Homework 4

Data Structures and Algorithms in C++
Shlomo Hershkop
Department of Computer Science
Columbia University
Summer 2006

Due Sunday, Aug 6 11pm

Theory (100 points)

1) What is the maximum number of inversions that can be removed by swapping a pair of distinct elements in a list? Express the result in terms of the distance between the elements in the list.

2) Suppose we are given a sequence S of n elements, each of which is colored red or blue. Assuming S is represented as an array, give an in-place method for ordering S so that all the blue elements are listed before all the red elements. Can you extend your approach to three colors?

3) Graph $f1(n) = n \log n$, $f2(n) = n^{1.5}$, and $f3(n) = n^2$ in the range $1 <= n <= 1000$ to visually compare their growth rates. Typically, the multiplicative constant factor in the running-time expression for an implementation of insertion sort will be less than the constant factors for Shell-sort or quicksort. How many times greater can the constant factor be for Shell sort to be faster than insertion sort when n = 1000? How many times greater can the constant factor be for quicksort to be faster than insertion sort when n = 1000?

4) Let A be a collection of objects. Describe an efficient method for converting A into a set. That is, remove all duplicates from A. what is the running time of this method?

5) Starting with a list containing the integers between 0 and 16 [0..16], show the result of the following sequence of instructions:
   union(14,15), union(12,13), union(9,15),
   union(10,13), union(7,8), union(8,15), union(6,13),
   union(5,13), union(3,13), union(2,1), union(16,0),
   union(0,2), union(13,15), union(2,15)

   a. performed so that union(x,y) makes y a child of x,
   b. performed by height
   c. performed by size

6) Shaker sort is an adoption of bubble sort. It alternates the direction in which the array elements are scanned during each pass. The first pass starts its scan with the first element, moving the larger element in each pair down the array. The second pass starts its scan with the next to last element, moving the smaller element in each pair up the array, and so on. Indicate what are the advantages of using a shaker sort over bubble sort.

7) If you found yourself in a maze, which graph traversal would you use, BFS (breadth first search) or DFS (depth first search) and why?

8) If you want to the Disjointed Set ADT new operations :
   a. Delete(x) – this will delete x from whatever set it is in
   b. Find_all(x) – returns a list of all members who are part of x's set

      analyze the run time for each of these operations and show a way how to speed them up as much as possible

9) When implementing quicksort, if the array contains lots of duplicates, is it better to perform a three-way partition (less than, equal to, greater than the pivot), to make smaller recursive calls instead of the standard 2 partitions(less than,greater than) Is this true or false? Justify your answer.

10) If you represent a graph as an adjacency list, how would you compute the in-degree and out-degree for any single node n ? what is the runtime ?