The Eval Monad and Strategies

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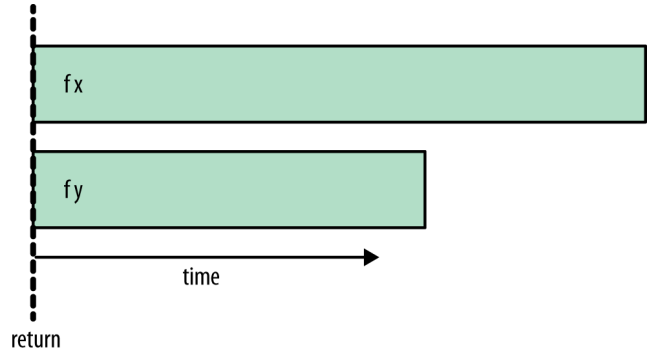
The Eval Monad: An Alternative to *seq* and *par*

When in doubt, create a Monad

```
module Control.Parallel.Strategies where

data Eval a = ...

instance Monad Eval where ...

runEval :: Eval a -> a   -- Get the result

rpar :: a -> Eval a      -- Spark evaluation
rseq :: a -> Eval a      -- Wait for evaluation to WHNF
```

rpar/rpar

```
runEval $ do
  a <- rpar (f x)
  b <- rpar (f y)
  return (a, b)
```

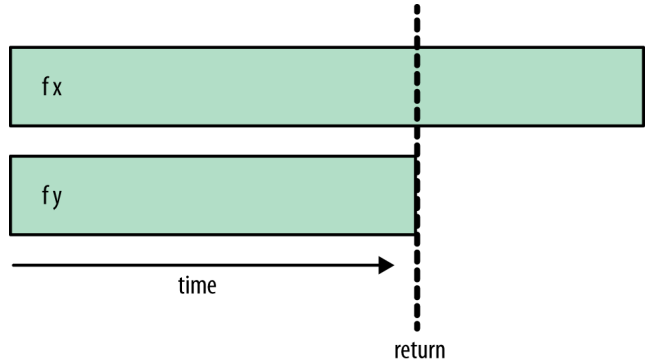


Start parallel evaluation of f x and f y

Return immediately

rpar/rseq

```
runEval $ do
  a <- rpar (f x)
  b <- rseq (f y)
  return (a, b)
```



Start parallel evaluation of f x and f y

Wait for f y to finish, then return

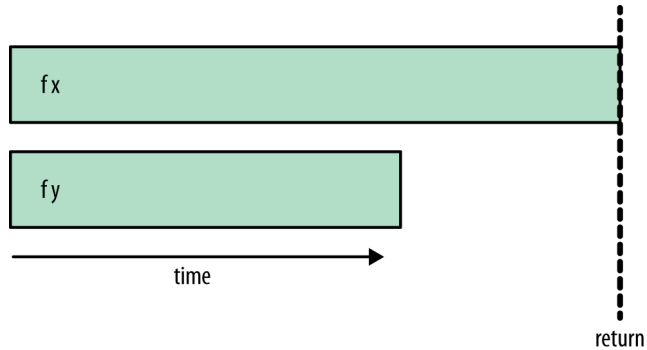
Do we really know f y is faster?

rpar/rpar/rseq/rseq

```
runEval $ do
  a <- rpar (f x)
  b <- rseq (f y)
  rseq a
  return (a, b)
```

Equivalent and symmetrical:

```
runEval $ do
  a <- rpar (f x)
  b <- rpar (f y)
  rseq a
  rseq b
  return (a, b)
```



Start parallel evaluation of f x and f y

Wait for both to finish, then return

Marlow's Sudoku Solver Example (single-threaded)

```
$ wget https://github.com/simonmar/parconc-examples/archive/master.tar.gz
$ tar --strip-components=1 -zxf master.tar.gz \
   parconc-examples-master/Sudoku.hs \
   parconc-examples-master/sudoku1.hs \
   parconc-examples-master/sudoku17.1000.txt
```

```
import Sudoku ; import Control.Exception
import System.Environment ; import Data.Maybe

main :: IO ()
main = do [f] <- getArgs
          file <- readFile f
          let puzzles   = lines file
              solutions = map solve puzzles
          print (length (filter isJust solutions))
```

```
$ stack ghc -- -O2 sudoku1.hs -rtsopts
$ ./sudoku1 sudoku17.1000.txt +RTS -s
```

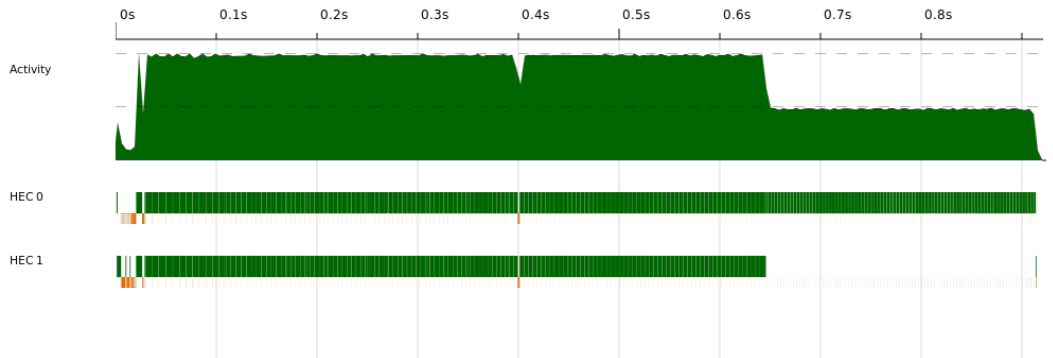
Static Partitioning

```
import Sudoku
import Data.Maybe ; import System.Environment
import Control.Parallel.Strategies ; import Control.DeepSeq
main :: IO ()
main = do [f] <- getArgs
          puzzles <- lines <$> readFile f
          let (as,bs) = splitAt (length puzzles `div` 2) puzzles

              solutions = runEval $ do
                as' <- rpar (force (map solve as)) -- To normal
                bs' <- rpar (force (map solve bs)) -- form
                _ <- rseq as'
                _ <- rseq bs'
                return (as' ++ bs')

          print (length (filter isJust solutions))
```

The Problem with Static Partitioning

```
$ stack install parallel
$ stack ghc -- -O2 -Wall sudoku2.hs -threaded -rtsopts
$ ./sudoku2 sudoku17.1000.txt +RTS -N2 -s -ls
$ threadscope sudoku2.eventlog
```

Speedup on two cores: $1.100 \div 0.910 = 1.21$



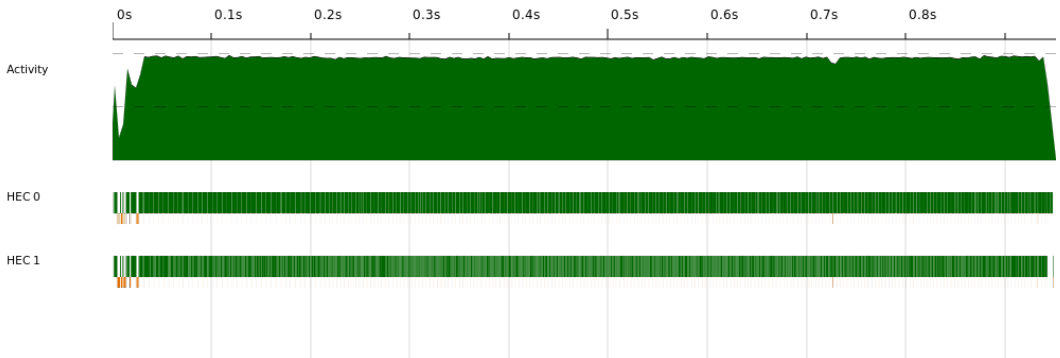
Better Balancing using parMap

```
import Sudoku
import System.Environment ; import Data.Maybe
import Control.Parallel.Strategies hiding (parMap)

parMap :: (a -> b) -> [a] -> Eval [b]
parMap _ []     = return []
parMap f (a:as) = do b  <- rpar  (f a) -- Spark evaluation of f a
                     bs <- parMap f as -- Recurse
                     return (b:bs)

main :: IO ()
main = do [f] <- getArgs
          puzzles <- lines <$> readFile f
          let solutions = runEval (parMap solve puzzles)

          print (length (filter isJust solutions))
```

Dynamic Partitioning

```
$ stack ghc -- -O2 -Wall sudoku3.hs -threaded -rtsopts
$ ./sudoku3 sudoku17.1000.txt +RTS -N2 -s -ls
$ threadscope sudoku3.eventlog
```

Speedup on two cores: $1.100 \div 0.950 = 1.16$ versus 1.21 with with static partitioning. What went wrong?



Amdahl's Law: Why Parallelism is Difficult

Validity of the single processor approach to achieving large scale computing capabilities

by DR. GENE M. AMDAHL
International Business Machines Corporation
Sunnyvale, California

AFIPS Joint Computer Conference, 1967

P is the parallel fraction of the task

N is the degree of parallelism

S is the speedup

$$S = \frac{1}{(1-P)+P/N}$$

$\lim_{N\to\infty} S$ is the depressing part:

If $P = 0.5$, $S \to 2$ as $N \to \infty$

If $P = 0.95$, $S \to 20$ as $N \to \infty$

DeepSeq and Normal Form

Weak Head Normal Form = Top is data constructor or lambda, not application

Normal Form = Data constructors or lambdas all the way down

In Control.Deepseq,

```
class NFData a where
  rnf :: a -> ()        -- "Reduce to Normal Form"
  rnf a = a `seq` ()    -- Default, e.g., for numbers

deepseq :: NFData a => a -> b -> b
deepseq a b = rnf a `seq` b -- Evaluate a to NF; return b

force   :: NFData a => a -> a
force a = a `deepseq` a     -- Evaluate a to NF; return it

($!!)   :: NFData a => (a -> b) -> a -> b
f $!! x = x `deepseq` f x   -- evaluate x then f x
```

Normal Form vs. Weak Head Normal Form

```
Prelude> import Control.DeepSeq
Prelude Control.DeepSeq> let x = [1..10] :: [Int]
Prelude Control.DeepSeq> :sprint x
x = _
Prelude Control.DeepSeq> :sprint x
x = _
Prelude Control.DeepSeq> x `seq` ()
()
Prelude Control.DeepSeq> :sprint x
x = 1 : _
Prelude Control.DeepSeq> (last $ take 3 x) `seq` ()
()
Prelude Control.DeepSeq> :sprint x
x = 1 : 2 : 3 : _
Prelude Control.DeepSeq> x `deepseq` ()
()
Prelude Control.DeepSeq> :sprint x
x = [1,2,3,4,5,6,7,8,9,10]
```

Roll-your-own NFData

```
import Control.DeepSeq
data Tree a = Empty | Branch (Tree a) a (Tree a)
instance NFData a => NFData (Tree a) where
  rnf Empty = ()
  rnf (Branch l a r) = rnf l `seq` rnf a `seq` rnf r
```

```
*Main> let singleton x = Branch Empty x Empty
*Main> let x = Branch (singleton 'a') 'b' (singleton 'c')
*Main> :sprint x
x = Branch _ 'b' _
*Main> x `seq` ()
()
*Main> :sprint x
x = Branch _ 'b' _
*Main> rnf x
()
*Main> :sprint x
x = Branch (Branch Empty 'a' Empty) 'b' (Branch Empty 'c' Empty)
```

Evaluation Strategies

"Strategies are a means for modularizing parallel code by separating the algorithm from the parallelism. Sometimes they require you to rewrite your algorithm, but once you do so, you will be able to parallelize it in different ways just by substituting a new Strategy." —Simon Marlow

1. Build a lazy data structure representing the computation
2. Apply a Strategy that traverses the computation applying *rpar* and *rseq*

A Strategy: an identity function in the *Eval* monad (Control.Parallel.Strategies)

```
type Strategy = a -> Eval a
```

An Example: A Parallel Strategy for Pairs

```
rpar :: Strategy a                  -- Spark evaluation
runEval :: Eval a -> a              -- Evaluate; return value

parPair :: Strategy (a,b)           -- Simple parallel strategy for pairs
parPair (a,b) = do
  a' <- rpar a                      -- Spark parallel evaluation of
  b' <- rpar b                      --   a and b to WHNF
  return (a',b')
```

```
runEval (parPair (fib 35, fib 36))
```

More elegantly,

```
using :: a -> Strategy a -> a       -- In Control.Parallel.Strategies
x `using` s = runEval (s x)         -- Apply s, return x
```

```
(fib 35, fib 36) `using` parPair
```

Basic Strategies

In Control.Parallel.Strategies:

```
r0        :: Strategy a
r0 x      = return x      -- Do not evaluate x

rseq      :: Strategy a   -- Evaluate to WHNF; wait for completion

rdeepseq :: NFData a => Strategy a -- Fully evaluate then proceed
rdeepseq x = rseq (rnf x) >> return x

rpar      :: Strategy a   -- Spark evaluation (in parallel) to WHNF

rparWith :: Strategy a -> Strategy a
rparWith s x              -- Spark evaluation of x ‘using‘ s
```

Building Strategies from Strategies

A skeleton for expressing stategies for evaluating tuples:

```
-- In Control.Parallel.Strategies,
evalTuple2 :: Strategy a -> Strategy b -> Strategy (a, b)
evalTuple2 sa sb (a, b) = do  a' <- sa a
                              b' <- sb b
                              return (a', b')
```

```
parPair :: Strategy (a, b)
parPair = evalTuple2 rpar rpar -- Spark elements' evaluation to WHNF
```

What if we wanted to fully evaluate the two elements in parallel?

```
parPair :: Strategy a -> Strategy b -> Strategy (a, b)
parPair sa sb = evalTuple2 (rparWith sa) (rparWith sb)
```

```
parPair rdeepseq rdeepseq (fib 25, fib 26)
```

parPair rdeepseq rdeepseq (a, b)

"Spark two parallel threads that fully evaluate a and b to normal form"

A cartoon of how this works:

```
parPair rdeepseq rdeepseq (a, b)
  = evalTuple2 (rparWith rdeepseq) (rparWith rdeepseq) (a, b)

  = do a' <- (rparWith rdeepseq) a
       b' <- (rparWith rdeepseq) b
       return (a', b')

  = do a' <- rpar (a `using` \x -> rseq (rnf x) >> return x)
       b' <- rpar (b `using` \x -> rseq (rnf x) >> return x)
       return (a', b')
```

Evaluating a List in Parallel

In Control.Parallel.Strategies,

```
evalList :: Strategy a -> Strategy [a] -- Apply a strategy
evalList _ []     = return []          -- to each list element
evalList s (x:xs) = do x'  <- s x
                       xs' <- evalList s xs
                       return (x':xs')

parList :: Strategy a -> Strategy [a] -- Evaluate each list element
parList s = evalList (rparWith s)     -- in parallel with strategy
```

Combining these to evaluate all list elements to WHNF in parallel:

```
parMap :: (a -> b) -> [a] -> [b]
parMap f xs = map f xs `using` parList rseq
```

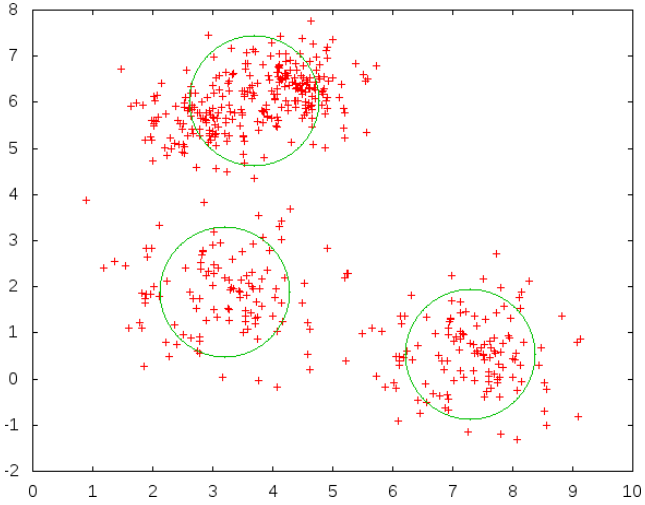
Parallel Sudoku with a Strategy

```
import Sudoku(solve)
import System.Environment(getArgs)
import Data.Maybe(isJust)
import Control.Parallel.Strategies(using, parList, rseq)

main :: IO ()
main = do [fname] <- getArgs
          puzzles <- lines <$> readFile fname

          let solutions = map solve puzzles `using` parList rseq

          print $ length $ filter isJust solutions
```

Note that *rseq* only evaluates to WHNF, but that suffices for Sudoku

About the same performance as the “parMap” version presented earlier

Example: The K-Means Problem



Lloyd's (approximation) algorithm

Give a number of clusters *k*,

1. Guess a center for each cluster
2. Group points by closest centerpoint
3. Calculate the centroid (average) of each group
4. Repeat steps 3–4 until satisfied

Example: The K-Means Problem

```
$ wget https://github.com/simonmar/parconc-examples/archive/master.tar.gz
$ tar --strip-components=1 -zxf master.tar.gz \
   parconc-examples-master/kmeans
```

2D points (to simplify visualization) and clusters, in KMeansCore.hs,

```
data Point = Point !Double !Double -- ! disables laziness

zeroPoint :: Point
zeroPoint = Point 0 0

sqDistance :: Point -> Point -> Double -- Distance squared for speed
sqDistance (Point x1 y1) (Point x2 y2) = ((x1-x2)^2) + ((y1-y2)^2)
```

```
data Cluster = Cluster { clId    :: Int    -- number of this cluster
                       , clCent  :: Point  -- centroid of this cluster
                       }
```

Example: The K-Means Problem

For computing the centroids (average of all points in a cluster), in kmeans.hs,

```
data PointSum = PointSum !Int !Double !Double

addToPointSum :: PointSum -> Point -> PointSum
addToPointSum (PointSum count xs ys) (Point x y)
     = PointSum (count+1) (xs + x) (ys + y)

pointSumToCluster :: Int -> PointSum -> Cluster
pointSumToCluster i (PointSum count xs ys) =  Cluster {
    clId    = i
  , clCent  = Point (xs / fromIntegral count)
                    (ys / fromIntegral count)
  }
```

1. Accumulate Points in PointSums for Nearest Centroid

```
assign :: Int -> [Cluster] -> [Point] -> Vector PointSum
assign nclusters clusters points = Vector.create $ do
    vec <- MVector.replicate nclusters (PointSum 0 0 0)

    let addpoint p = do
          let c = nearest p ; cid = clId c
          ps <- MVector.read vec cid
          MVector.write vec cid $! addToPointSum ps p

    mapM_ addpoint points
    return vec
 where
  nearest p = fst $ minimumBy (compare `on` snd)
                [ (c, sqDistance (clCent c) p) | c <- clusters ]
```

Vectors are Haskell's fixed-length, random-access arrays that are "mutable" in the right monad. See Data.Vector and Data.Vector.Mutable

2. Create New Clusters from PointSums

```
makeNewClusters :: Vector PointSum -> [Cluster]
makeNewClusters vec =
  [ pointSumToCluster i ps
  | (i,ps@(PointSum count _ _)) <- zip [0..] (Vector.toList vec)
  , count > 0
  ]
```

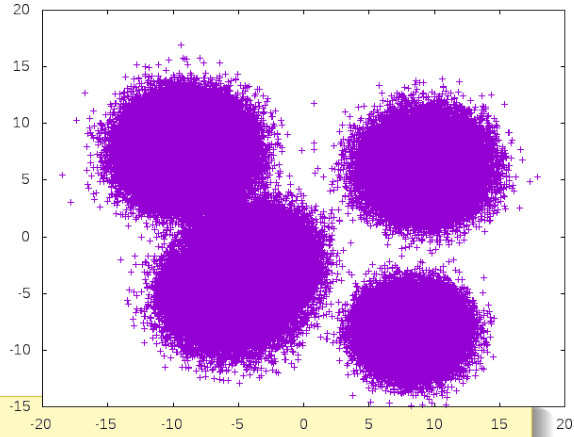
One step of the algorithm: group by nearest centroid; calculate new centroids

```
step :: Int -> [Cluster] -> [Point] -> [Cluster]
step nclusters clusters points
   = makeNewClusters (assign nclusters clusters points)
```

The Sequential Loop: step until converged or give up

```
kmeans_seq :: Int -> [Point] -> [Cluster] -> IO [Cluster]
kmeans_seq nclusters points clusters =
  let loop :: Int -> [Cluster] -> IO [Cluster]
      loop n clusters | n > tooMany = do
        putStrLn "giving up."
        return clusters
      loop n clusters = do
        printf "iteration %d\n" n
        putStr (unlines (map show clusters))
        let clusters' = step nclusters clusters points
        if clusters' == clusters
           then return clusters
           else loop (n+1) clusters'
  in loop 0 clusters

tooMany = 80
```

Generating a Data Set



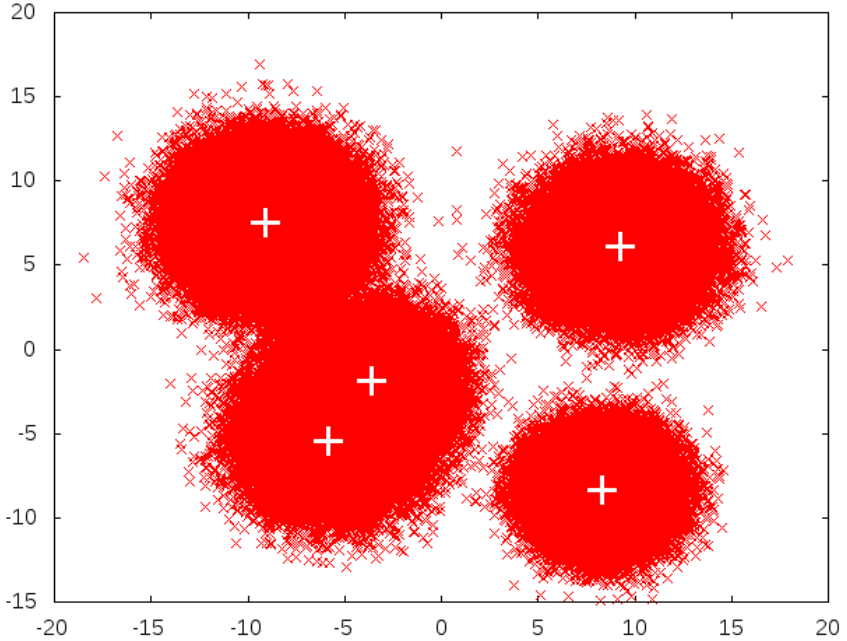

```
$ stack install normaldistribution
$ stack ghc -- -O2 -Wall GenSamples.hs
$ ./GenSamples 5 50000 100000 1010
$ ls -lh points.bin
--rw-rw-r-- 1 sedwards sedwards 16M Nov 23 14:58 points.bin
$ gnuplot -e 'set terminal png;set nokey;plot "points"' > points.png
```

Compiling and Running K-Means

```
$ stack install monad-par
$ stack ghc -- -O2 -threaded -rtsopts kmeans.hs
```

Run it in sequential mode:

```
$ ./kmeans seq
...
iteration 20
Cluster {clId = 0, clCent = Point -5.84359465 -5.46502314}
Cluster {clId = 1, clCent = Point 8.316354592 -8.33043084}
Cluster {clId = 2, clCent = Point -9.06455081 7.561852464}
Cluster {clId = 3, clCent = Point 9.243597731 6.138576051}
Cluster {clId = 4, clCent = Point -3.62170911 -1.82458124}
Total time: 0.73
```


20
15
10
5
0
-5
-10
-15
-20
-15
-10
-5
0
5
10
15
20

Parallelizing K-Means

Computing nearest center for each point is the main operation to parallelize. This is a fold with an associative accumulation function *addToPointSum*.

Too many points and not enough work per point for per-point parallelism; overhead would dominate. Better to split work into coarser chunks.

```
split :: Int -> [a] -> [[a]]        -- Divide into numChunks chunks
split numChunks xs = chunk (length xs `quot` numChunks) xs

chunk :: Int -> [a] -> [[a]]        -- Split into n-point chunks
chunk _ [] = []
chunk n xs = let (as,bs) = splitAt n xs in   as : chunk n bs

addPointSums :: PointSum -> PointSum -> PointSum -- Accumulate PointSums
addPointSums (PointSum c1 x1 y1) (PointSum c2 x2 y2)
  = PointSum (c1+c2) (x1+x2) (y1+y2)

combine :: Vector PointSum -> Vector PointSum -> Vector PointSum
combine = Vector.zipWith addPointSums          -- Accumulate vectors
```

Code for a Parallel *step*

Analyze the chunks in parallel; merge; and make new clusters:

```
parSteps_strat :: Int -> [Cluster] -> [[Point]] -> [Cluster]
parSteps_strat nclusters clusters pointss
  = makeNewClusters $
      foldr1 combine $          -- Merge the results from each chunk
       (map (assign nclusters clusters) pointss -- Analyze chunks
         `using` parList rseq)                  -- in parallel
```

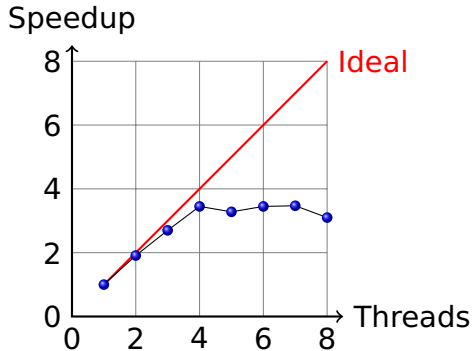
The Parallel Loop: Divide into chunks; apply parSteps_strat

```
kmeans_strat :: Int -> Int -> [Point] -> [Cluster] -> IO [Cluster]
kmeans_strat numChunks nclusters points clusters =
  let chunks = split numChunks points      -- One big change

      loop :: Int -> [Cluster] -> IO [Cluster]
      loop n clusters | n > tooMany = do
        printf "giving up."
        return clusters
      loop n clusters = do
        printf "iteration %d\n" n
        putStr (unlines (map show clusters))
        let clusters' = parSteps_strat nclusters clusters chunks
        if clusters' == clusters
           then return clusters
           else loop (n+1) clusters'
  in loop 0 clusters
```

Performance of `kmeans_strat` on 1–8 Cores

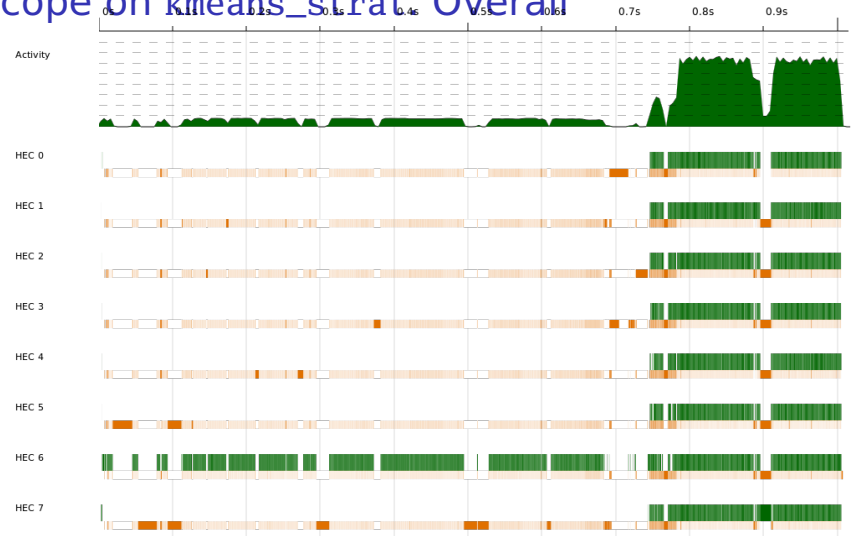
```
./kmeans strat 64 +RTS -N1
./kmeans strat 64 +RTS -N8
```

Cores	Time (s)	Speedup
1	0.77	1.00
2	0.40	1.91
3	0.29	2.70
4	0.22	3.45
5	0.23	3.28
6	0.22	3.45
7	0.22	3.47
8	0.25	3.10



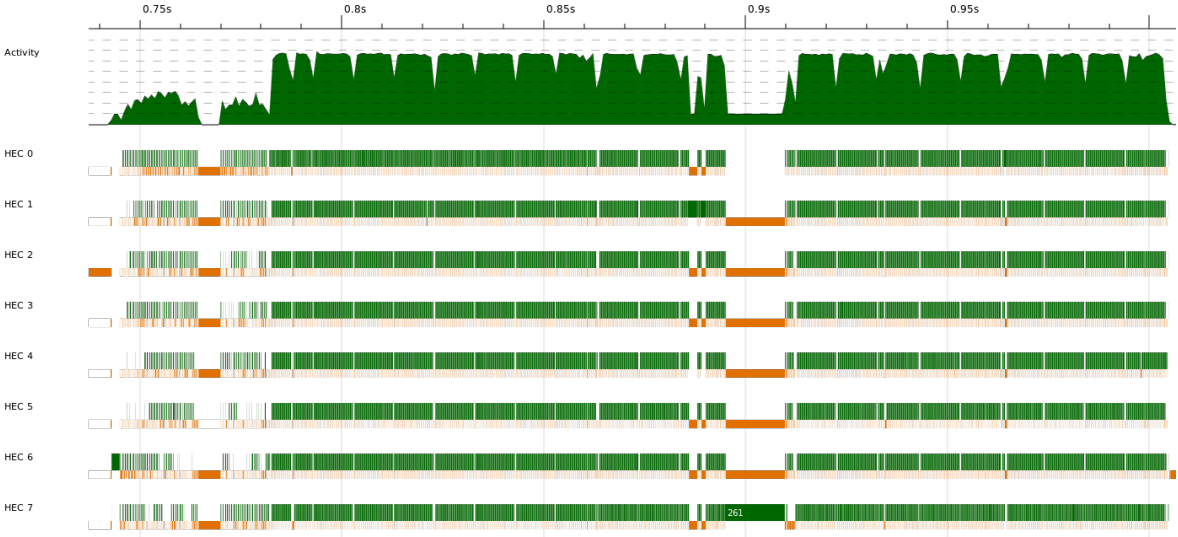
Using “Total time” reported by the program; ignores reading point data

Threadscope on `kmeans_strat`: Overall



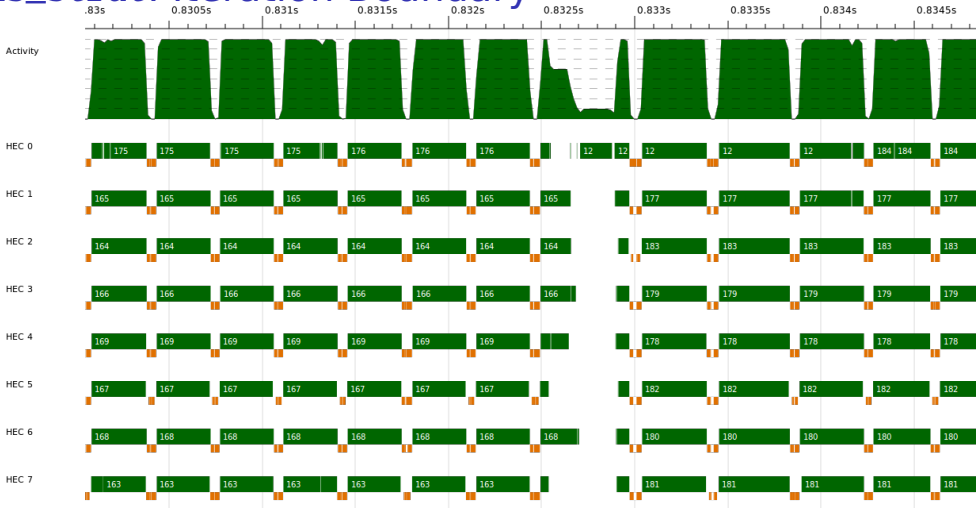
Lots of sequential file processing start: not being counted against speedup

kmeans_strat: Parallel Section Only



Off to a difficult start; iterations are periodic hiccups; big garbage collect

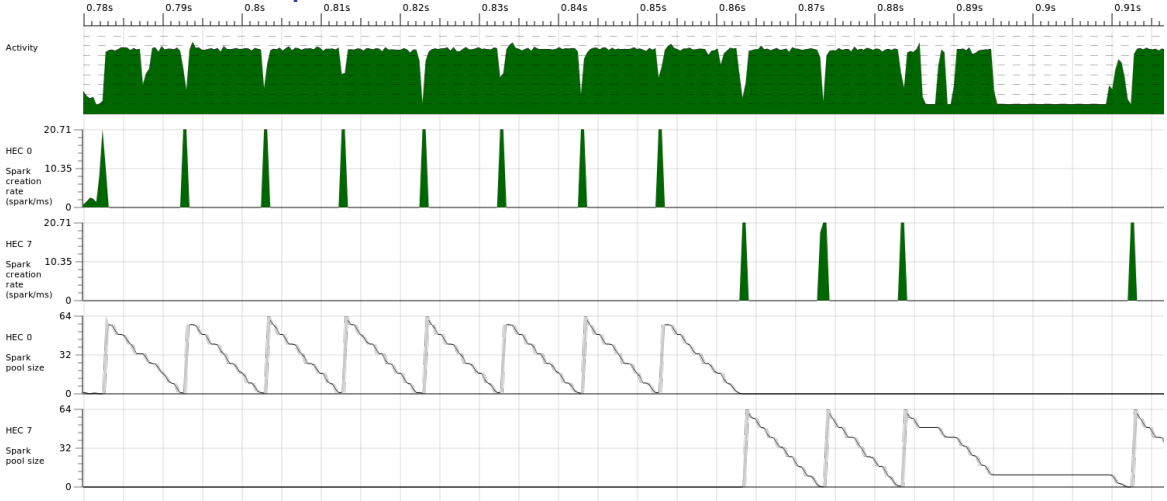
`kmeans_strat`: Iteration Boundary



Program suddenly turns completely sequential; darn Amdahl.

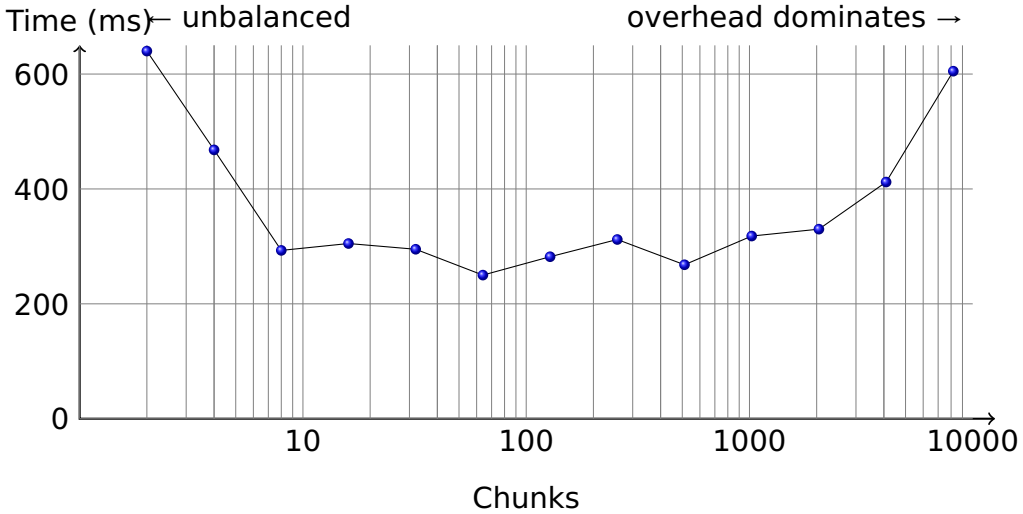
Marlow found printing was a major culprit, but removing it didn’t matter

`kmeans_strat`: Spark creation and conversion



Iteration: sudden spark creation activity in single HEC pool, then slow conversion. Main thread migrated after 8 iterations. (Under the “Traces” tab)

The Effects of Granularity (N=8)



GC'ed Sparks and Speculative Parallelism [Marlow p. 48]

parList/evalList creates a new list, which seems wasteful

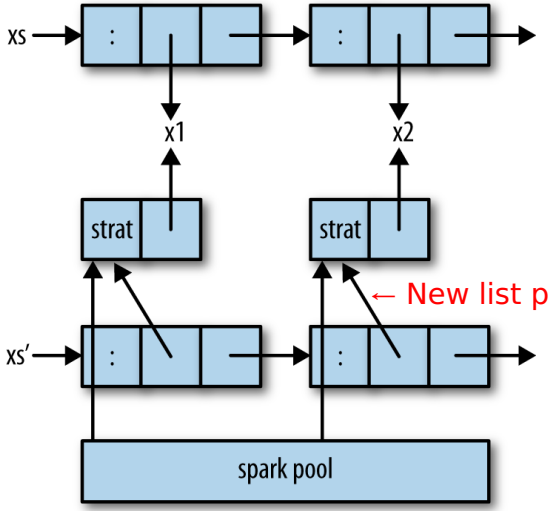
```
parList :: Strategy a
        -> Strategy [a]
parList s =
     evalList (rparWith s)

evalList :: Strategy a
         -> Strategy [a]
evalList _ []     = return []
evalList s (x:xs) =
  do x'  <- s x
     xs' <- evalList s xs
     return (x':xs') -- Cons
```

Consider this walk-the-list alternative that "touches" elements with *rparWith*

```
parList :: Strategy a
        -> Strategy [a]
parList strat xs = do go xs
                      return xs
 where
  go []     = return ()
  go (x:xs) = do rparWith strat x
                 go xs
```

Doesn't work: each spark created by *rparWith* is garbage-collected because it is never used. Critical that the result of *rpar/rparWith* be returned.

Heap Layout of Working parList: New List Inhibits Spark GC



Original list

List elements

Sparks: apply the strategy
to evaluate list elements

← New list provides second pointer to each spark

New list

Array of pointers to sparks

Marlow, fig. 3-8

Parallelizing Lazy Streams: RSA Encoder/Decoder from Marlow

```
$ stack ghc -- -O2 -Wall -rtsopts rsa
$ ./rsa encrypt /usr/share/dict/words > /dev/null +RTS -s
   5,089,757,232 bytes allocated in the heap
       3,043,360 bytes copied during GC
         107,888 bytes maximum residency (3 sample(s))
          27,968 bytes maximum slop
               0 MB total memory in use

  Total   time    5.740s  (  5.767s elapsed)

$ ls -sh /usr/share/dict/american-english
920K /usr/share/dict/american-english
```

Dictionary file is about 1 MB, but runtime only uses 107,888 bytes maximum because of Data.ByteString.Lazy.Char8

Parallelizing RSA

Sequential implementation:

```
encrypt :: Integer -> Integer -> ByteString -> ByteString
encrypt n e = B.unlines                                  -- Join results
            . map (B.pack . show . power e n . code)     -- Encrypt
            . chunk (size n)                             -- Split
```

First try (rsa1.hs): use *parList rdeepseq*

```
encrypt n e = B.unlines
            . withStrategy (parList rdeepseq)
            . map (B.pack . show . power e n . code)
            . chunk (size n)
```

`withStrategy` $s\ e = e$ `` `using` `` s

Speedup using *parList rdeepseq*

```
$ stack ghc -- -O2 -Wall -threaded -rtsopts rsa1
$ ./rsa1 encrypt /usr/share/dict/words > /dev/null +RTS -N8 -s
   5,319,033,432 bytes allocated in the heap
      18,619,728 bytes copied during GC
       3,029,464 bytes maximum residency (10 sample(s))
         570,920 bytes maximum slop
               2 MB total memory in use

 SPARKS: 9988
        (8254 converted, 1734 overflowed, 0 dud, 0 GC'd, 0 fizzled)

  Total   time   14.403s  (  2.991s elapsed)
```

Speedup of 1.92 over sequential (rsa.hs) (4.8× itself)

Maximum memory use now 3 MB (cf. 107 KB): *parList* traverses the whole list.

Control.Parallel.Strategies.parBuffer: Regulate number of outstanding sparks

parBuffer 100 creates 100 outstanding sparks; sparks more once consumed

```
parBuffer :: Int -> Strategy a -> Strategy [a]
```

```
encrypt n e = B.unlines    -- rsa2.hs
            . withStrategy (parBuffer 100 rdeepseq) -- 100 max
            . map (B.pack . show . power e n . code)
            . chunk (size n)
```

```
$ ./rsa2 encrypt /usr/share/dict/words > /dev/null +RTS -N8 -s
     506,640 bytes maximum residency (18 sample(s))
  SPARKS: 9988
          (9987 converted, 0 overflowed, 0 dud, 0 GC'd, 1 fizzled)
  Total   time   12.160s  (  1.641s elapsed)
```

Down to 500 KB residency, 3.5× over sequential, excellent 7.4×

Spark Creation and Pool with parBuffer

HEC5 got the spark creation process

Spark pool remains around 100

Hiccups primarily garbage collection

No creation or pools on other HECs

Gray on graph denotes variance

About 11% overhead

RSA Strategies (parList, parBuffer) Compared

Technique	Memory (K)	Sparks					Time (s)	Speedu
		Converted	Overflowed	Dud	GC'ed	Fizzled		
Sequential	105						5.77	1
parList	2958	8254	1734	0	0	0	2.99	1.92
parBuffer	495	9987	0	0	0	1	1.64	3.52

Both generate the same number of sparks

parList forces the entire file to be loaded (memory consumption) and generates all the sparks at the beginning (spark pool overflow).