3137 Data Structures and Algorithms in C++

Lecture 9

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Announcements

- ok we will be wrapping up with graphs and advanced data structures
- please check your grades/work submission online
- Final Exam : online (same as midterm). Will be available from Thursday 5pm - Friday 11pm
  - should be able to be completed in two and half hours or less

Outline

- Graph
  - search
  - sort
  - expansion
  - path exploration
- algorithm designs
- game trees
- homework help

- Reading:
  - Chapter 9, 10.1, 10.2

3

Definition

- A graph G=(V,E) consists of a set of vertices (nodes) V and edges E where each edge is a pair (v,w) such that v,w in V

4

- Directed Graph – A graph where vertices have direction
- Undirected Graph
- Weights – in addition can associate a weight with each edge

5

Example

- think of a map
  - cities (nodes)
  - roads (edges)
  - mileage (weights)
  - path between two cities
  - total road length
  - shortest path between any two points
  - capacity

6

Definition

- Path = a sequence of vertices v_1, v_2, v_3.. such that the vertices are adjacent
- Length = number of edges
- Loop = edges that starts and end at the same vertex

7

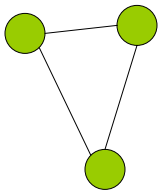
More definitions

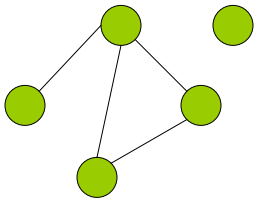
- Simple path = a path with distinct vertices
  - that doesn't cross itself
- Cycle = path with the first and last vertex being the same

8

definitions

- connected graph = undirected graph where there is a path from any vertex to any other

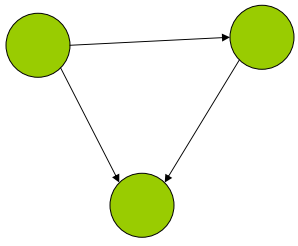




9

Strongly Connected

- directed graph where path from any vertex to any other



?????

10

more definitions

- weakly connected – directed graph which would have been connected if it was undirected
- Complete graph = every vertex connected to every other

11

Point

- Very general DS
  - very useful for real world problem representation
- Air transit
  - airports
  - flights
  - weight = cost
- Can answer:
  - what are the cheapest flights ?
  - shorted flights ?
  - where we should add flights ?
  - can we reach every city ?

12

one last definition

- Degree of a node:
  - undirected
    - number of connections of the node
  - Directed
    - in degree
    - out degree
- Sparse Graph
  - E <<<<< V
- Dense Graph
  - E <= V

13

questions ?

- so how to code graph so arbitrary node labels ?
- how to grow over time ? (no preset number of nodes ?)

14

code

```
DirectedWGraph dirG(20);
dirG.add("a","b",10);
dirG.add("b","c",5);
…
dirG.isDirEdge("a","c");
dirG.getW("a","b");

dirG.getEdgeCount();
dirG.getNodeCount();
```

15

Arrays implementation

- 2 dimensional n by n array
- weights are in location M,N to show directed edge from M to N
- -1 for no edge (if possible 0 weight)
- diagonal is M to itself
- great for dense graphs

16

Linked List implementation

- List representation called an adjacency list
- Array of nodes
- linked list of edges
- how hashtable fits in here?

17

Resources

- how much space (in terms of V and E) would it take to represent a graph on either implementation ?
- what does that tell you about your implementation choice ??

18

further on…

- once you have your information in the graph, how do you save it ??
  - serialization
  - file representation
    - compact
    - dump
    - considerations
- Data in graphs can be operated on in a variety of ways
  - Printing
  - Would like to support is searching the graph
    - We want to visit nodes without wasting time or missing any

19

Expansion

- starting from any node on a graph, we would like to reach others
- List method
- DFS
  - depth first search
- BFS
  - breadth first search

20

Expansion Algorithm

- have a list of vertices and nodes
- choose one from the list
- put into DS X
- while X is not empty
  - choose y out of X
    - if not visited, visit
      - for each adj of y if not visited put into X

- what is X ??

21

- STACK
  - DFS
- Queue
  - BFS

22

Topological sorting

- sometimes its useful to see an ordered output of a graph, so there is some type of relationship information present in the output
- Topological sorting
  - an ordering of nodes in a DAG such that if exists a path from v to w, then w appears after v in ordering

23

Algorithm

```
for(i=0;i< num_vertices; i++) {
   v = findnewIndegreeZero()
   if (v==NULL)
      return;
   cout<<v;

   for(j=0;j<children(v);j++)
      nextchild(v).indegree--;
      }
```

24

complexity ?

- O(v+e)
- what do you do about zero degree nodes ?

25

bottom line

- ok so what else can we do with graphs ??
  - relationship algorithms
  - path algorithms
  - minimum connectivity

26

path exploration

- We mentioned finding the shortest path between two nodes
  - price line tickets
  - so how would you do it ?? if you hadn't read the chapter ??

27

goal

- Given a weighted graph G(V,E) and a distinguished vertex s, find shortest path from s to any other node in the graph
- although usually want s to a specific t it's the same computation to every other one

28

simplest

- easiest is if only look at segments (unweighted edges) and want the least number of edges between s and every other node

29

un weighted shortest path

- O(|V|+|E|)

```
int pathlen = 0;
foreach(v)
   v.distance = ∞;
s.distance =0;
while(pathlen < numvertices) {
   foreach node.distance == pathlen;
      foreach adj node.distance= ∞
         distance = pathlen +1
   pathlen++;
}
```

30

- what expansion strategy was this ?
- how to get the actual cheapest path ?

31

- now for weighted path
  - we want cheapest weighted path
  - Dijksta's algorithm
    - greedy algorithm
    - doesn't deal with negative weighted paths….

32

code

33

definition

- spanning tree
  - set of edges in the graph which will connect all nodes
  - let us use a directed graph as example
  - what is the simplest example ?

34

harder

- most of the time will have weight associated with the graph
- will want the minimum cost spanning tree connecting all nodes
- Example : laying down electric lines

35

- any ideas on generating this ?

36

solutions

- prim's algorithm
  - greedy solution
  - $O(V^2)$ but can be done in O(E log V) with heap
- kruskals algorithm
  - bottom up solution
  - O(E log V)

37

prim's algorithm

- V is your list of nodes
- choose random node to start MST
- while V notEmpty()
  - choose cheapest edge outgoing from your MST
  - remove from V, add to MST
  - avoid cycles

38

kruskal's algorithm

- look at all edges
- for use the cheapest edge to connect two nodes
  - as long as no cycles created

39

graph issues

- make sure you are clear on:
- how would you represent a graph from the code point of view
- how would you save a graph?
- load a graph?

40

flow networks

- sometimes we are interested in graph as a capacity problem:
  - Represent graph of capacity between two points
  - Want to see what is the max flow the graph can carry between the points
- application: oil pipeline, electricity grid, etc

41

- Given a graph with a flow capacity, what strategies can you think will allow you
  - any flow
  - max flow
  - min flow

42

max flow algorithm

- hand out
- will be using extra graph for book keeping
- will review next time

43

algorithm design

- for solving any problem the algorithm is important
- but so is the data structure
- so how do you invent an algorithm ??
- different classes of algorithms we have seen

44

Greedy Algorithm

- many examples that we have seen
- “grab what you can and run “ philosophy
- settle for an answer not quite best, hoping we get close

45

problem

- how would you program a cash register to spit out correct change for a transaction ?
- lets say we want to minimize the number of coins we are returning

46

Approaches

- usually largest coin first
- so for 15 cents ?
- runtime ?
- what would happen if we have a 12 cent coin ?

47

from the past

- huffman was an example of a greedy algorithm
- that is why we needed to keep the tree local to the encryption so we can decrypt the data

48

packing problem

- say you are writing the software system for UPS/FEDEX how do you load the trucks going to destination x ??
- given N items of size s_1, s_2, ..s_n
- fewest number of bins to fit them in which each bin has a capacity
- example capacity 1,
- .2 .5 .4 .7 .1 .3 .8

49

constraints

- online vs offline
- real world online
  - what would be an algorithm
  - can you prove its optimal ?

50

strategies

- next fit
  - fit in last or create new bin
  - runtime ?
  - we can prove that if optimal is M, it will not use more than 2M bins
  - can you do the proof ??

51

proof

- consider 2 adjacent bins
- the sum must be greater than capacity (else they would be placed in one bin)
- which means we are wasting < ½ the capacity

52

first fit

- scan all current bin and place in first bin which can hold the current item
  - runtime ?
  - can you improve it ?
  - it wont use more than 17/10 M bins ☺

53

best fit

- find where it fits in best
  - runtime ?
  - worst case is exactly as first fit
  - easy to code

54

- what can you say about the offline version of the problem ?
- can we get to optimal ?
- how ?

55

divide and conquer

- we've seen many examples
- runtime typical ?

56

problem

- given a set of points in 2 dimensional space
- can you find closest pair of points to any point ?
- hmmm sounds familiar ??
- will talk about it next time
  - do reading, catch up to homeworks please

57