updated)

By Dan Goodin | Published 7 days ago

One-Time Pad (Vernam Cipher)



Key is a random bit sequence as long as the plaintext

Encrypt by bitwise XOR of plaintext and key:
ciphertext = plaintext ⊕ key

Decrypt by bitwise XOR of ciphertext and key:
ciphertext ⊕ key =
(plaintext ⊕ key) ⊕ key =
plaintext ⊕ (key ⊕ key) =
plaintext

Cipher achieves perfect secrecy if and only if
there are as many possible keys as possible plaintexts, and
every key is equally likely (Claude Shannon, 1949)

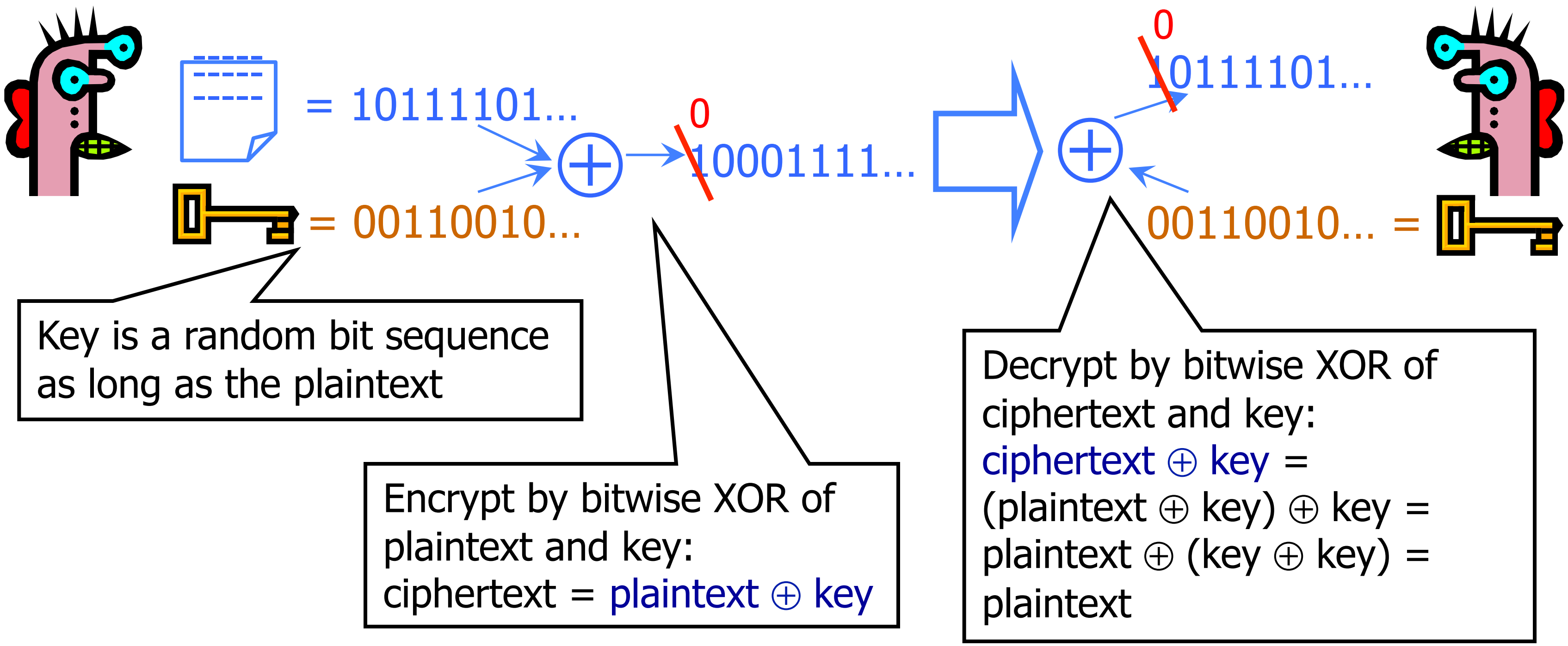
Advantages of One-Time Pad

- Easy to compute
  - Encryption and decryption are the same operation
  - Bitwise XOR is very cheap to compute
- As secure as theoretically possible
  - Given a ciphertext, all plaintexts are equally likely, regardless of attacker's computational resources
  - …if and only if the key sequence is truly random
    - True randomness is expensive to obtain in large quantities
  - …if and only if each key is as long as the plaintext
    - But how do the sender and the receiver communicate the key to each other? Where do they store the key?

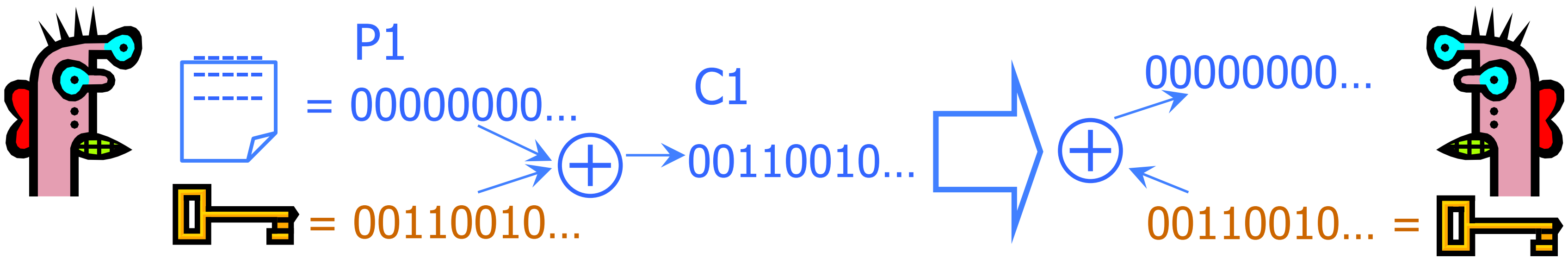
Problems with One-Time Pad

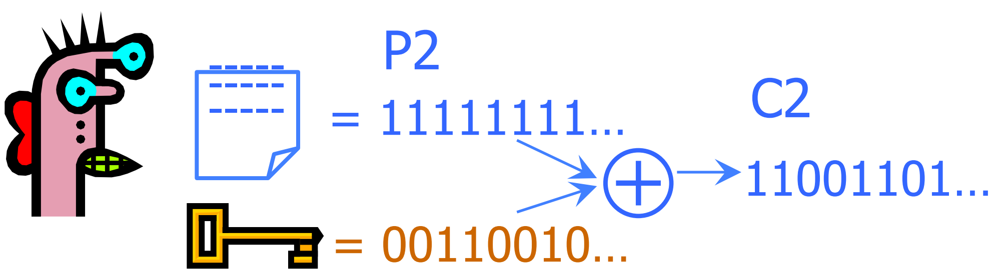
- Key must be as long as the plaintext
  - Impractical in most realistic scenarios
  - Still used for diplomatic and intelligence traffic
- Does not guarantee integrity
  - One-time pad only guarantees confidentiality
  - Attacker cannot recover plaintext, but can easily change it to something else
- Insecure if keys are reused
  - Attacker can obtain XOR of plaintexts

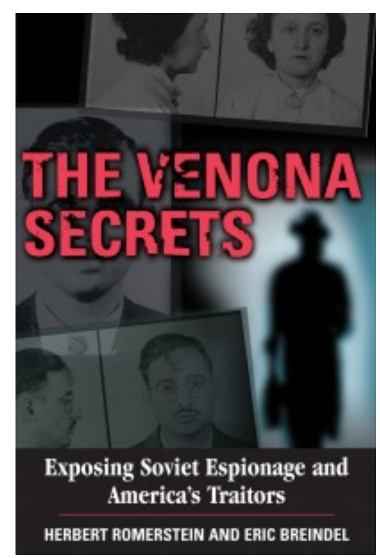
No Integrity



Dangers of Reuse







Learn relationship between plaintexts

$C1 \oplus C2 = (P1 \oplus K) \oplus (P2 \oplus K) =$

$(P1 \oplus P2) \oplus (K \oplus K) =$ $P1 \oplus P2$

Reducing Key Size

- What to do when it is infeasible to pre-share huge random keys?
- Use special cryptographic primitives:

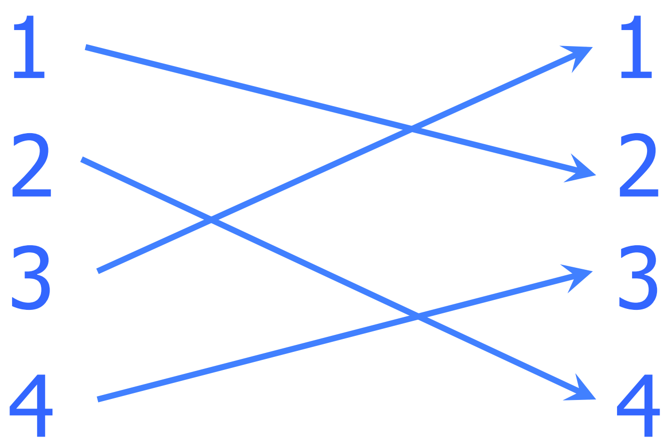
  block ciphers, stream ciphers

  - Single key can be re-used (with some restrictions)
  - Not as theoretically secure as one-time pad

Block Ciphers

- Operates on a single chunk ("block") of plaintext
  - For example, 64 bits for DES, 128 bits for AES
  - Same key is reused for each block (can use short keys)
- Result should look like a random permutation
- Not impossible to break, just very expensive
  - If there is no more efficient algorithm (unproven assumption!), can only break the cipher by brute-force, try-every-possible-key search
  - Time and cost of breaking the cipher exceed the value and/or useful lifetime of protected information

Permutation



CODE becomes DCEO

- For N-bit input, N! possible permutations
- Idea: split plaintext into blocks, for each block use secret key to pick a permutation, rinse and repeat
    - Without the key, permutation should "look random"

A Bit of Block Cipher History

- Playfair and variants (from 1854 until WWII)
- Feistel structure

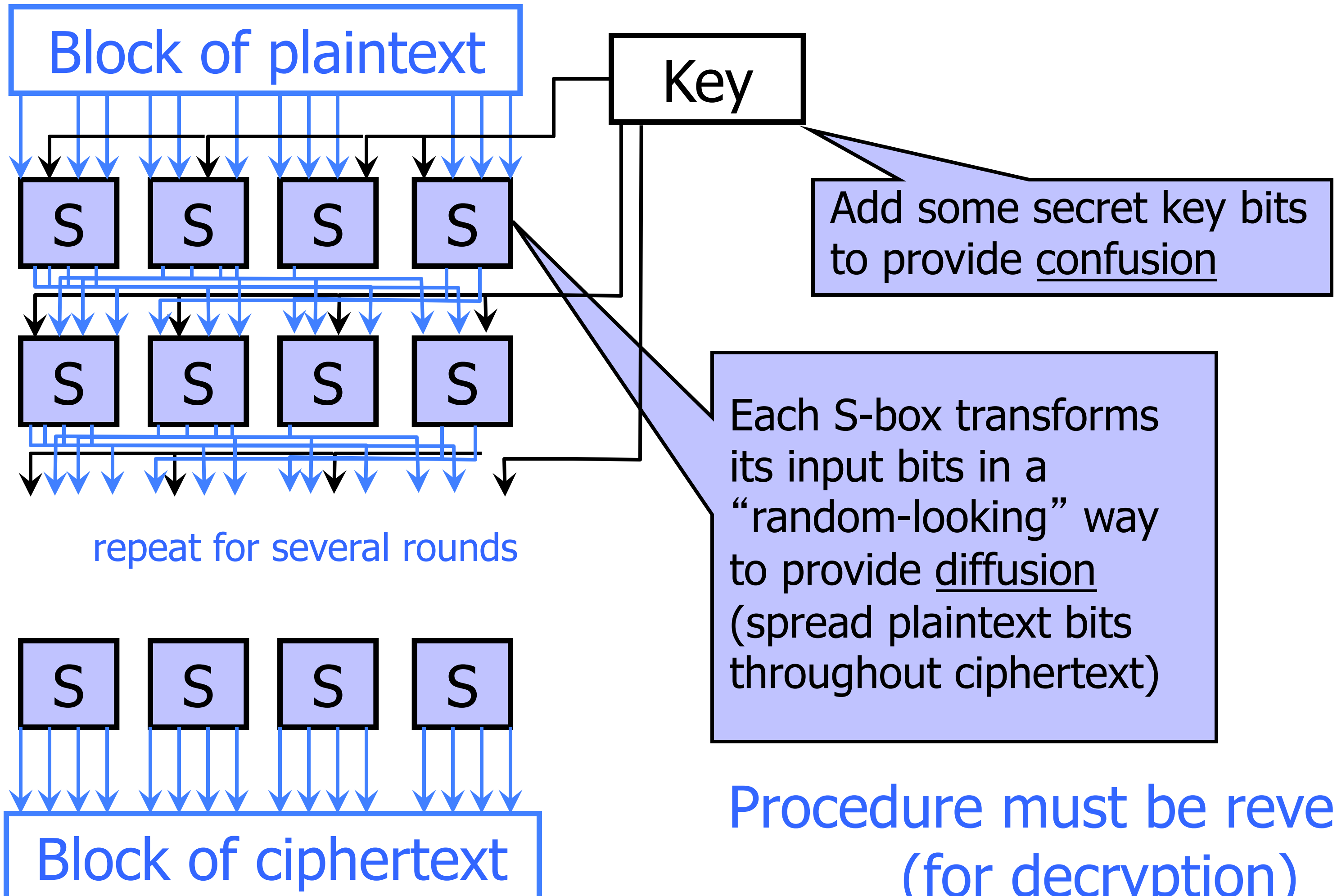
  Textbook

  - "Ladder" structure: split input in half, put one half through the round and XOR with the other half
  - After 3 random rounds, ciphertext indistinguishable from a random permutation
- DES: Data Encryption Standard

  Textbook

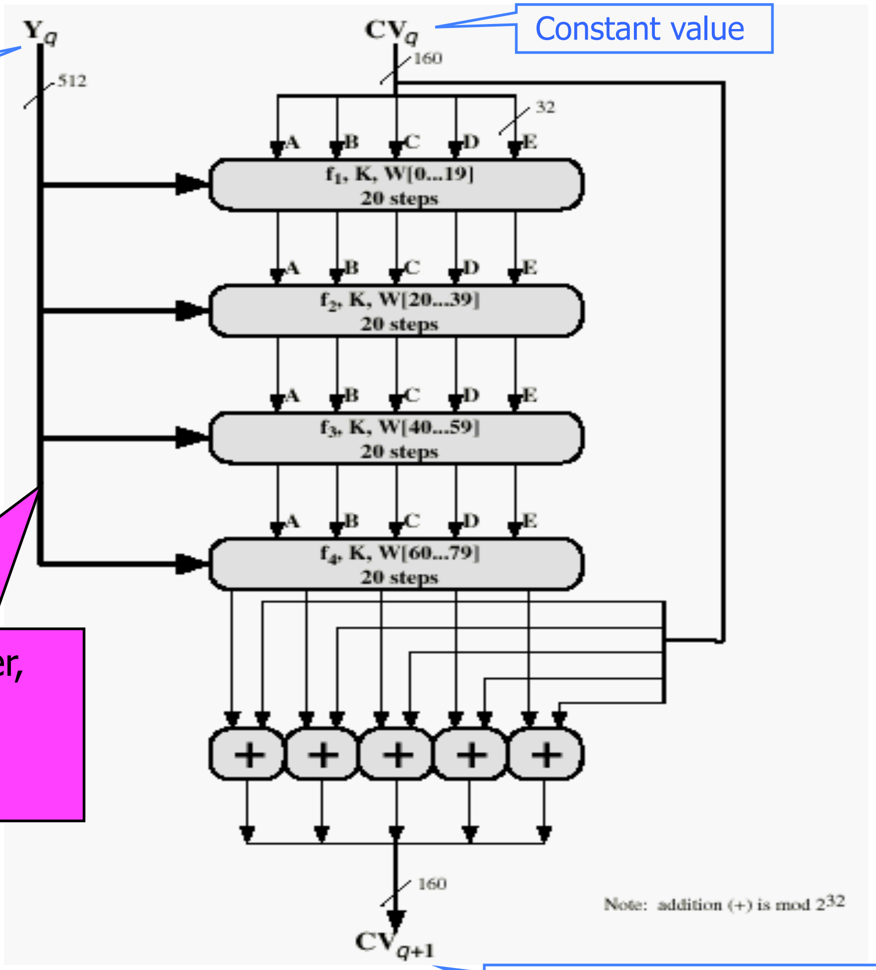
  - Invented by IBM, issued as federal standard in 1977
  - 64-bit blocks, 56-bit key + 8 bits for parity
  - Very widely used (usually as 3DES) until recently
    - 3DES: DES + inverse DES + DES (with 2 or 3 different keys)

DES Operation (Simplified)



Procedure must be reversible
(for decryption)

Remember SHA-1?

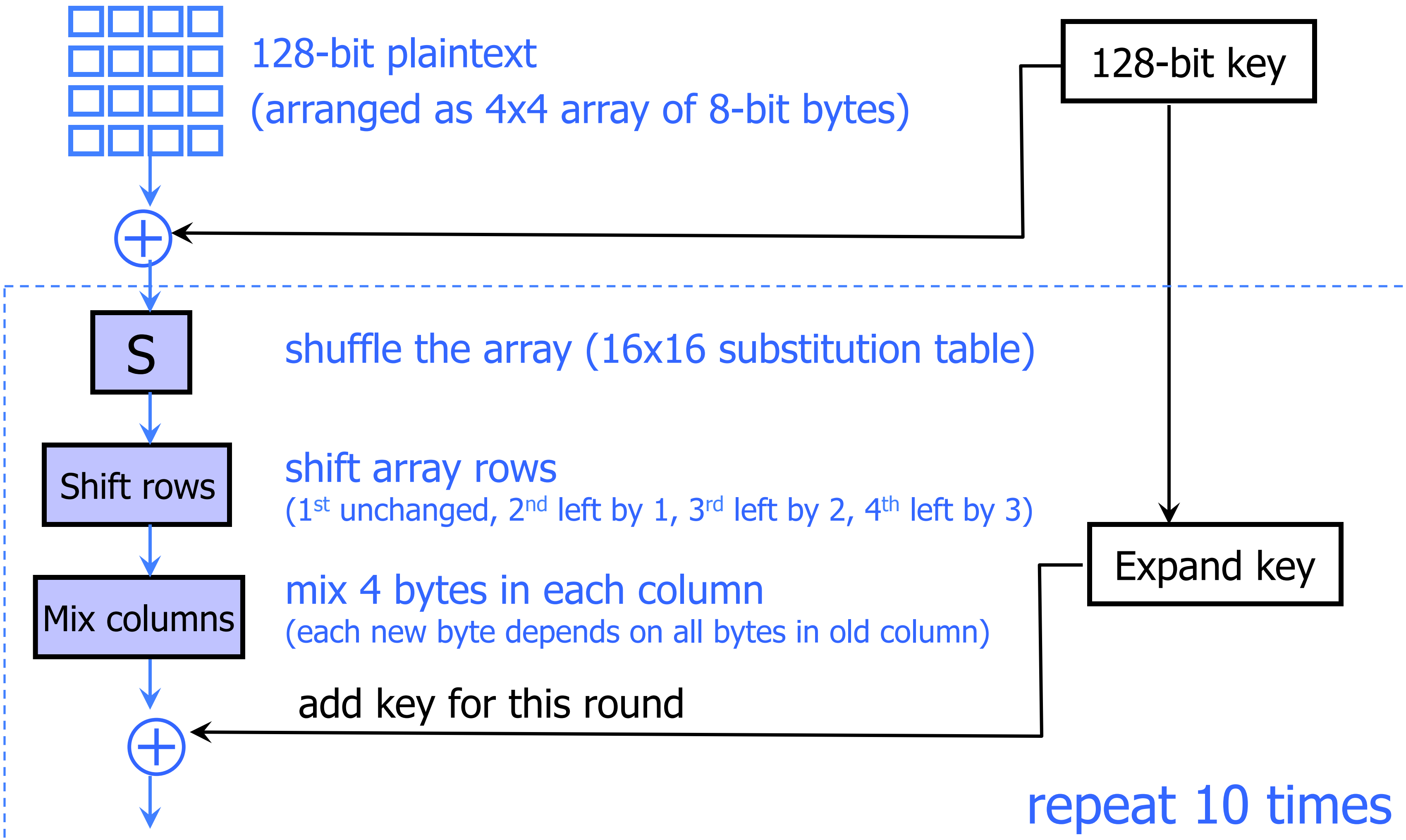


Very similar to a block cipher, with message itself used as the key for each round

Advanced Encryption Standard (AES)

- US federal standard as of 2001
- Based on the Rijndael algorithm
- 128-bit blocks, keys can be 128, 192 or 256 bits
- Unlike DES, does not use Feistel structure
  - The entire block is processed during each round
- Design uses some clever math
  - See section 8.5 of the textbook for a concise summary

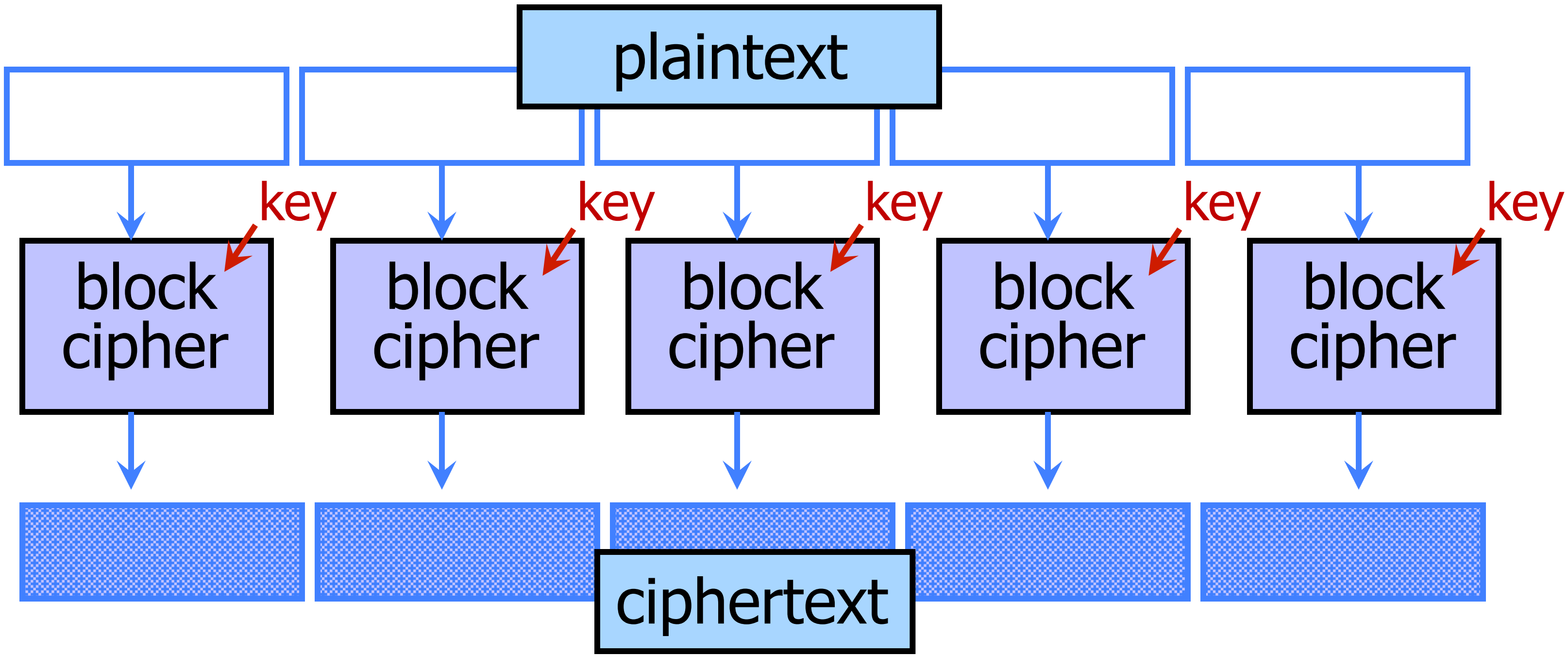
Basic Structure of Rijndael



Encrypting a Large Message

- So, we've got a good block cipher, but our plaintext is larger than 128-bit block size
- Electronic Code Book (ECB) mode
  - Split plaintext into blocks, encrypt each one separately using the block cipher
- Cipher Block Chaining (CBC) mode
  - Split plaintext into blocks, XOR each block with the result of encrypting previous blocks
- Also various counter modes, feedback modes, etc.

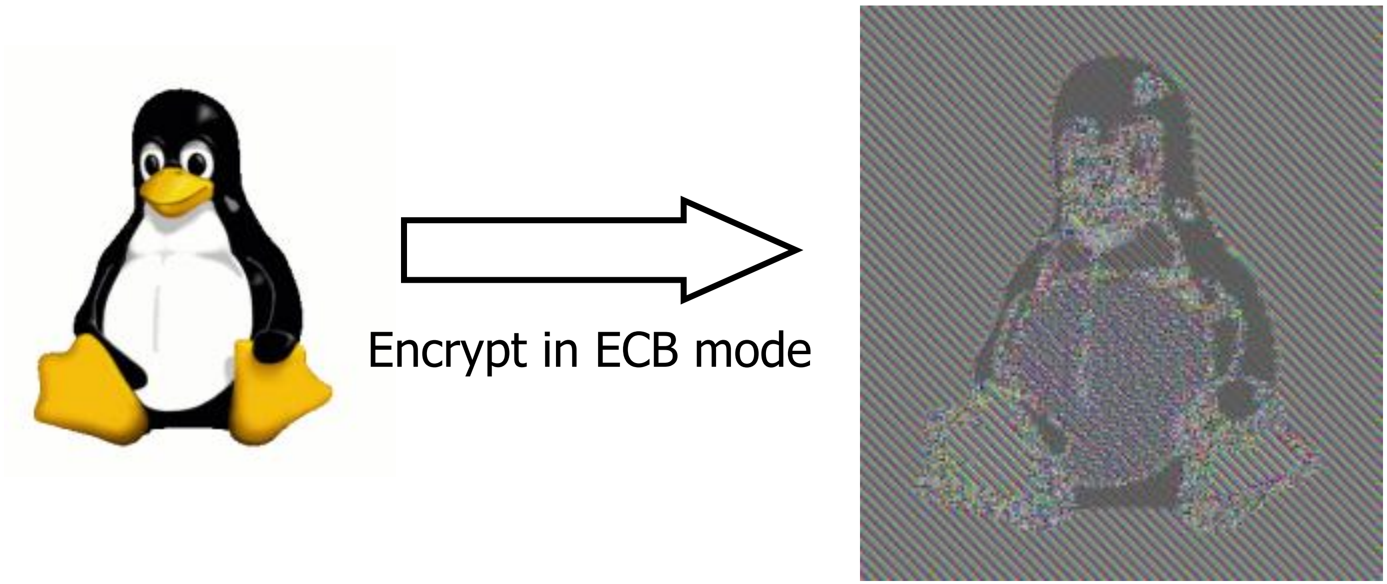
ECB Mode



- Identical blocks of plaintext produce identical blocks of ciphertext
- No integrity checks: can mix and match blocks

Information Leakage in ECB Mode

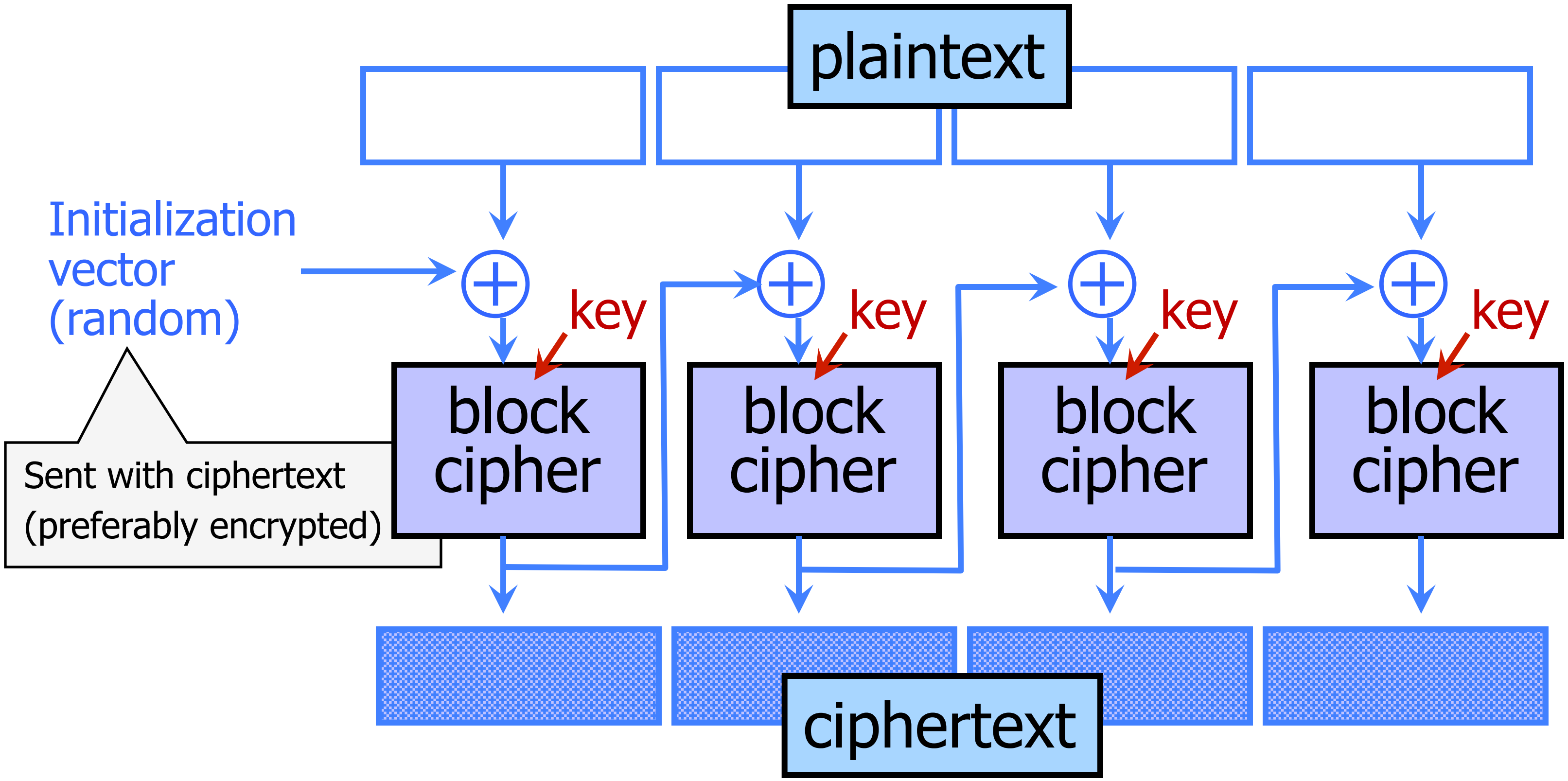
[Wikipedia]



Adobe Passwords Stolen (2013)

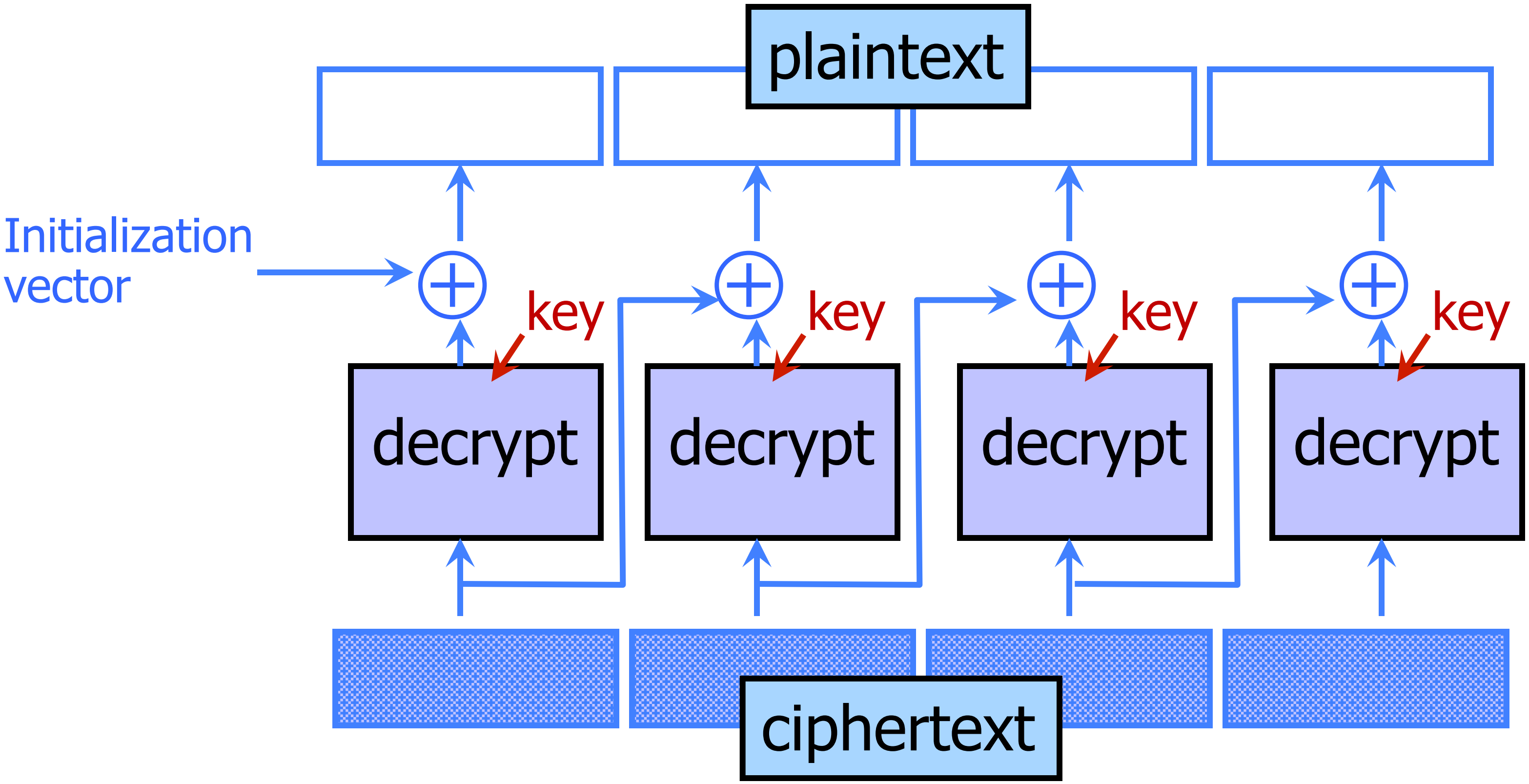
- 153 million account passwords
  - 56 million of them unique
- Encrypted using 3DES in ECB mode rather than hashed

CBC Mode: Encryption



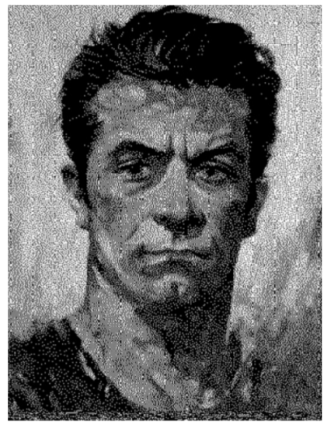
- Identical blocks of plaintext encrypted differently
- Last cipherblock depends on entire plaintext
  - Still does not guarantee integrity

CBC Mode: Decryption



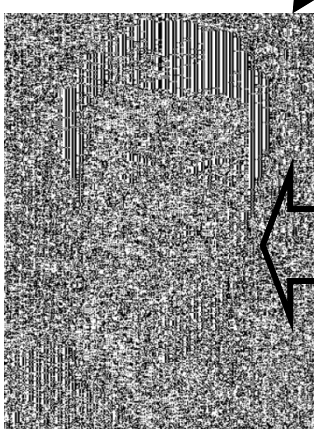
ECB vs. CBC

[Picture due to Bart Preneel]

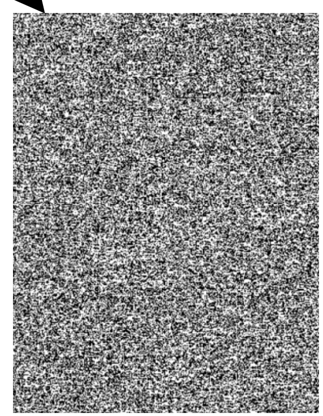


AES in ECB mode

AES in CBC mode



Similar plaintext
blocks produce
similar ciphertext
blocks (not good!)

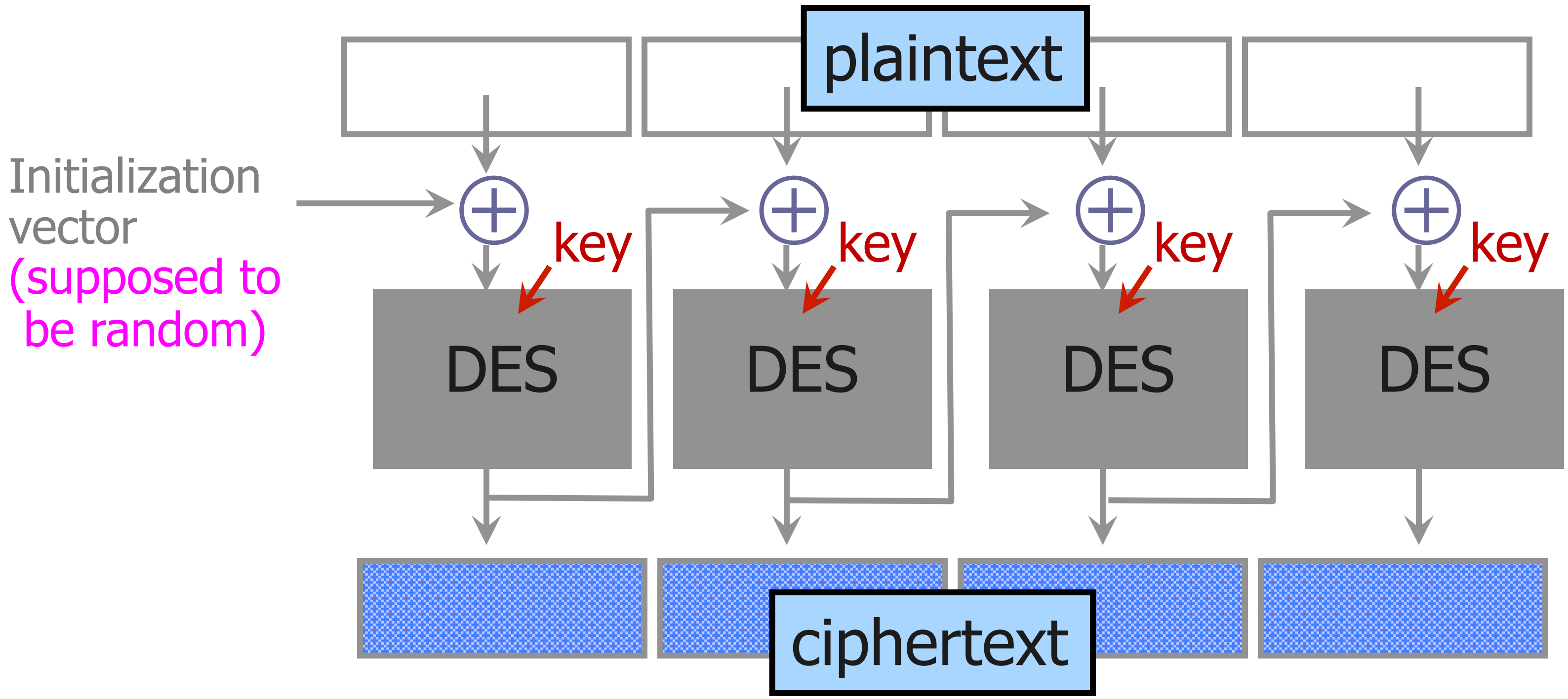


Choosing the Initialization Vector

- Key used only once
  - No IV needed (can use IV=0)
- Key used multiple times
  - Best: fresh, random IV for every message
  - Can also use unique IV (eg, counter), but then the first step in CBC mode must be IV' $\leftarrow$ E(k, IV)
    - Example: Windows BitLocker
    - May not need to transmit IV with the ciphertext
- Multi-use key, unique messages
  - Synthetic IV: IV $\leftarrow$ F(k', message)
    - F is a cryptographically secure keyed pseudorandom function

CBC and Electronic Voting

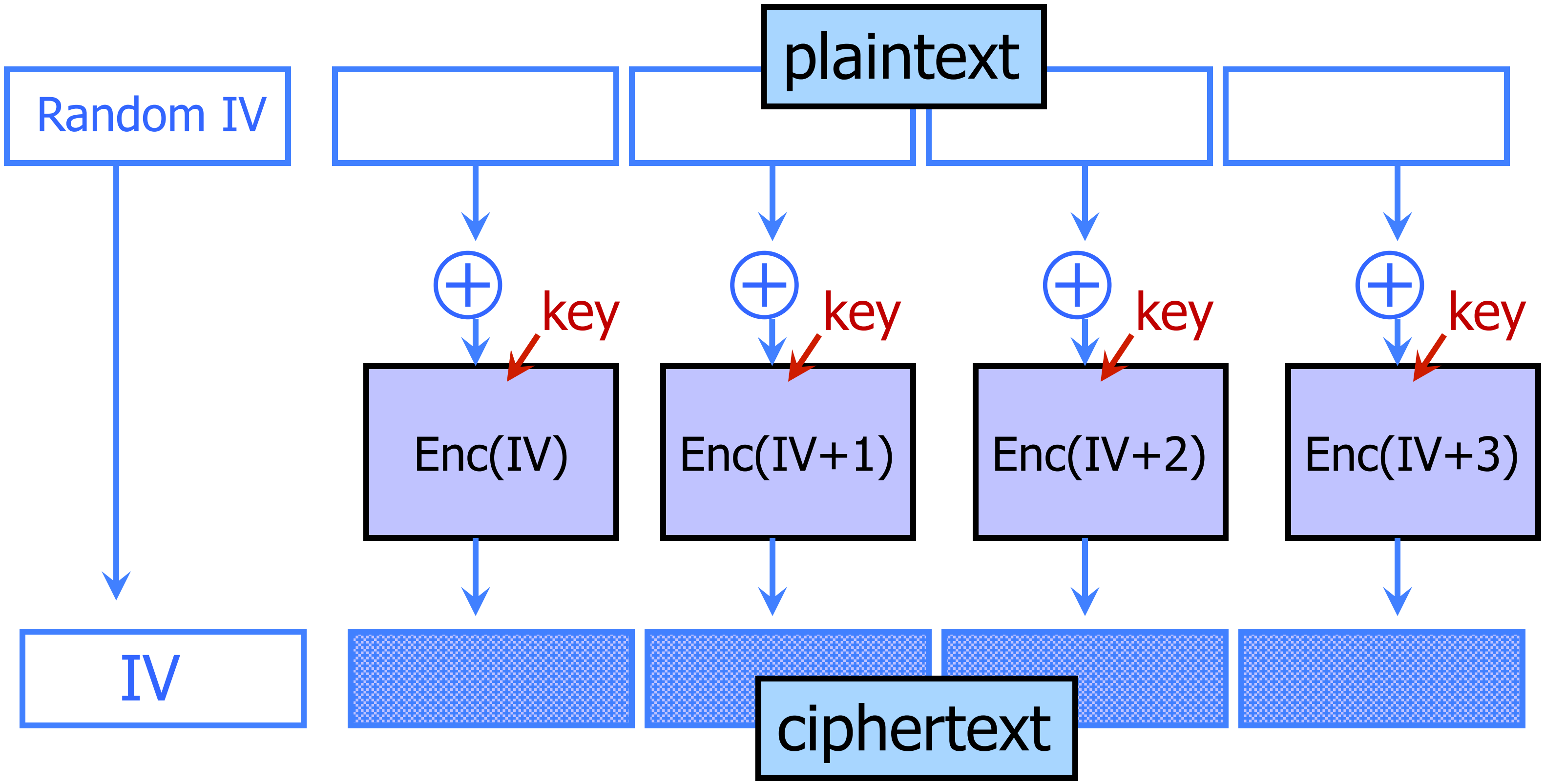
[Kohno, Stubblefield, Rubin, Wallach]



Found in the source code for Diebold voting machines:

```
DesCBCEncrypt((des_c_block*)tmp, (des_c_block*)record.m_Data,
        totalSize, DESKEY, NULL, DES_ENCRYPT)
```

CTR (Counter Mode)



- Still does not guarantee integrity
- Fragile if counter repeats

When Is a Cipher "Secure"?

- Hard to recover plaintext from ciphertext?
  - What if attacker learns only some bits of the plaintext? Some function of the bits? Some partial information about the plaintext?
- Fixed mapping from plaintexts to ciphertexts?
  - What if attacker sees two identical ciphertexts and infers that the corresponding plaintexts are identical?
  - What if attacker guesses the plaintext – can he verify his guess?
  - Implication: encryption must be randomized or stateful

How Can a Cipher Be Attacked?

- Attackers knows ciphertext and encryption algthm
  - What else does the attacker know? Depends on the application in which the cipher is used!

- Known-plaintext attack (stronger)
  - Knows some plaintext-ciphertext pairs
- Chosen-plaintext attack (even stronger)
  - Can obtain ciphertext for any plaintext of his choice
- Chosen-ciphertext attack (very strong)
  - Can decrypt any ciphertext <u>except</u> the target
  - Sometimes very realistic

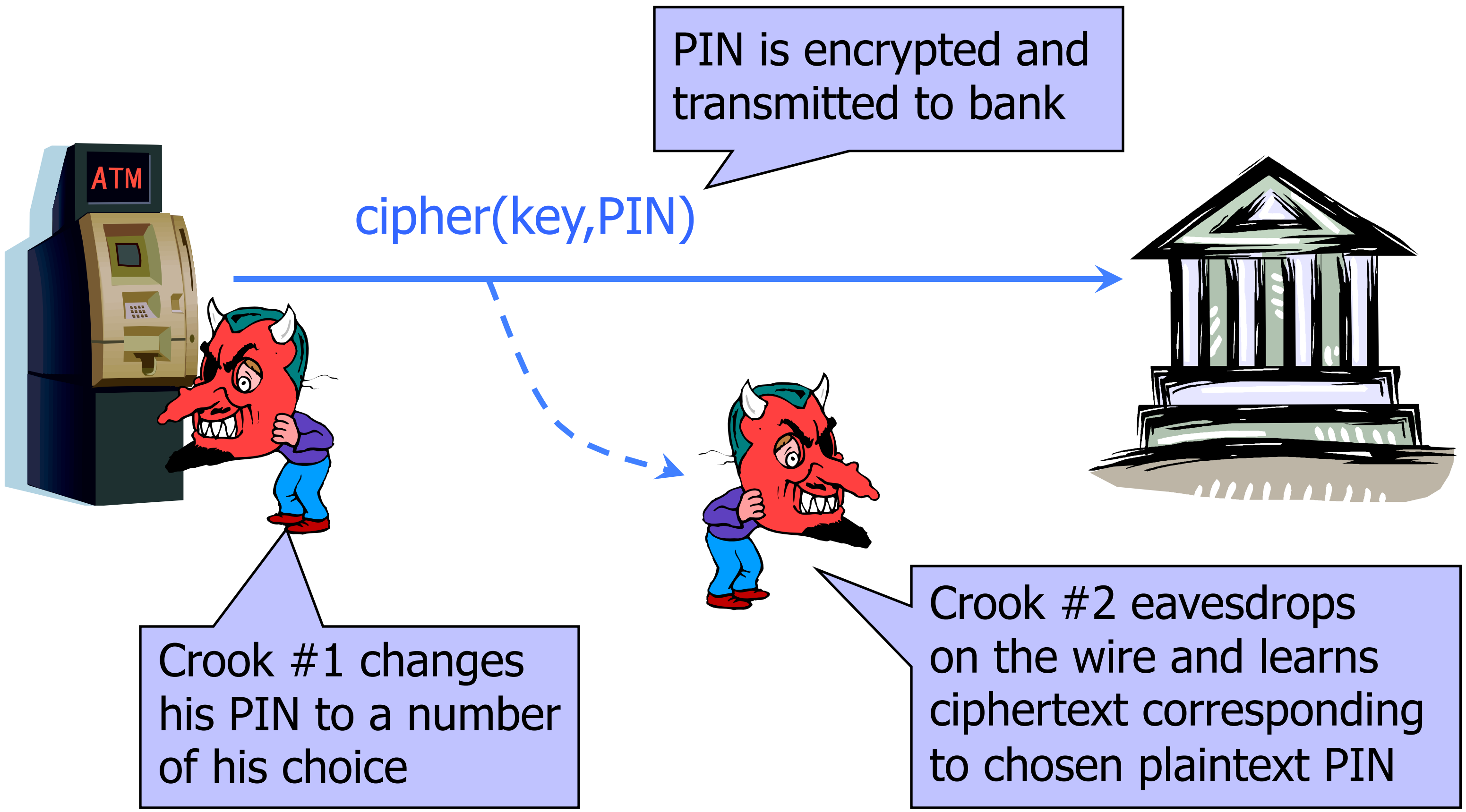
Known-Plaintext Attack

[From “The Art of Intrusion”]

Extracting password from an encrypted PKZIP file …

- “… I opened the ZIP file and found a `logo.tif’ file, so I went to their main Web site and looked at all the files named `logo.tif.’ I downloaded them and zipped them all up and found one that matched the same checksum as the one in the protected ZIP file”
- With known plaintext, PkCrack took 5 minutes to extract the key
  - Biham-Kocher attack on PKZIP stream cipher

Chosen-Plaintext Attack



… repeat for any PIN value

Very Informal Intuition

Minimum security requirement for a modern encryption scheme

- Security against chosen-plaintext attack
  - Ciphertext leaks no information about the plaintext
  - Even if the attacker correctly guesses the plaintext, he cannot verify his guess
  - Every ciphertext is unique, encrypting same message twice produces completely different ciphertexts
- Security against chosen-ciphertext attack
  - Integrity protection – it is not possible to change the plaintext by modifying the ciphertext

The Chosen-Plaintext Game

- Attacker does not know the key
- He chooses as many plaintexts as he wants, and receives the corresponding ciphertexts
- When ready, he picks two plaintexts M_0 and M_1
  - He is even allowed to pick plaintexts for which he previously learned ciphertexts!
- He receives either a ciphertext of M_0, or a ciphertext of M_1
- He wins if he guesses correctly which one it is

Meaning of "Leaks No Information"

- Idea: given a ciphertext, attacker should not be able to learn even a single bit of useful information about the plaintext
- Let Enc(M_0,M_1,b) be a "magic box" that returns encrypted M_b

  0 or 1

  - Given two plaintexts, the box always returns the ciphertext of the left plaintext or right plaintext
  - Attacker can use this box to obtain the ciphertext of any plaintext M by submitting M_0=M_1=M, or he can try to learn even more by submitting $M_0 \neq M_1$
- Attacker's goal is to learn just this one bit b

Chosen-Plaintext Security

- Consider two experiments (A is the attacker)

  Experiment 0
  A interacts with Enc(-,-,0) and outputs his guess of bit b

  Experiment 1
  A interacts with Enc(-,-,1) and outputs his guess of bit b

  - Identical except for the value of the secret bit
  - b is attacker's guess of the secret bit
- Attacker's advantage is defined as

  | Prob(A outputs 1 in Exp0) - Prob(A outputs 1 in Exp1)) |
- Encryption scheme is chosen-plaintext secure if this advantage is negligible for any efficient A

Simple Example

- Any deterministic, stateless symmetric encryption scheme is insecure
  - Attacker can easily distinguish encryptions of different plaintexts from encryptions of identical plaintexts
  - This includes ECB mode of common block ciphers!

<u>Attacker A interacts with Enc(-,-,b)</u>

Let X,Y be any two different plaintexts

$C_1 \leftarrow Enc(X,X,b)$; $C_2 \leftarrow Enc(X,Y,b)$;

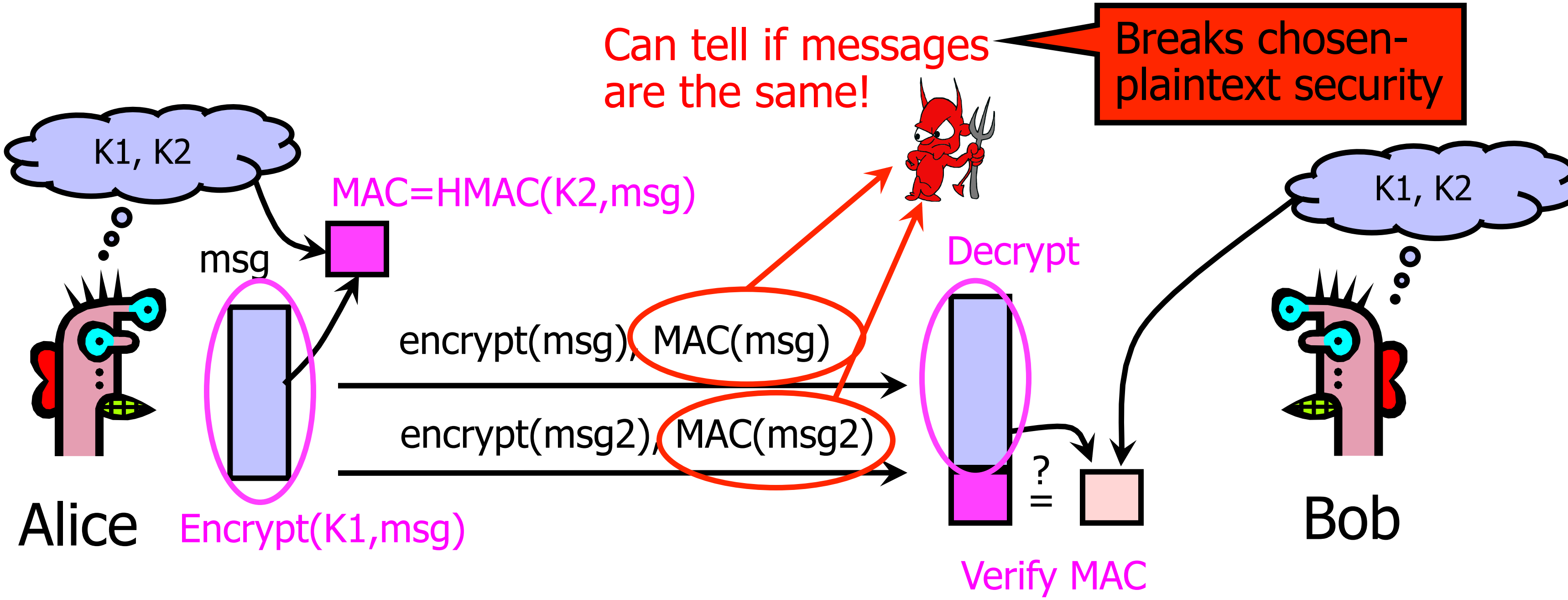
If $C_1=C_2$ then b=0 else b=1

- The advantage of this attacker A is 1

Prob(A outputs 1 if b=0)=0   Prob(A outputs 1 if b=1)=1

Encrypt + MAC

Goal: confidentiality + integrity + authentication

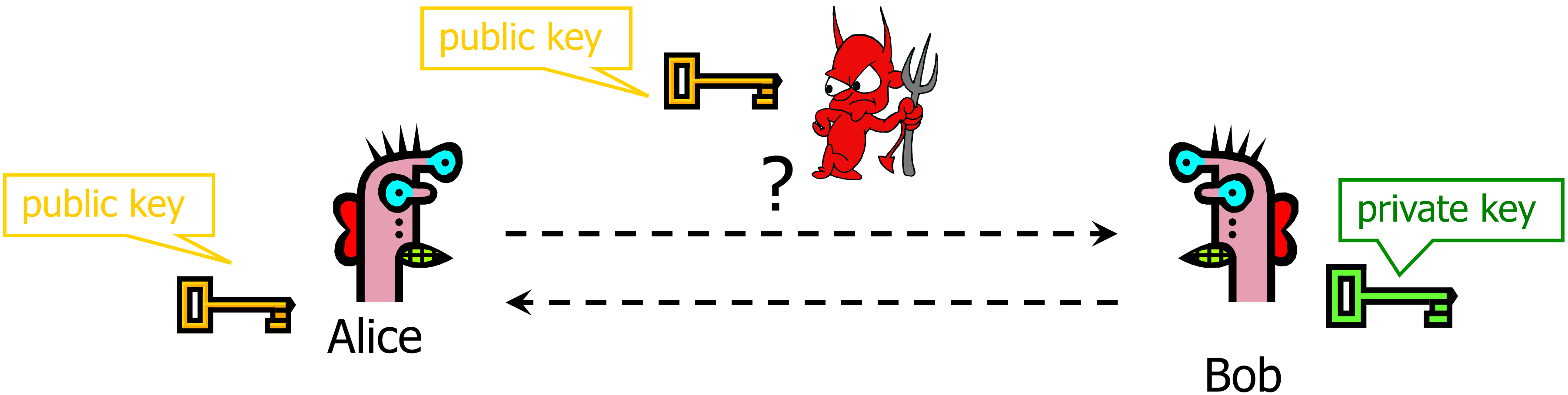


MAC is deterministic: messages are equal $\Rightarrow$ their MACs are equal

Solution: Encrypt, <u>then</u> MAC (or MAC, then encrypt)

Overview of Public-Key Cryptography

Public-Key Cryptography



Given: Everybody knows Bob's public key

- How is this achieved in practice?

Only Bob knows the corresponding private key

Goals: 1. Alice wants to send a message that only Bob can read

2. Bob wants to send a message that only Bob could have written

Applications of Public-Key Crypto

- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know the secret key to encrypt
  - Only someone who knows the private key can decrypt
  - Secret keys are only stored in one place
- Digital signatures for authentication
  - Only someone who knows the private key can sign
- Session key establishment
  - Exchange messages to create a secret session key
  - Then switch to symmetric cryptography (why?)

Public-Key Encryption

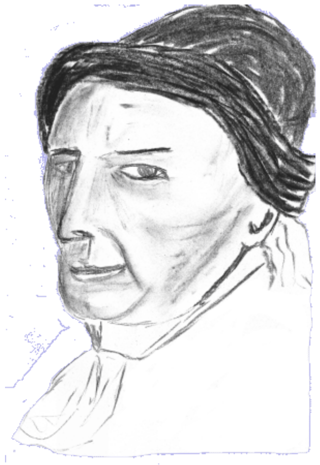
- **Key generation:** computationally easy to generate a pair (public key PK, private key SK)
- **Encryption:** given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- **Decryption:** given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to learn anything about M from C without SK
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M

Some Number Theory Facts

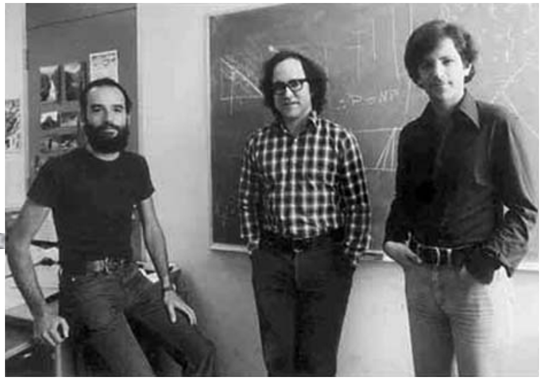
- Euler totient function $\varphi(n)$ where $n \geq 1$ is the number of integers in the $[1,n]$ interval that are relatively prime to n
  - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
- Euler's theorem:

  if $a \in Z_n^*$, then $a^{\varphi(n)} \equiv 1 \bmod n$
- Special case: <u>Fermat's Little Theorem</u>

  if p is prime and $\gcd(a,p)=1$, then $a^{p-1} \equiv 1 \bmod p$



RSA Cryptosystem



[Rivest, Shamir, Adleman 1977]

- **Key generation:**
  - Generate large primes p, q
    - At least 2048 bits each… need primality testing!
  - Compute n=pq
    - Note that $\varphi(n)=(p-1)(q-1)$
  - Choose small e, relatively prime to $\varphi(n)$
    - Typically, e=3 (may be vulnerable) or $e=2^{16}+1=65537$ (why?)
  - Compute unique d such that $ed \equiv 1 \bmod \varphi(n)$
  - Public key = (e,n); private key = d
- **Encryption** of m: $c = m^e \bmod n$
- **Decryption** of c: $c^d \bmod n = (m^e)^d \bmod n = m$

Why RSA Decryption Works

- $e \cdot d \equiv 1 \bmod \varphi(n)$
- Thus $e \cdot d = 1 + k \cdot \varphi(n) = 1 + k(p-1)(q-1)$ for some k
- If gcd(m,p)=1, then by Fermat's Little Theorem, $m^{p-1} \equiv 1 \bmod p$
- Raise both sides to the power k(q-1) and multiply by m, obtaining $m^{1+k(p-1)(q-1)} \equiv m \bmod p$
- Thus $m^{ed} \equiv m \bmod p$
- By the same argument, $m^{ed} \equiv m \bmod q$
- Since p and q are distinct primes and $p \cdot q = n$, $m^{ed} \equiv m \bmod n$

Why Is RSA Secure?

- RSA problem: given c, n=pq, and e such that gcd(e,(p-1)(q-1))=1, find m such that $m^e=c \bmod n$
  - In other words, recover m from ciphertext c and public key (n,e) by taking e^{th} root of c modulo n
  - There is no known efficient algorithm for doing this
- Factoring problem: given positive integer n, find primes $p_1, \ldots, p_k$ such that $n=p_1^{e_1}p_2^{e_2}\ldots p_k^{e_k}$
- If factoring is easy, then RSA problem is easy, but may be possible to break RSA without factoring n

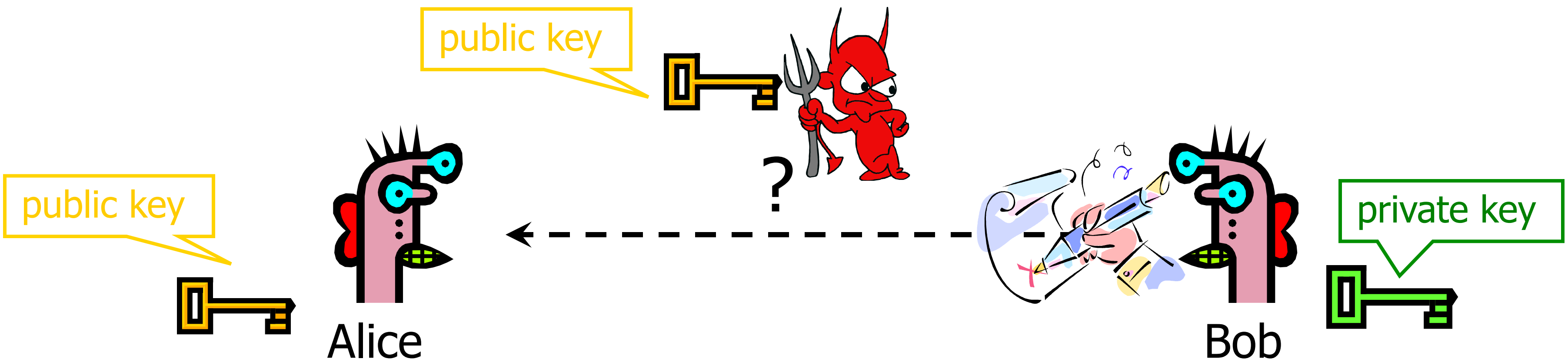
“Textbook” RSA Is Bad Encryption

- Deterministic
  - Attacker can guess plaintext, compute ciphertext, and compare for equality
  - If messages are from a small set (for example, yes/no), can build a table of corresponding ciphertexts
- Can tamper with encrypted messages
  - Take an encrypted auction bid c and submit $c(101/100)^e \bmod n$ instead
- Does not provide semantic security (security against chosen-plaintext attacks)

Integrity in RSA Encryption

- "Textbook" RSA does not provide integrity
  - Given encryptions of m_1 and m_2, attacker can create encryption of $m_1 \cdot m_2$
    - $(m_1^e) \cdot (m_2^e) \bmod n \equiv (m_1 \cdot m_2)^e \bmod n$
  - Attacker can convert m into m^k without decrypting
    - $(m^e)^k \bmod n \equiv (m^k)^e \bmod n$
- In practice, OAEP is used: instead of encrypting M, encrypt $M \oplus G(r) \, ; \, r \oplus H(M \oplus G(r))$
  - r is random and fresh, G and H are hash functions
  - Resulting encryption is plaintext-aware: infeasible to compute a valid encryption without knowing plaintext
    - … if hash functions are "good" and RSA problem is hard

Digital Signatures: Basic Idea



<u>Given</u>: Everybody knows Bob's public key

Only Bob knows the corresponding private key

<u>Goal</u>: Bob sends a "digitally signed" message

1. To compute a signature, must know the private key
2. To verify a signature, only the public key is needed

RSA Signatures

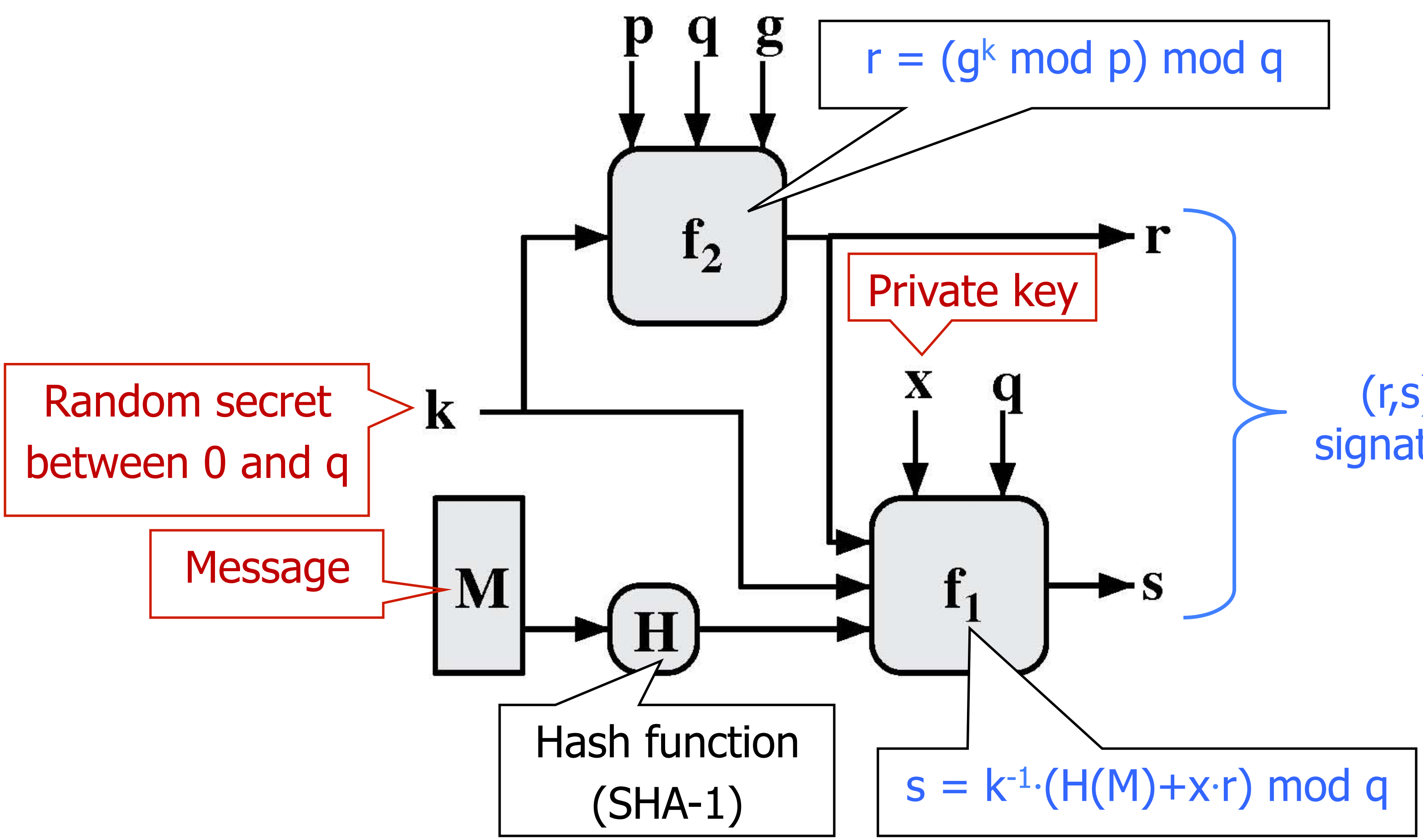
- Public key is (n,e), private key is d
- To sign message m: $s = hash(m)^d \bmod n$
  - Signing and decryption are the same mathematical operation in RSA
- To verify signature s on message m:

  $s^e \bmod n = (hash(m)^d)^e \bmod n = hash(m)$
  - Verification and encryption are the same mathematical operation in RSA
- Message must be hashed and padded (why?)

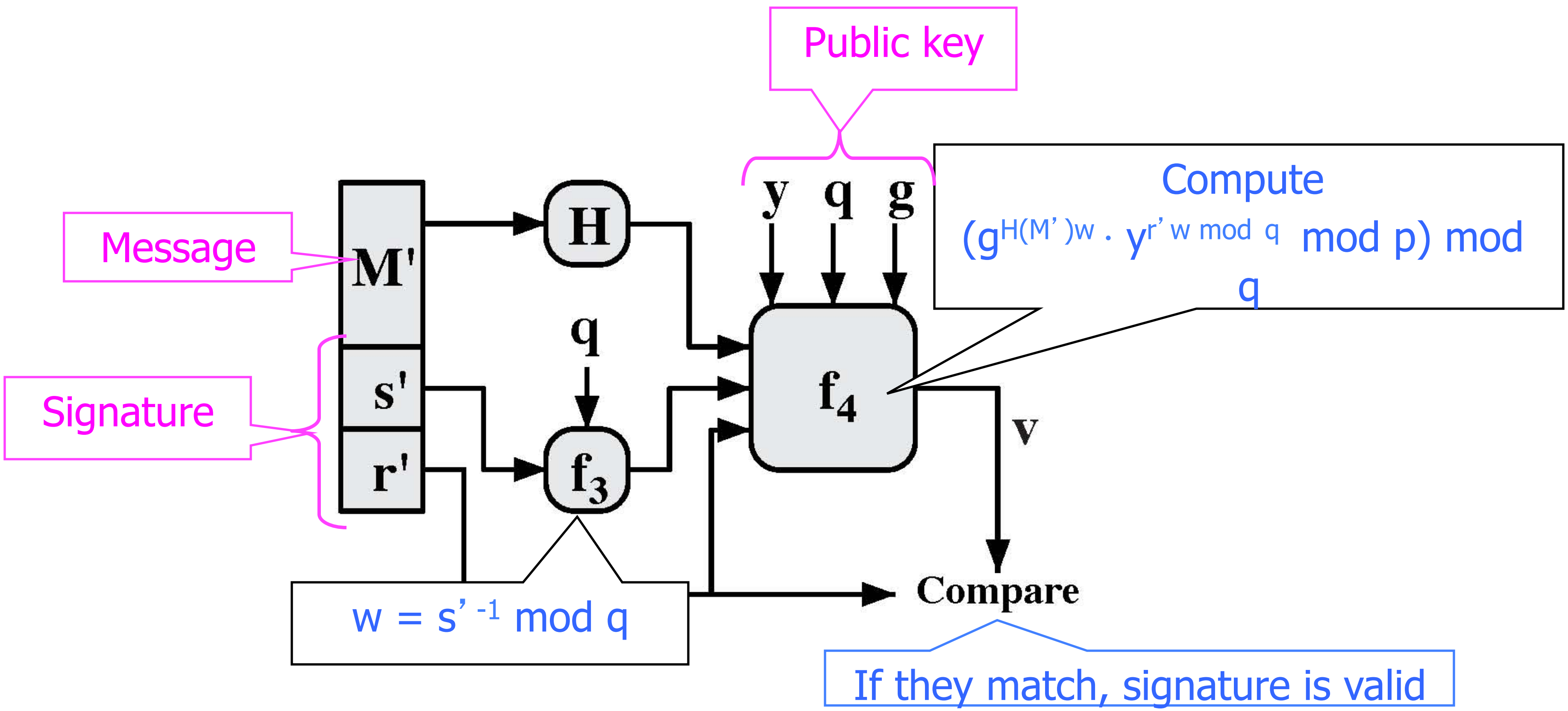
Digital Signature Algorithm (DSA)

- U.S. government standard (1991-94)
  - Modification of the ElGamal signature scheme (1985)
- Key generation:
  - Generate large primes p, q such that q divides p-1
    - $2^{159} < q < 2^{160}$, $2^{511+64t} < p < 2^{512+64t}$ where $0 \le t \le 8$
  - Select $h \in Z_p^*$ and compute $g = h^{(p-1)/q} \bmod p$
  - Select random x such $1 \le x \le q-1$, compute $y = g^x \bmod p$
- Public key: $(p, q, g, g^x \bmod p)$, private key: x
- Security of DSA requires hardness of discrete log
  - If one can take discrete logarithms, then can extract x (private key) from g^x mod p (public key)

DSA: Signing a Message



DSA: Verifying a Signature



Why DSA Verification Works

- If (r,s) is a valid signature, then
  $r \equiv (g^k \bmod p) \bmod q$; $s \equiv k^{-1} \cdot (H(M)+x \cdot r) \bmod q$
- Thus $H(M) \equiv -x \cdot r+k \cdot s \bmod q$
- Multiply both sides by $w=s^{-1} \bmod q$
- $H(M) \cdot w + x \cdot r \cdot w \equiv k \bmod q$
- Exponentiate g to both sides
- $(g^{H(M) \cdot w + x \cdot r \cdot w} \equiv g^k) \bmod p \bmod q$
- In a valid signature, $g^k \bmod p \bmod q = r$, $g^x \bmod p = y$
- Verify $g^{H(M) \cdot w} \cdot y^{r \cdot w} \equiv r \bmod p \bmod q$

Security of DSA

- Can't create a valid signature without private key
- Can't change or tamper with signed message
- If the same message is signed twice, signatures are different
  - Each signature is based in part on random secret k
- Secret k must be different for each signature!
  - If k is leaked or if two messages re-use the same k, attacker can recover secret key x and forge any signature from then on

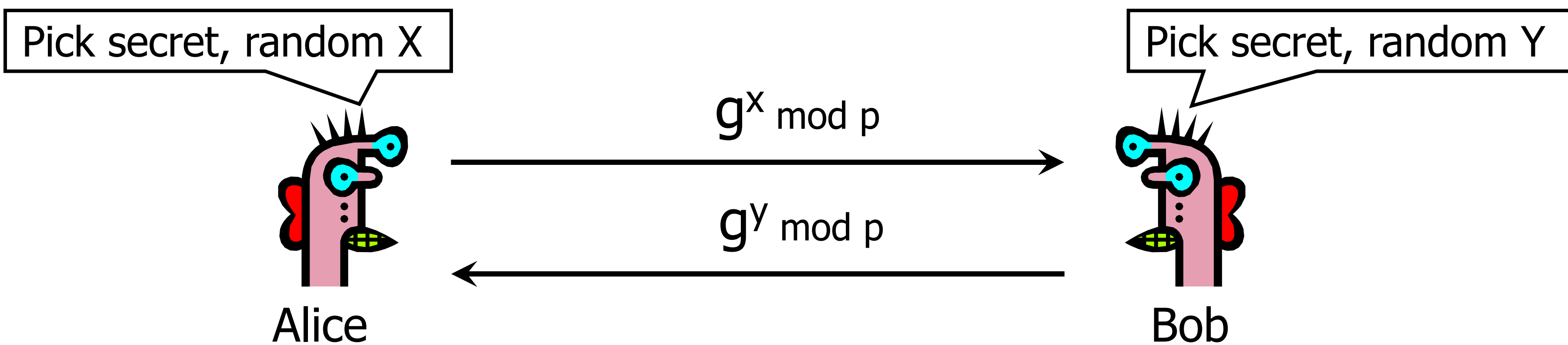
PS3 Epic Fail

- Sony uses ECDSA algorithm to sign authorized software for Playstation 3
  - Basically, DSA based on elliptic curves
    … with the same random value in every signature
- Trivial to extract master signing key and sign any homebrew software – perfect "jailbreak" for PS3
- Announced by George "Geohot" Hotz and Fail0verflow team in Dec 2010

Q: Why didn't Sony just revoke the key?

Diffie-Hellman Protocol


- Alice and Bob never met and share no secrets
- Public info: p and g
  - p is a large prime number, g is a generator of Z_p^*
    - $Z_p^*=\{1, 2 \ldots p-1\}$; $\forall a \in Z_p^*\ \exists i$ such that $a=g^i \bmod p$



Compute $k=(g^y)^x=g^{xy} \bmod p$

Compute $k=(g^x)^y=g^{xy} \bmod p$

Why Is Diffie-Hellman Secure?

- Discrete Logarithm (DL) problem:
  given g^x mod p, it's hard to extract x
  - There is no known efficient algorithm for doing this
  - This is not enough for Diffie-Hellman to be secure!
- Computational Diffie-Hellman (CDH) problem:
  given g^x and g^y, it's hard to compute g^{xy} mod p
  - … unless you know x or y, in which case it's easy
- Decisional Diffie-Hellman (DDH) problem:

  given g^x and g^y, it's hard to tell the difference between g^{xy} mod p and g^r mod p where r is random

Properties of Diffie-Hellman

- Assuming DDH problem is hard, Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  - Eavesdropper can't tell the difference between the established key and a random value
  - Can use the new key for symmetric cryptography
- Basic Diffie-Hellman protocol does not provide authentication
  - IPsec combines Diffie-Hellman with signatures, anti-DoS cookies, etc.

Advantages of Public-Key Crypto

- Confidentiality without shared secrets
  - Very useful in open environments
  - Can use this for key establishment, avoiding the "chicken-or-egg" problem
    - With symmetric crypto, two parties must share a secret before they can exchange secret messages
- Authentication without shared secrets
- Encryption keys are public, but must be sure that Alice's public key is really <u>her</u> public key
  - This is a hard problem… Often solved using public-key certificates

Disadvantages of Public-Key Crypto

- Calculations are 2-3 orders of magnitude slower
  - Modular exponentiation is an expensive computation
  - Typical usage: use public-key cryptography to establish a shared secret, then switch to symmetric crypto
    - SSL, IPsec, most other systems based on public crypto
- Keys are longer
  - 2048 bits (RSA) rather than 128 bits (AES)
- Relies on unproven number-theoretic assumptions
  - Factoring, RSA problem, discrete logarithm problem, decisional Diffie-Hellman problem…