The SR Domain

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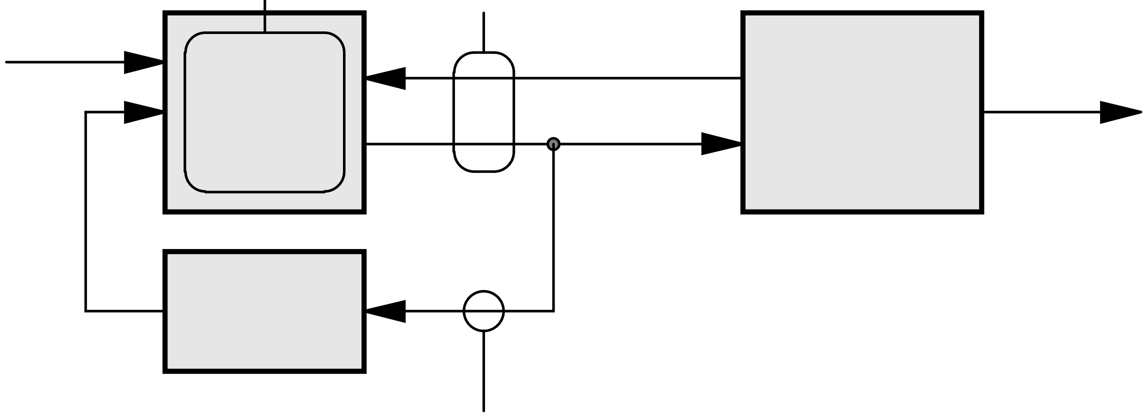
The SR Domain

- A specification scheme
  - Synchronous model of time
    - Predictable temporal behavior
    - Easier to design
    - Easier to analyze
  - Heterogeneous: compiler cannot see inside blocks
    - Mixing languages made easy
    - Allows separate compilation
    - Large designs are tractable
- Deterministic
  - Guaranteed by fixed-point semantics
- Fast, predictable execution time
  - Chaotic iteration-based scheme
  - Fully static scheduling

SR Systems

Zero-delay blocks compute
continuous functions

Instantaneous communication
with feedback

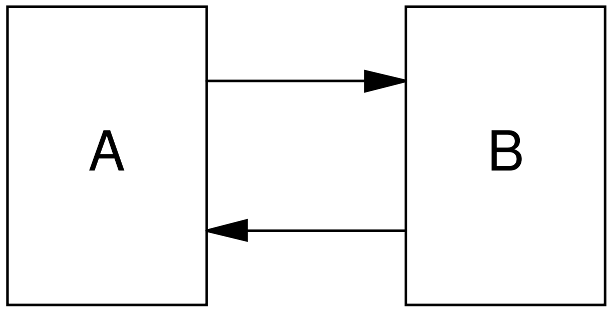


Single driver, multiple receiver wires
with values from flat CPOs

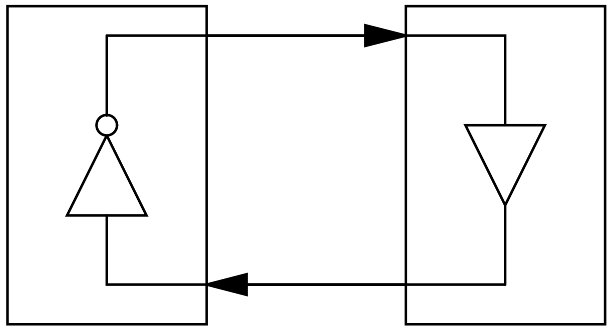
- Block functions may change between instants for time-varying behavior
- Block functions may be specified in any language

Zero Delay and Feedback

How to maintain determinism?



Which goes first?
Need an order-invariant semantics



Contradictory!
Need to attach meaning to such systems.

Dealing with Feedback

Why bother at all?
Answer: *Heterogeneity*

- Cycles are usually broken by delay elements *at the lowest level*
- Some schemes (e.g., Lustre) insist on this
- False feedback often appears at higher levels
- Data dependent cycles can appear when sharing resources
- *Virtually all cycles are "false," yet must be dealt with.*

Fixed-point Semantics are Natural for Synchronous Specifications with Feedback

Why a fixed point?

Self-reference:
The essence of a cycle

$$f(x_t) = x_t$$

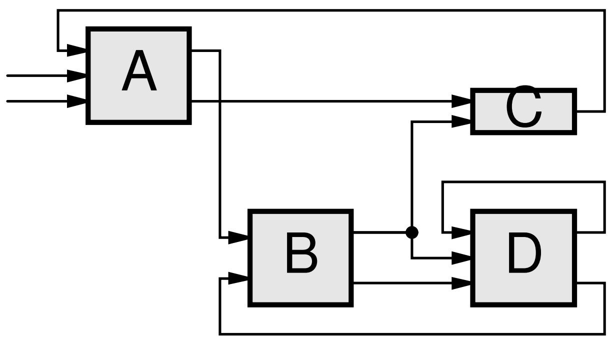
System function (composition of block functions)

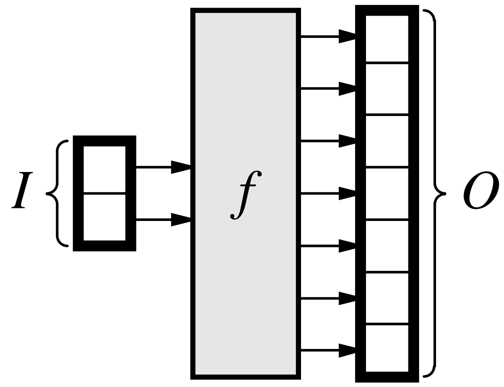
Vector of signals at time t (zero delay)

fixed point $\iff$ stable state

determinism $\iff$ unique solution

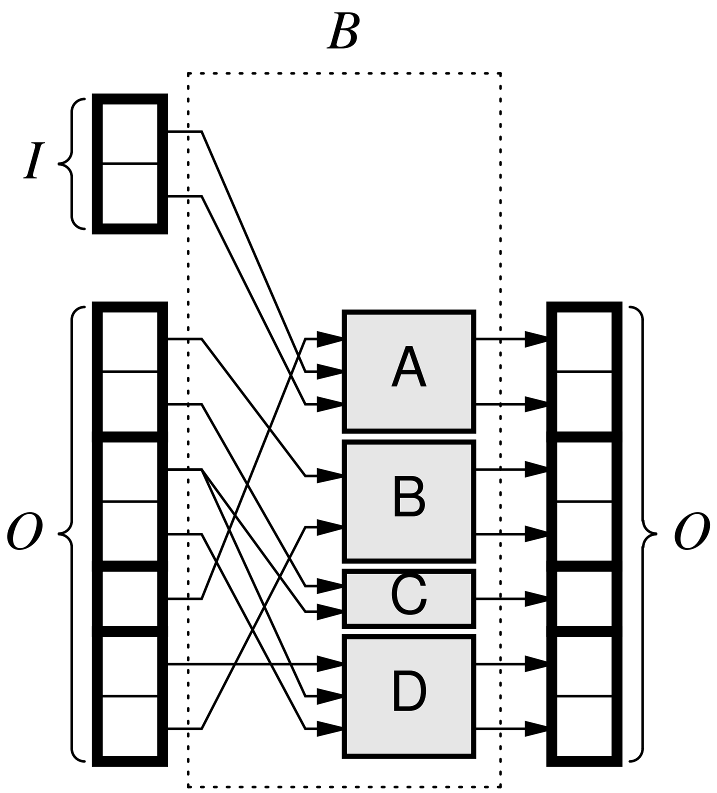
The Least Fixed Point of What?





Interpret as ↘ ↗ Take LFP

$$B(I, f(I)) = f(I)$$



Unique Least Fixed Point Theorem

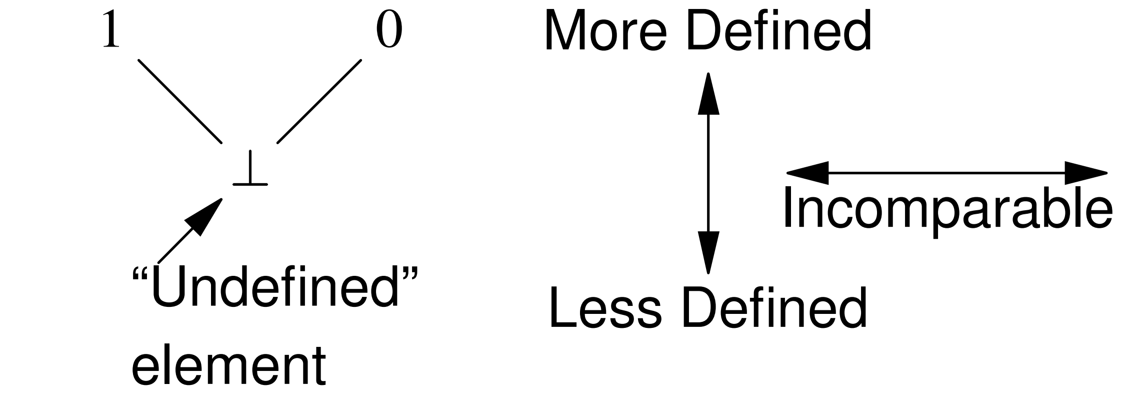
Recall:

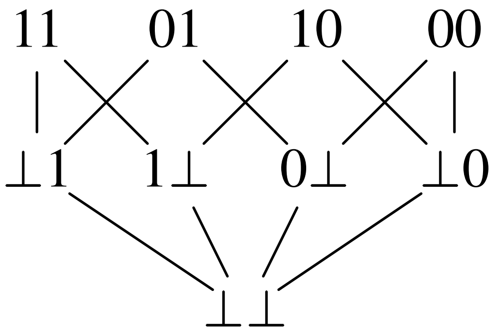
> A monotonic function on a complete partial order (with $\perp$) has a unique least fixed point.

What does it mean to make the system function f monotonic and the signal values a CPO?

Vector of Signals is a CPO

Values along an upward path grow more defined.



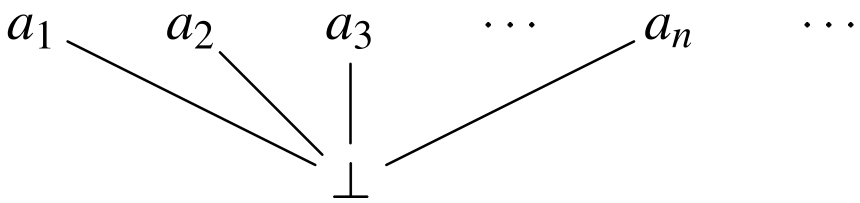


vector-valued extension

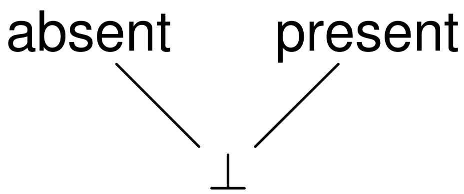
Formally, $x \sqsubseteq y$ if y is at least as defined as x.

Adding $\perp$ Is Enough

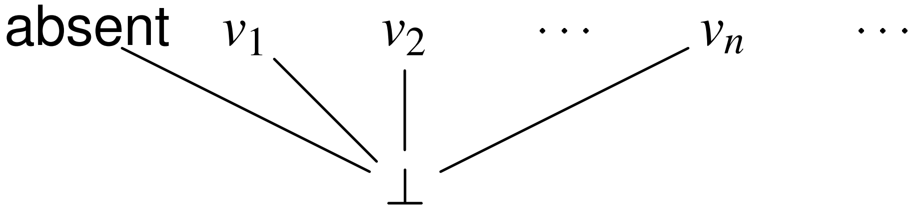
Any set $\{a_1, a_2, \ldots, a_n, \ldots\}$ can easily be "lifted" to give a flat partial order:



A CPO for signals with pure events:



A CPO for valued events:



Why not absent $\sqsubseteq$ present?

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present A then ... else ... end
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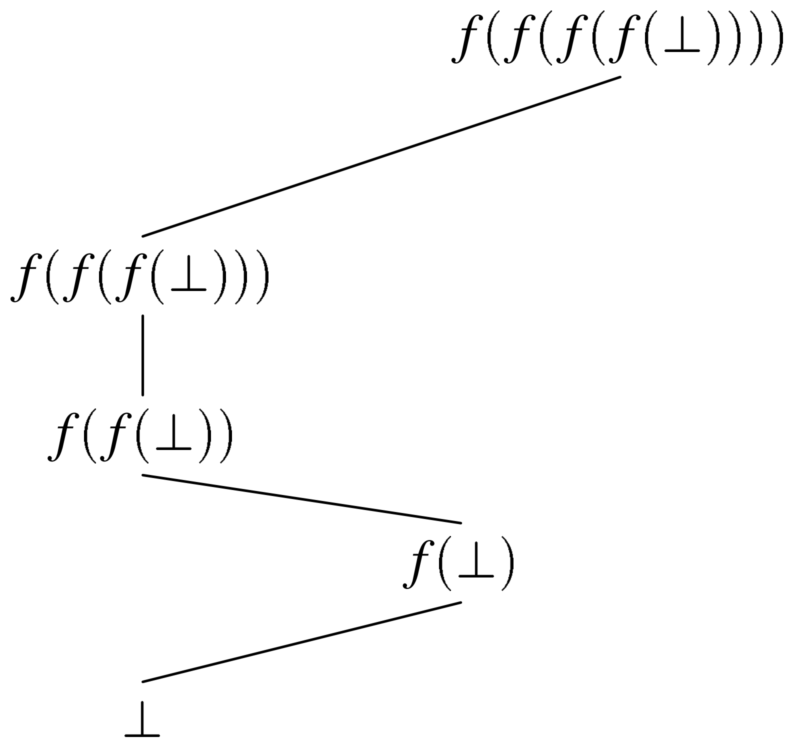
Violates monotonicity

Monotonic Block Functions

Giving a more defined input to a monotonic function always gives a more defined output.



Formally, $x \sqsubseteq y$ implies $f(x) \sqsubseteq f(y)$.

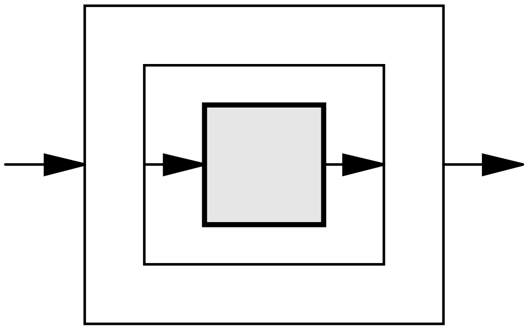
A monotonic function never recants ("changes its mind").

Many Languages Use Strict Functions, Which Are Monotonic

A strict function:

$$g(\underbrace{\ldots, \perp, \ldots}_{\text{inputs}}) = (\underbrace{\perp, \ldots, \perp}_{\text{outputs}})$$

Outside:
A strict monotonic function



Inside:
Simple "function call" semantics

Most common imperative languages only compute strict functions.

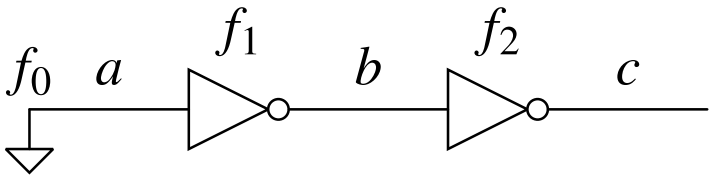
Danger: *Cycles of strict functions deadlock—fixed point is all ⊥—need some non-strict functions.*

A Simple Way to Find the Least Fixed Point

$$\perp \sqsubseteq f(\perp) \sqsubseteq f(f(\perp)) \sqsubseteq \cdots \sqsubseteq \text{LFP} = \text{LFP} = \cdots$$

For each instant,

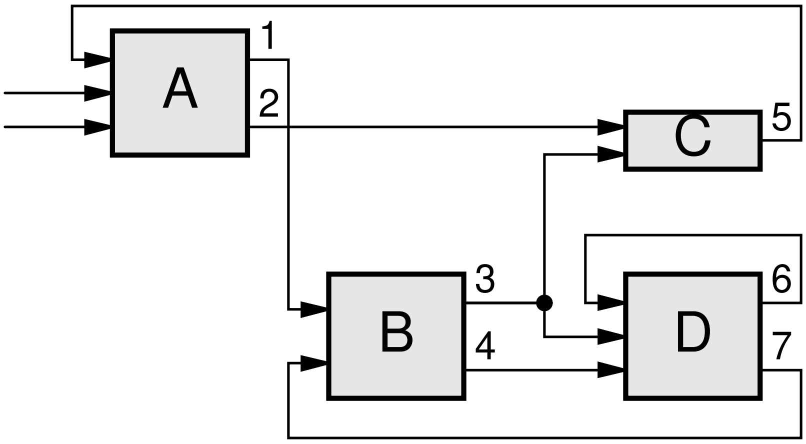
1. Start with all signals at $\perp$
2. Evaluate all blocks (in some order)
3. If any change their outputs, repeat Step 2



$$\begin{aligned}
(a,b,c) &= (\perp,\perp,\perp) \\
f_0(\perp,\perp,\perp) &= (0,\perp,\perp) \\
f_1(0,\perp,\perp) &= (0,1,\perp) \\
f_2(0,1,\perp) &= (0,1,0) \\
f_2(f_1(f_0(0,1,0))) &= (0,1,0)
\end{aligned}$$

The Dependency Graph

Transform into single-output functions:



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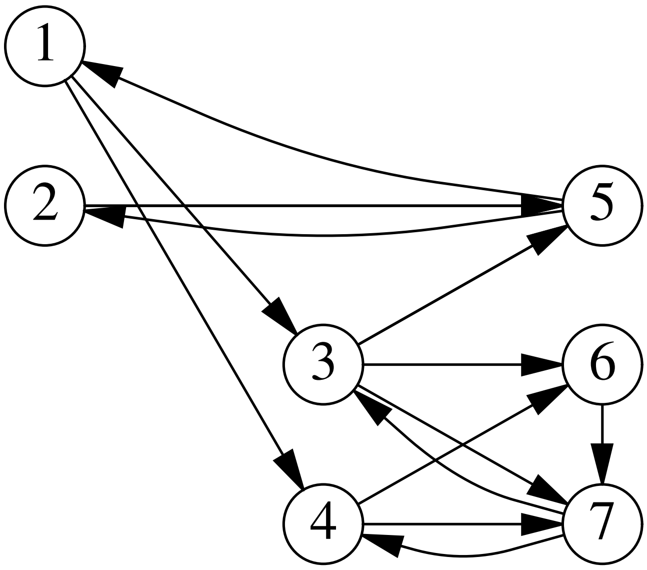
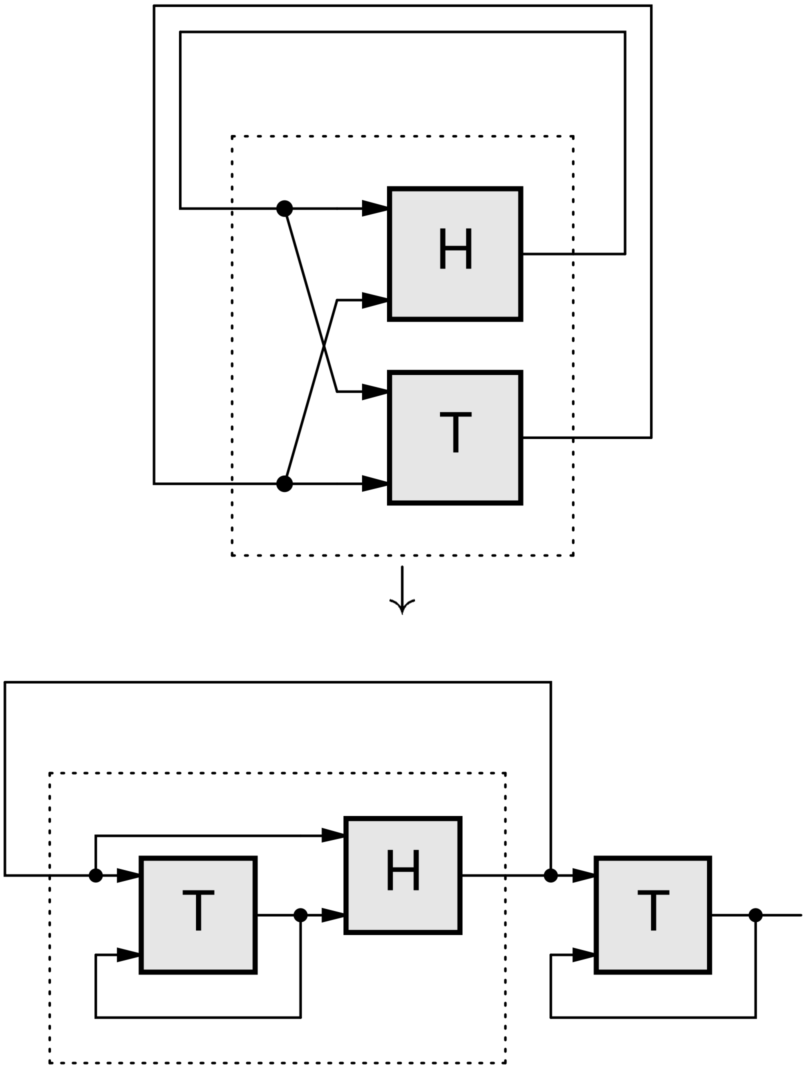


The Scheduling Algorithm

1. Decompose into strongly-connected components
2. Remove a head (set of vertices) from each SCC, leaving a tail
3. Recurse on each tail

Evaluating SCCs

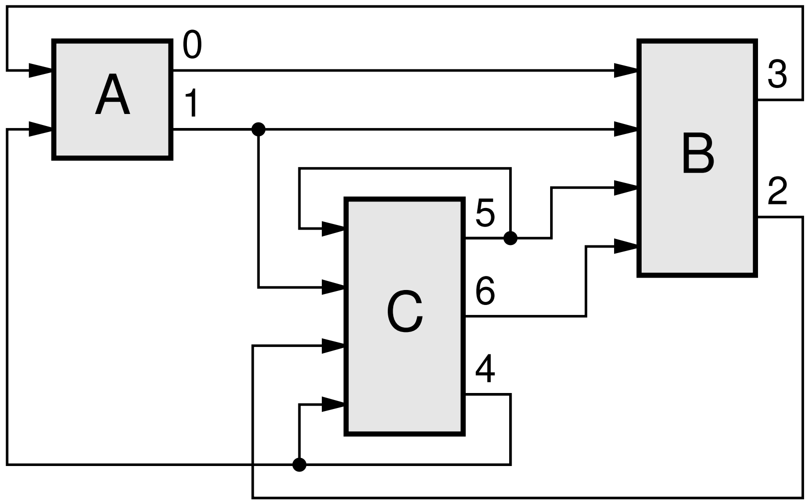
Split a strongly-connected graph into a head and tail:



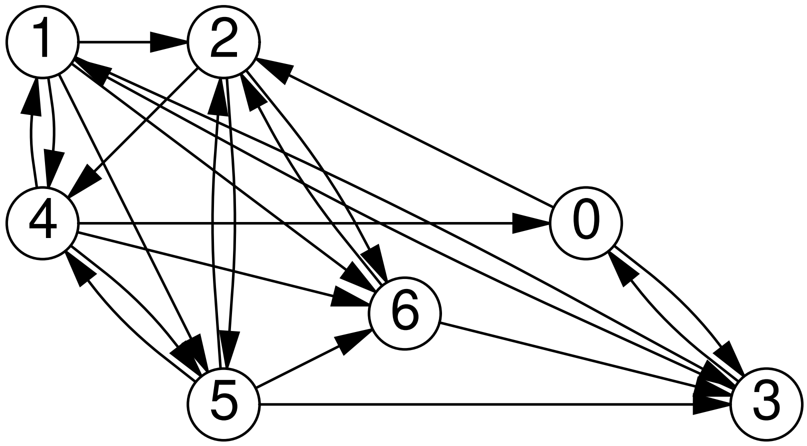
Good heads break T's strong connectivity.

Example

System

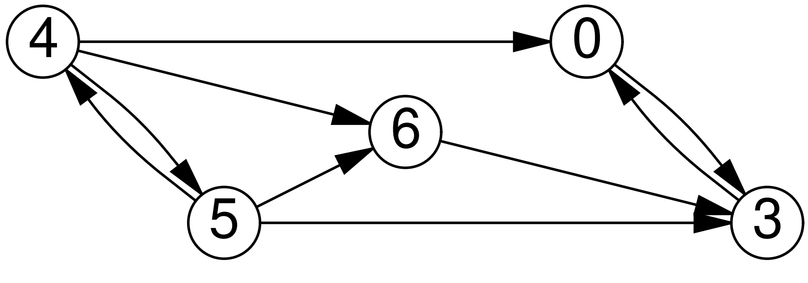


Graph

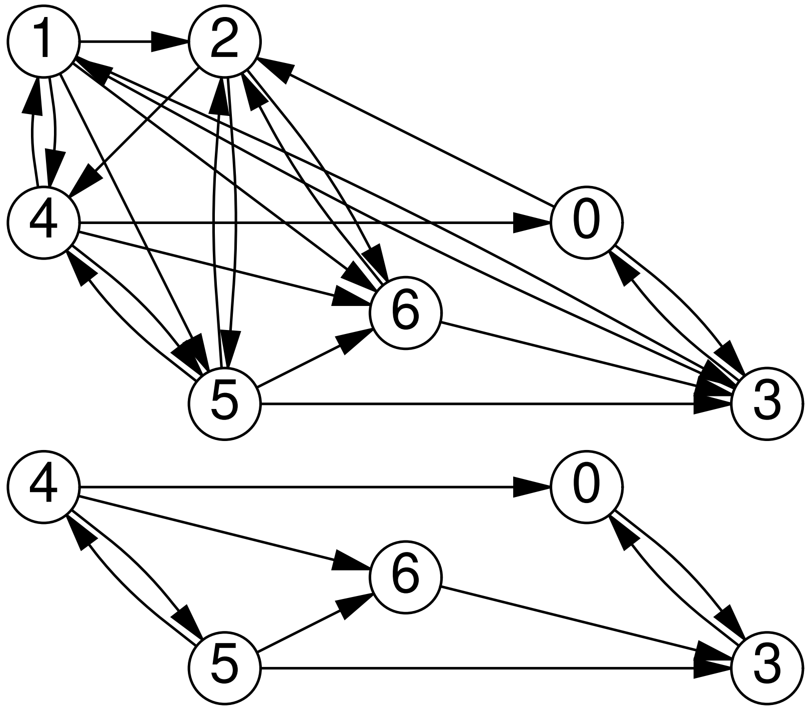


Head

Tail



Schedules

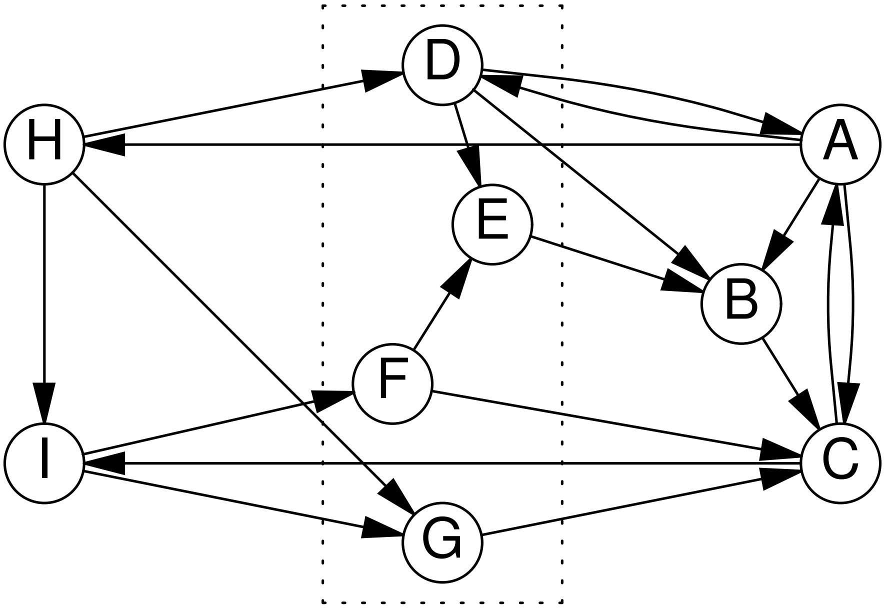


$$(\overbrace{1\ 2}^{\text{head}} \; . \; \overbrace{\underbrace{(\overbrace{4}^{\text{head}} \; . \; \overbrace{5}^{\text{tail}})}_{\text{SCC}} \; 6 \; \underbrace{(\overbrace{0}^{\text{head}} \; . \; \overbrace{3}^{\text{tail}})}_{\text{SCC}}}^{\text{tail}})$$

5 4 5 6 3 0 3 1 2 5 4 5 6 3 0 3 1 2 5 4 5 6 3 0 3

Finding Good Heads

Must break strong connectivity—remove a border of a set of vertices:

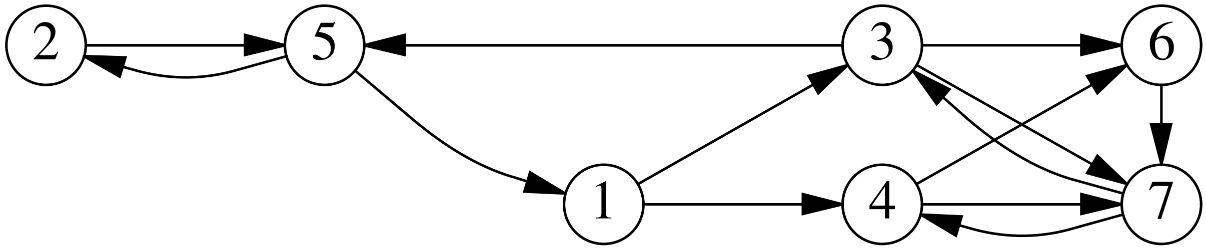


border of { A, B, C }

(vertices with incoming edges)

Choosing Good Border Sets

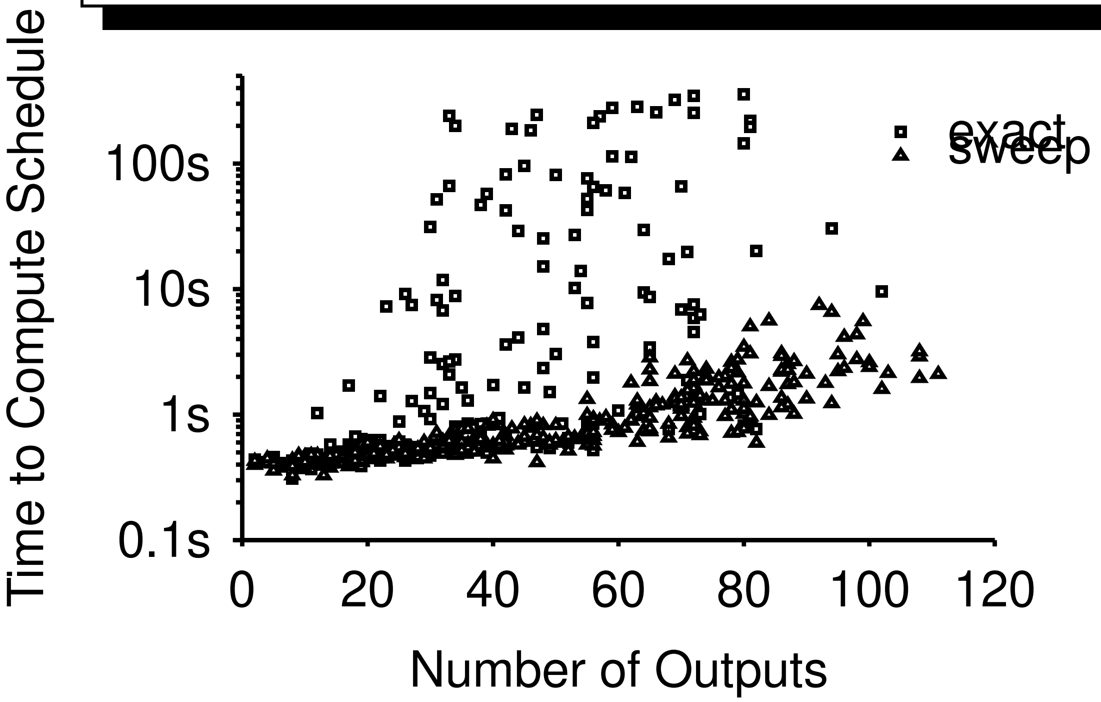
Heuristic: “Grow” a set starting from a vertex and greedily include the best border vertex:

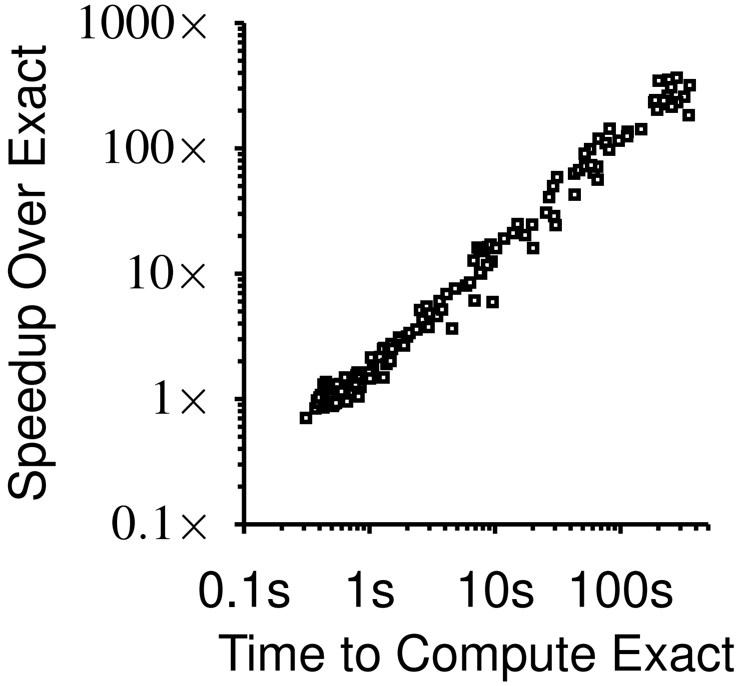


Set	Border
1	5
1 5	2 3
1 5 2	3
1 5 2 3	7
1 5 2 3 7	4 6
1 5 2 3 7 4	6

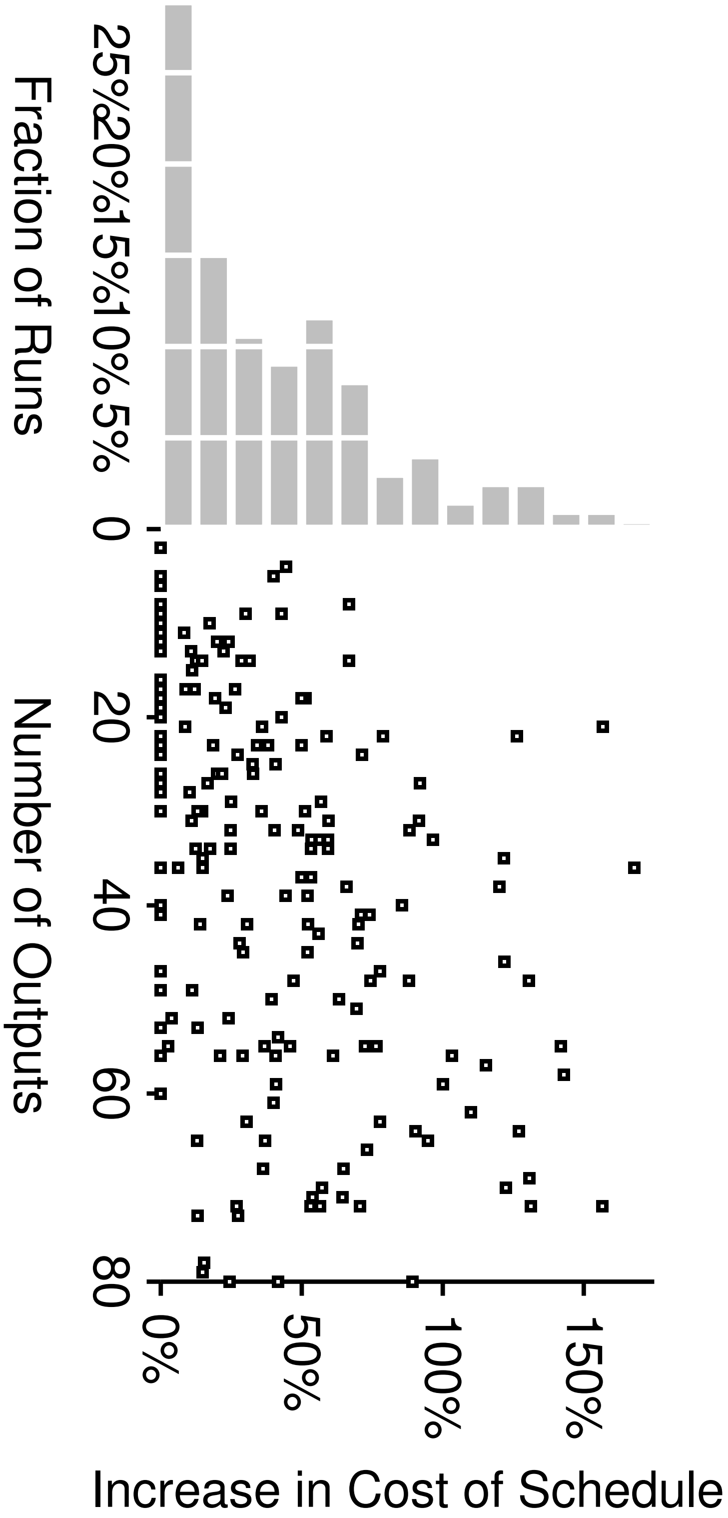
2 is better (3 would increase border)

Scheduling Results

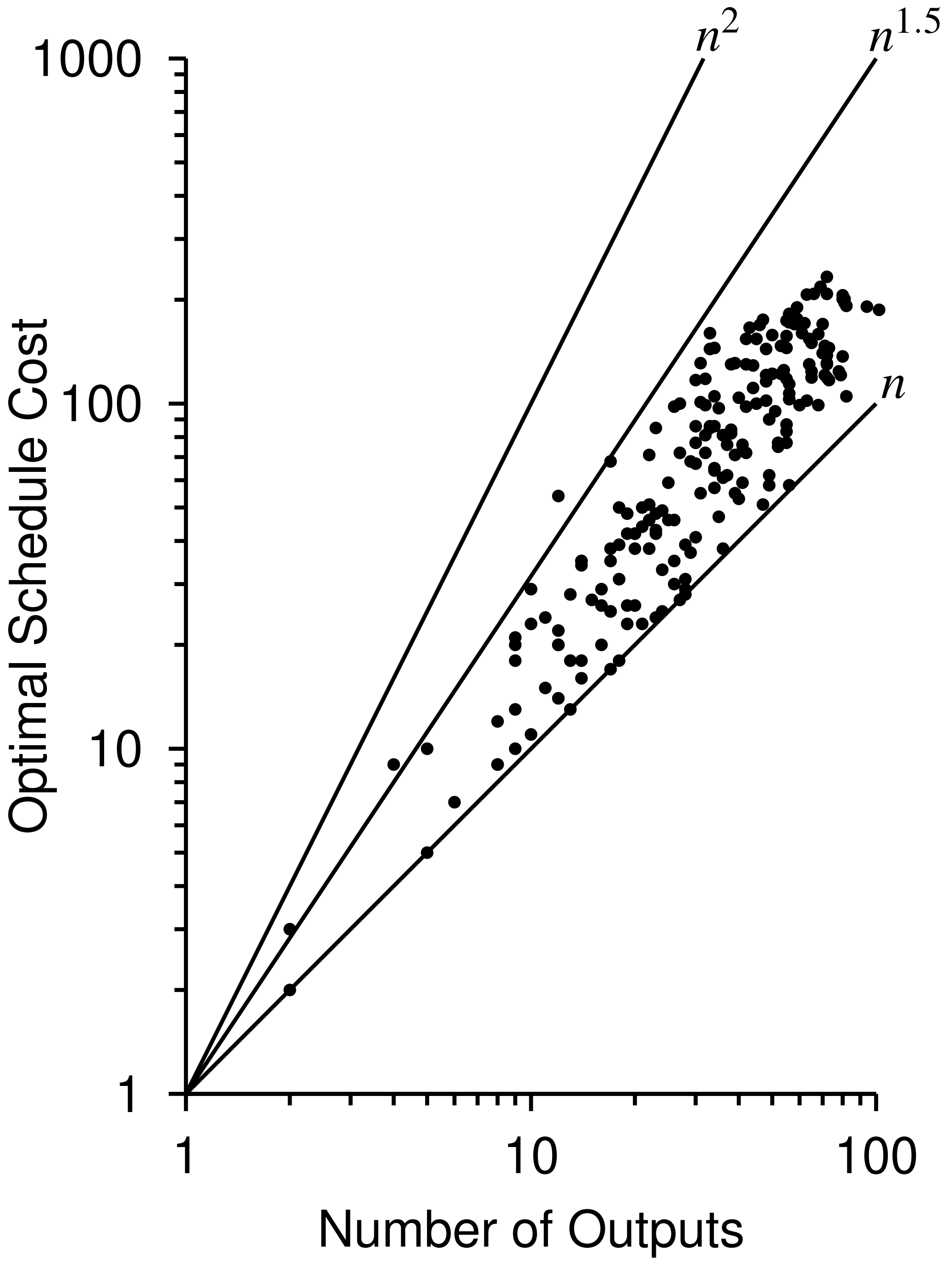




The Cost of Using the Heuristic



Asymptotic Schedule Cost



Conclusions

- Deterministic specification scheme combining synchrony and heterogeneity
- Semantics: the least fixed point of a continuous function on a CPO
- Iterative execution scheme based on recursive divide-and-conquer
- Exact scheduling practical for small graphs
- Heuristic practical for very large graphs
- Execution time for random graphs growing slower than $n^{1.5}$