CSEE W4840 Embedded Systems AccelReg: An Accelerator for Linear Regression Final Report

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Introduction

Linear regression is a simple and widely used method for modeling the relationship between two variables: one input (x) and one output (y). In this project, we focus on the 1D case, where we fit a straight line to data points so that it best predicts y from x. The best line is found by minimizing the squared differences between the actual y values and the predicted values, a method known as least squares. We use the closed-form solution, which means we directly calculate the optimal line using matrix operations, instead of using iterative methods like gradient descent. This approach is efficient and gives an exact answer for the model parameters.

In this implementation, the dataset containing the paired input and output values is first stored on the SD card, which also holds the operating system for the FPGA. A key aspect of this project is the design of the linear regression training process on the FPGA using a 4-bit quantization-aware approach. This involves representing the numerical values with reduced precision, which is crucial for efficient hardware implementation. The software module reads the dataset from the SD card and sends it to the FPGA. The FPGA then processes this data to compute the intermediate values needed for calculating the optimal slope and intercept for the linear regression model through matrix operations, all while adhering to the 4-bit quantization constraints. The effectiveness of this quantization strategy is enhanced by utilizing a representative dataset during the quantization-aware training phase, which allows the model to achieve performance closely comparable to its full-precision counterpart.

As the results demonstrate, this approach yields an R2 score of 0.9490 and a Mean Squared Error (MSE) of 0.3402 for the 4-bit quantized model. This is visually represented



Figure 1: (a) Scatter plot of processed data with 4-bit quantized linear regression fit. (b) Scatter plot of original data with full-precision linear regression fit.

in Figure 1(a), which shows the scatter plot of the processed data with the quantized linear fit. These metrics are very close to the R2 score of 0.9515 and MSE of 0.3232 achieved by the original, non-quantized model, depicted in Figure 1(b) (Scatter plot of original data). Once trained, the quantized model can then be used by the FPGA to make predictions on new input data, enabling efficient and automated linear regression directly in hardware with minimal loss in accuracy.

These fit weights were generated using an implementation in C to highlight the efficacy of the model before the implementation in FPGA. This approach allows for a highly efficient implementation as all operations can be performed in integer format. The algorithm for this implementation will be detailed in a subsequent section.

Algorithm

For 1D linear regression with n observations (x_i, y_i) , the model is:

$$y = w_0 + w_1 x$$

For a single observation, this can be written as:

$$y_i = \begin{bmatrix} 1 & x_i \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$$

The weights $\mathbf{w} = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}$ are typically found by solving the normal equation:

$$\mathbf{w} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$$

where:

$$\mathbf{X} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

The key matrix computations involved are:

$$\mathbf{X}^T \mathbf{X} = \begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix}$$

and

$$\mathbf{X}^T \mathbf{y} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix}$$

The inverse of $\mathbf{X}^T\mathbf{X}$ is calculated as:

$$(\mathbf{X}^T \mathbf{X})^{-1} = \frac{1}{n \sum x_i^2 - (\sum x_i)^2} \begin{bmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{bmatrix}$$

For datasets where x_i and y_i are 4-bit unsigned integers (i.e., $x_i, y_i \in \{0, 1, ..., 15\}$), the computation can be optimized by first calculating five critical integer accumulations:

$$S_{1} = n \qquad (\text{count of observations})$$

$$S_{2} = \sum_{i=1}^{n} x_{i} \qquad (\text{sum of } x_{i})$$

$$S_{3} = \sum_{i=1}^{n} y_{i} \qquad (\text{sum of } y_{i})$$

$$S_{4} = \sum_{i=1}^{n} x_{i}^{2} \qquad (\text{sum of } x_{i}^{2})$$

$$S_{5} = \sum_{i=1}^{n} x_{i}y_{i} \qquad (\text{sum of } x_{i}y_{i})$$

Considering the constraints that n is bounded by 512 and x_i, y_i are 4-bit unsigned integers, the bit-width requirements for these sums are as follows:

- $S_1 = n$: Requires 9 bits, as $2^8 < 512 \le 2^9$.
- $S_2 = \sum x_i$ and $S_3 = \sum y_i$: The maximum value for x_i or y_i is 15. In the worst case, these sums can reach $512 \times 15 = 7680$. This requires $\lceil \log_2(7680 + 1) \rceil = 13$ bits. This can be conceptually understood as the sum of bits for the maximum input data (4 bits) and the maximum count (9 bits).

• $S_4 = \sum x_i^2$ and $S_5 = \sum x_i y_i$: The product x_i^2 or $x_i y_i$ can be up to $15 \times 15 = 225$. This intermediate product requires $\lceil \log_2(225 + 1) \rceil = 8$ bits (conceptually, the sum of bits of the two 4-bit operands, 4 + 4 = 8). The accumulated sum can then reach $512 \times 225 = 115200$. This requires $\lceil \log_2(115200 + 1) \rceil = 17$ bits (conceptually, the sum of bits for the intermediate product (8 bits) and the maximum count (9 bits)).

These bit-width calculations are essential for designing the hardware accumulators with sufficient capacity to prevent overflow. Hence, we will design each of the sum registers in 17 bits.

These sums can be computed efficiently, for instance, using dedicated parallel blocks or pipelined reduction circuits, processing data as it streams (e.g., from an SD card). Since x_i and y_i are small integers, these sums will also be integers. The only floating-point operations required are the final divisions. The scaling factor, which is the reciprocal of the determinant $det(\mathbf{X}^T\mathbf{X}) = S_1S_4 - S_2^2$, is applied after all integer accumulations are complete.

We can implement a pipelined accumulation scheme where, for each incoming data pair (x_i, y_i) , four parallel compute units update the sums in real-time:

$$S_2 \leftarrow S_2 + x_i$$

$$S_3 \leftarrow S_3 + y_i$$

$$S_4 \leftarrow S_4 + x_i^2$$

$$S_5 \leftarrow S_5 + x_i y_i$$

Simultaneously, S_1 is incremented by a counter with each data pair processed.

After all data points have been processed and the accumulations are complete, the regression coefficients w_0 and w_1 are calculated as:

$$\begin{bmatrix} w_0 \\ w_1 \end{bmatrix} = \frac{1}{S_1 S_4 - S_2^2} \begin{bmatrix} S_4 S_3 - S_2 S_5 \\ S_1 S_5 - S_2 S_3 \end{bmatrix}$$

Let $D = S_1S_4 - S_2^2$ (the determinant), $N_0 = S_4S_3 - S_2S_5$, and $N_1 = S_1S_5 - S_2S_3$. All D, N_0 , and N_1 are integers. Then:

$$w_0 = \frac{N_0}{D}$$
$$w_1 = \frac{N_1}{D}$$

The individual sum accumulators $(S_1, S_2, S_3, S_4, S_5)$ stored in registers can be processed by a combinational logic block to compute the three integer values (N_0, N_1, D) . These three values are then used to compute the final floating-point weights w_0 and w_1 .

Avalon System Components

Fig. 2 shows the system configuration inside Intel's Quartus Platform Designer (formerly Qsys) for a project named soc_system. This system integrates three key components: a

13	System	n Contents 🛛 🛛 🛛 🛛 🖾	dress Map 💠 Interc	onnect Requirements 🛛 🕅						
	≤ ▲	System: soc_s	ystem Path: clk_0							
+	Use	Connections	Name	Description	Export	Clock	Base	End	IRQ	Tags
4	~	0 D	□ clk_0	Clock Source	clk	exported				
×		CIK_III_reset CIK_III_reset CIK_III_reset CIK_III_RESET		Reset Input Clock Output Reset Output	reset Double-click to Double-click to	clk_0				
				Arria V(Syclone V Hard Proce Clock Output Conduit Reset Output Clock input AXI Master Clock input AXI Slave Clock input AXI Slave Clock input AXI Slave	Double-click to hps_ddr3 hps Double-click to Double-click to Double-click to Double-click to Double-click to Double-click to Double-click to	hps_0_h2 clk_0 [h2f_axi clk_0 [f2h_axi [b2f_lw_a	£			
	2		Ir_acc_0 clock reset avalon_slave_0	Ir_acc Clock Input Reset Input Avalon Memory Mapped Slave	Double-click to Double-click to Double-click to	clk_0 [clock] [clock]	© 0x0000_0000	0x0000_0fff		
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	Туре	Path				Messag	e			
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		soc_system.hps_	0 HPS Main PLL counter	r settings: n = 0 m = 73						
		soc_system.hps	0 HPS peripherial PLL c	ounter settings: n = 0 m = 39						

Figure 2: Avalon System Components

clock source (clk_0), a Hard Processor System (hps_0), and a custom hardware accelerator for linear regression (lr_acc_0). These components are connected using Avalon and AXI interconnects to enable communication between the HPS and FPGA fabric, allowing software running on the ARM processor to configure and interact with the hardware accelerator.

The clock source clk_0 provides the main clock and reset signals to all other components in the design. These signals are exported to the top-level module so they can be driven externally, for example, from a PLL or onboard oscillator. The hps_0 block represents the Cyclone V HPS (Hard Processor System), which includes dual ARM Cortex-A9 processors and a rich set of memory and IO interfaces. Through the lightweight AXI master interface (h2f_lw_axi_master), the processor can access memory-mapped peripherals in the FPGA fabric.

The lr_acc_0 module is a custom Avalon slave component that implements a linear regression accelerator. It receives clock and reset from the clock source and is memory-mapped into the system address space starting at base address $0x0000_0000$ with a 4KB range (ending at $0x0000_0FFF$). The HPS accesses this accelerator using the lightweight AXI master interface for tasks like sending training data, starting computations, and reading regression outputs.

- clk_0 (Clock Source):
 - Provides clock and reset signals to the system.
 - Signals clk and reset are exported to the top-level design.
 - Drives the clock inputs of both the lr_acc_0 and the HPS.

- hps_0 (Hard Processor System):
 - Represents the ARM Cortex-A9 subsystem embedded in Cyclone V SoCs.
 - Interfaces include:
 - * h2f_axi_master: For general AXI-based communication from HPS to FPGA.
 - * h2f_lw_axi_master: Lightweight interface for mem-mapped peripheral access.
 - * f2h_axi_slave: For FPGA-to-HPS communication.
 - * Clock inputs/outputs for synchronization with the fabric.
 - Also interfaces with external memory (e.g., DDR) and IO via exported conduits.
- lr_acc_0 (Linear Regression Accelerator):
 - Custom Avalon memory-mapped slave IP core.
 - Receives clock and reset from clk_0.
 - Connected to the HPS via h2f_lw_axi_master.
 - Mapped to address space $0x0000_0000$ to $0x0000_0FFF$ (4KB).
 - Implements logic to perform linear regression computations.

Software-Hardware Avalon Interface

Fig. 5 shows the **Signals & Interfaces** tab for the avalon_slave_0 interface in Intel Quartus Platform Designer (Qsys) defines the boundary between software and hardware. This interface is responsible for connecting the ARM processor subsystem (HPS) to a custom hardware peripheral, which in this case is likely the *Linear Regression Accelerator*.

The avalon_slave_0 is configured as an Avalon Memory-Mapped Slave Interface, which allows the processor to communicate with FPGA logic using standard memory read and write instructions. Through this interface, software running on the processor can send commands, write input data, and read computation results directly from the hardware peripheral via mapped memory addresses. The interface includes all the standard signals required for such communication, including read, write, chipselect, address, writedata, and readdata. Each of these signals plays a critical role in enabling synchronous, low-latency, memory-mapped access between the processor and the accelerator logic within the FPGA. In the center block diagram of avalon_slave_0, the following signals are defined. These represent the interface between the processor (HPS) and a custom hardware peripheral, such as a Linear Regression Accelerator.

Signal Descriptions

- writedata[31:0]: 32-bit data written by the processor to the hardware module.

Component Type 🛛 Block Symbol 🖇 F	Files 🕺 Parameters 🛱 Signals & Interfaces	8	- 5 🗆
 About Signals 			
Name	Name: avalon_slave_0 Type: Avalon Memory Mapped Slave Associated Clock: clock Associated Reset: reset Assignments: Edit		
<add interface="">></add>	avalon_slave_0 writedata[31.0] writedata[31.0] write address[3.0] read read read read read read read read	Associated clock: Associated reset: Bts per symbol: Burstcount units: Explicit address span: Timing Read walt: Hold: Timing units: Cycles	
	4	Pipelined Transfers Read latency: Maximum pending read transactions: Maximum pending write transactions: Burst on burst boundaries only Linewrap bursts	
Messages 💠			
Info: No errors or warnings.			

Figure 3: Software-Hardware Avalon Interface

- write: Single-bit control signal that indicates a write transaction is requested.
- chipselect: Asserted when the memory-mapped region assigned to this slave is being accessed by the processor.
- address [9:0]: A 10-bit address line allowing access to 1024 unique words within the module.
- read: Single-bit control signal indicating a read transaction is requested.
- readdata[31:0]: 32-bit data read from the hardware module by the processor.

These signals together allow the CPU (typically an ARM Cortex-A9 core in the HPS) to communicate with the custom logic through memory-mapped I/O. Standard processor instructions such as ldr, str, or Linux-based MMIO access can be used to interact with the peripheral.

i. Software-Hardware Interface Configuration (Right Panel)

The configuration settings in the Parameters panel define how the Avalon Memory-Mapped Slave interface behaves:

 Associated Clock: clock — All signal activities on this interface are synchronized to this signal.

- Associated Reset: reset Used to reset the hardware registers inside the peripheral.
- Address Units: Set to WORDS, meaning each address increment maps to a 32-bit word rather than a byte.
- Bits per Symbol: 8 Byte-level granularity, although accesses are word-based.
- Explicit Address Span: Left empty in this instance; can be filled to override auto-assigned address range.

ii. Timing Characteristics

The following parameters specify the cycle-level behavior of transactions over this interface:

- Setup: 0 cycles No setup delay is required.
- Read Wait: 1 cycle The readdata will be valid one cycle after asserting read.
- Write Wait: 0 cycles Write transactions complete in the same cycle they are issued.
- Hold: 0 cycles No additional hold time required after a transaction.
- **Timing Units**: All timing values are expressed in clock cycles.

Block Diagrams

Linear Regression Main Pipeline

From Fig. 4, the linear regression accelerator uses a set of output stages to convey partial and final computation results through a memory-mapped interface. Each of these outputs is accessible via specific addresses decoded using bits a_9 to a_0 . These output registers—such as NO, N1, D, and S1--S5—hold values calculated across the regression pipeline, which ultimately computes the coefficients w_0 and w_1 . This address mapping is crucial for software to retrieve intermediate results or verify convergence across the stages.

Linear Regression Accelerator 8b Block

Fig. 5 illustrates a **pipelined binary adder tree** designed to compute the sum of 16 unsigned 18-bit inputs, labeled op0 through opF. The architecture employs a hierarchical reduction approach, where pairs of inputs are incrementally summed using multiple



Figure 4: Linear Regression Main Pipeline



Figure 5: Linear Regression Accelerator 8b Block

layers of adders. Registers are inserted between each stage to form a fully pipelined structure, significantly enhancing throughput and enabling higher clock frequencies. This is especially beneficial for FPGA implementations, where timing closure and parallelism are critical.

Each stage reduces the number of operands by half. Initially, eight pairwise adders reduce the 16 inputs into 8 intermediate values. These values are then fed into four second-level adders, reducing them to 4 values, and so on, until the final result is produced. The output, labeled **result**, is a single 18-bit value representing the total sum.

Hierarchy and Pipelining Structure

Inputs:

- 16 inputs: op0 to opF
- Each input is 18 bits wide

Stage 1 — Pairwise Addition:

- 8 adders compute:

(op0 + op1), (op2 + op3), ..., (opE + opF)

- Each output is stored in a pipeline register

Stage 2 — Second-Level Addition:

- 4 adders compute:

$$Sum1 = (op0 + op1) + (op2 + op3)$$

$$Sum2 = (op4 + op5) + (op6 + op7)$$

$$Sum3 = (op8 + op9) + (opA + opB)$$

$$Sum4 = (opC + opD) + (opE + opF)$$

- Outputs are again registered

Stage 3 — Third-Level Addition:

-2 adders compute:

$$SumA = Sum1 + Sum2$$

 $SumB = Sum3 + Sum4$

- Outputs are pipelined to the final stage



Figure 6: Linear Regression Accelerator 8b Block 512

LR Accumulator 8b 512

Fig. 6 illustrates a detailed architecture of a Linear Regression Accelerator designed for hardware implementation, typically on an FPGA. The accelerator performs *feature accumulation*, *intermediate statistical computation* (S_1-S_5) , and *final coefficient evaluation* using a pipelined datapath composed of accumulators, adders, multipliers, and subtractors. The module is memory-mapped, allowing interaction with a processor through a standard Avalon or AXI-like interface using control and status registers.

The module operates under the control of a clock signal (clk) and is reset by a synchronous reset signal (reset). Memory-mapped I/O signals, such as address, writedata, read, write, and chipselect, facilitate communication between the processor and the accelerator. At the core of the design are a **Register Map** and a Write Block, which receive data from the processor and route it to 16 parallel Accumulator Pipelines, labeled from PC0 to PC15. These pipelines accumulate partial sums from incoming training data across multiple cycles.

The accumulated outputs from all pipelines feed into an **18-bit Adder Tree**, which computes global statistics such as:

$$S_1 = \sum x, \quad S_2 = \sum y, \quad S_3 = \sum x^2, \quad S_4 = \sum xy, \quad S_5 = \sum y^2$$

These intermediate statistical values are then passed through a series of **multipliers** and **subtractors** to calculate the higher-level regression terms:

 N_0 , N_1 , and D

where N_0 and N_1 are the numerators used in the computation of the slope and intercept of the regression line, and D is the common denominator.

Finally, all computed values, along with a status signal MD (Master Done), are routed to a **Multiplexer (MUX)**. This MUX connects to the **readdata** bus, enabling the processor to selectively access internal results based on the supplied address.

Final Stage — Root Addition:

• Final adder computes:

result = SumA + SumB

• The result is stored in a final register before output

Key Design Features

- Fully Pipelined: Every adder is followed by a register, enabling high-throughput processing.
- **Balanced Tree Structure:** Reduces fanout and critical path delay, ideal for timingoptimized implementations.
- Uniform Bit Width: All operations are performed with 18-bit unsigned values, maintaining consistency.
- **FPGA-Friendly Design:** Optimized for synthesis and implementation in modern FPGAs using DSP slices and logic blocks.

Combinational 18-bit Adder

Fig. 7 shows a **pipelined binary adder tree** designed to compute the sum of 16 parallel 18-bit unsigned input values, labeled from op0 to opF. Each input is first registered to align timing and introduce pipelining. The architecture groups these inputs into adjacent pairs and sums them using multiple stages of binary adders. The tree structure ensures that additions are performed with *logarithmic depth*, reducing the 16 inputs down to a single final output in four addition stages.

Each addition operation is followed by a pipeline register, allowing every stage to operate in a fully pipelined manner. This design ensures a high-throughput datapath in which one sum result can be computed on every clock cycle once the pipeline is filled. The final 18-bit result is produced at the **result** output port, making the design suitable for applications in real-time signal processing or statistical summation workloads.

The adder tree employs a balanced binary reduction strategy with:

- Stage 1: 8 parallel adders for 8 pairwise additions.
- Stage 2: 4 adders for the second-level reduction.
- Stage 3: 2 adders to compute two intermediate sums.
- Stage 4: Final adder to compute the total sum.

By fully pipelining each stage, the design achieves:



Figure 7: Combinational 18-bit Adder

- Minimal critical path delay between stages.
- High clock frequency compatibility for FPGA implementations.
- Improved timing closure with reduced combinational fanout.
- Scalability to wider bit-widths or more input operands with minor structural changes.

This module is used within the Linear Regression Accelerator as a core summation unit to compute statistical accumulations such as $\sum x$, $\sum y$, $\sum x^2$, and $\sum xy$, leveraging its efficient structure for fast, large-scale accumulation operations in hardware.

Register Map Tables

It defines how software (e.g., a CPU or host controller) can interact with hardware using memory-mapped registers. It acts as a bridge between the software control layer and the hardware datapath by exposing internal hardware states, controls, and outputs at specific memory addresses. This abstraction enables software to configure, monitor, and trigger hardware operations without direct intervention in signal-level control. It also facilitates easy integration into driver frameworks and high-level control applications.

a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0	
0	0	0	0	0	0	0	0	0	0	Master Done (inform completion of operation)
0	0	0	0	0	0	0	0	0	1	N_0 (Used for w_0 computation)
0	0	0	0	0	0	0	0	1	0	N_1 (Used for w_1 computation)
0	0	0	0	0	0	0	0	1	1	D (Used for both w_0 and w_1)
0	0	0	0	0	0	0	1	0	0	S_1 (Provided for users who want all outputs)
0	0	0	0	0	0	0	1	0	1	S_2 (Provided for users who want all outputs)
0	0	0	0	0	0	0	1	1	0	S_3 (Provided for users who want all outputs)
0	0	0	0	0	0	0	1	1	1	S_4 (Provided for users who want all outputs)
0	0	0	0	0	0	1	0	0	0	S_5 (Provided for users who want all outputs)

Figure 8: Read Address Mapping

Read Address Mapping

Fig 8 shows the 10-bit address encoding used to access the results. Each address corresponds to a specific pipeline output or flag:

- Address 0x000 selects the MASTER_DONE flag, used to indicate completion of the regression computation.
- Address 0x001 maps to NO, the partial sum relevant for w_0 's calculation.
- Address 0x002 selects N1, used for computing w_1 .
- Address 0x003 accesses D, a value reused by both coefficient computations.
- Addresses 0x004 through 0x008 represent stages S1 through S5, and are mainly provided for debug or verbose mode where all pipeline outputs are required.

Each of these registers is mapped via a read multiplexer, controlled by address bits a_3 to a_0 , and the result is driven onto the **READDATA** bus. The output is valid only when the CPU initiates a read operation (i.e., when WRITE = 0) to one of these addresses with CHIPSELECT asserted. Since the accelerator module is memory-mapped, the CPU communicates via standard read/write protocols, ensuring synchronization without interrupt handling.

To support the complete address decoding logic, the accelerator treats all addresses with $a_9 = 0$ and WRITE = 0 as read operations.

Read Register Map

fig. 9 provides a memory map of readable output registers of the accelerator, showing how each pipeline stage's result or status signal is placed in memory. These offsets represent addresses accessible via the Avalon interface by the processor.

The table clearly distinguishes between control/status bits (like Master Done) and result values (like N_0 , N_1 , D, and S_1-S_5). The Master Done flag is encoded at offset 0, bit 0, with

Offset (Hex)	Bit	Description
0	31:1	Unused
0	0	Master Done
1	31:0	Signed N_0
2	31:0	Signed N_1
3	31:0	Signed D
4	31:0	Unsigned S_1
5	31:0	Unsigned S_2
6	31:0	Unsigned S_3
7	31:0	Unsigned S_4
8	31:0	Unsigned S_5

Figure 9: Read Register Map

the remaining bits unused. The rest of the offsets (1 through 8) contain signed or unsigned 32-bit results from various computation stages of the accelerator.

- Offset 0:
 - Bit 0: Master Done (set to 1 if the accelerator has completed computation).
 - Bits 31–1: Unused.
- Offsets 1–3 (Signed Values):
 - Offset 1: Signed result of N_0 (used in computing w_0).
 - Offset 2: Signed result of N_1 (used in computing w_1).
 - Offset 3: Signed result D (common to both weights).
- Offsets 4–8 (Unsigned Values):
 - Represent unsigned outputs S_1 through S_5 .
 - Provided primarily for full-output tracing or debugging purposes.

a_9	a_8	a_7	a_6	a_5	a_4	a_3	a_2	a_1	a_0	
0	0	0	0	0	0	0	0	0	0	Write to Address 0 in Memory $(a_{8:0} = 1)$
0	0	0	0	0	0	0	0	0	1	Write to Address 1 in Memory $(a_{8:0} = 1)$
0	0	0	0	0	0	0	0	1	0	Write to Address 2 in Memory $(a_{8:0} = 2)$
÷	÷	÷	÷	÷	÷	÷	÷	÷	÷	:
0	1	1	1	1	1	1	1	1	1	Write to Address 511 in Memory $(a_{8:0} = 511)$
1	0	0	0	0	0	0	0	0	0	Write Reset
1	y_8	y_7	y_6	y_5	y_4	y_3	y_2	y_1	y_0	Write Go with Count of Data y

Figure 10: Write Address Encoding

Write Address Encoding

Fig. 10 illustrates the address decoding scheme for write operations in the Linear Regression Accelerator. The 10-bit address field $(a_9 \text{ to } a_0)$ determines whether a memory address is being written to, or a control operation (reset or go) is being issued. When the most significant bit $a_9 = 0$, the operation targets the training memory and stores input data (typically X and Y pairs). The address lines $a_8 : 0$ define the specific memory location, allowing for up to 512 entries. When $a_9 = 1$, the address is interpreted as a control command—either to reset the accelerator or to trigger the computation (go), where the lower 8 bits specify the number of input samples (count).

• Data Write Operations:

- $-a_9 = 0 \rightarrow$ Memory write mode.
- $-a_8:0$ defines the memory address (range: 0 to 511).
- Used to write training data into internal memory.

• Control Operations:

- $-a_9 = 1$ and all lower bits $= 0 \rightarrow$ Write Reset.
- $-a_9 = 1$ and remaining bits = count value $y[7:0] \rightarrow$ Write Go (start operation with input count y).

Write Register Map

Fig. 11 defines the memory layout for training data in the Linear Regression Accelerator. The range from offset 0x000 to 0x1FF is used to store input pairs (x, y) for regression computation. Each 32-bit word is partially used: only the lower 8 bits are valid, where

Offset (Hex)	Bit	Description
	31:8	Unused
$0-1\mathrm{FF}$	7:4	y
	3:0	x
200 - 2FF	31:0	Unused

Figure 11: Write Register Map

bits [3:0] encode x and bits [7:4] encode y. The upper bits [31:8] are unused and may be written as zeros. The subsequent address space from 0x200 to 0x2FF is completely unused and is likely reserved for padding, alignment, or future extension.

This compact representation is optimized for 4-bit quantized input values, minimizing memory usage while maintaining high data throughput. The structure also allows software to stream input pairs into the accelerator sequentially, simplifying buffer management and reducing logic complexity. Such a layout is particularly well-suited for FPGA-based accelerators, where memory efficiency and deterministic access patterns are critical.

- Offset Range 0x000 { 0x1FF:

- * Total of 512 entries.
- * Bits [3:0]: Encodes input x.
- * Bits [7:4]: Encodes input y.
- * Bits [31:8]: Unused.
- Offset Range 0x200 { 0x2FF:
 - * All bits [31:0] are unused.
 - * Reserved for future use or address padding.

Timing Diagrams

8-bit Block Simulation Analysis

The lr_acc_8b module implements a pipelined datapath designed to accumulate essential statistical values for linear regression, using 8-bit input streams. The included

Fitter Summary				
		EntityInstance	Table of Contents	Slow 1100mV 85C Model Fmax Summary
< <ri><<ri><<ri><<ri><</ri></ri></ri></ri>		Cyclone V: SCSEMASF31C6	E Flow Summary	 <<fiter>></fiter>
Fitter Status	Successful - Tue May 13 02:15:41 2025	y = 0 for on the set of the se	 Row Settings Flow Non-Default Global Settings Flow Elapsed Time 	Finax Restricted Finax Clock Name Note 1 335.23 MHz 310.08 MHz clk (um)n)
Quartus Prime Version	21.1.0 Build 842 10/21/2021 SJ Lite Edition		Flow OS Summary	
Revision Name	lr_acc_8b_wrapper		+ Analysis & Synthesis	
Top-level Entity Name	lr_acc_8b_wrapper		Fitter	
Family	Cyclone V		Parallel Compilation	
Device	5CSEMA5F31C6		Incremental Compilation Sect	
Timing Models	Final		Resource Section	
Logic utilization (in ALMs)	74 / 32,070 (< 1 %)		Device Options Operating Settings and Condit	
Total registers	193	t Participa - Road	Estimated Delay Added for Hc Messages	
Total pins	76 / 457 (17 %)	Task Time	Suppressed Messages Elow Messager	
Total virtual pins	0	Compile Design O0:00:45 Analysis & Synthesis O0:00:11	Flow Suppressed Messages Arcombler	
Total block memory bits	0 / 4,065,280 (0 %)	Fitter (Place & Route) O0:00:20	Timing Analyzer Summany	
Total RAM Blocks	0/397(0%)	Iming Analysis O0:00:07	Parallel Compilation	
Total DSP Blocks	2 / 87 (2 %)	EDA Natilist Writer Edit Settings	Clocks	
Total HSSI RX PCSs	0	Program Device (Open Programmer)	TT Fmax Summary	
Total HSSI PMA RX Deserializers	0		Setup Summary Hold Summary	
Total HSSI TX PCSs	0		Recovery Summary	
Total HSSI PMA TX Serializers	0		Minimum Pulse Width Sun	
Total PLLs	0/6(0%)		Slow 1100mV 0C Model Fast 1100mV 85C Model	This panel reports FMAX for every clock in the design, regardless that the duty cycle (in terms of a percentage) is maintained. Alter
Total DLLs	0/4(0%)	4	+ East 1100mV/0C Model	•

simulation results validate the correctness and efficiency of the internal datapath.







Figure 14: Timing Diagram

The lr_acc_8b module implements a pipelined datapath designed to accumulate essential statistical values for linear regression, using 8-bit input streams. The included waveform captures a representative simulation cycle-by-cycle, validating correct temporal behavior and functional correctness.

- Inputs (x, y): Sequential data pairs applied after a reset. Timing is aligned with the testbench vectors to ensure synchronization with pipeline stages.
- sum_x / sum_y : These signals accumulate the total of all observed x and y values, respectively, and are used to calculate the mean and other statistical moments.
- sum_x_squared: Validates the squaring and accumulation of each x sample, representing $\sum x^2$, needed in slope denominator computation.
- **sum_XY**: Captures the accumulated sum of products $x \cdot y$, forming the numerator for the slope $(m = \frac{\sum(xy) n\bar{x}\bar{y}}{\sum(x^2) n\bar{x}^2})$.

This block is essential for real-time regression analysis, offering throughput-optimized, low-bitwidth accumulation suitable for embedded and resource-constrained environments. The successful simulation confirms the block's readiness for integration into the full datapath of the regression accelerator.

Pipelined Multiplier Simulation Analysis

sful - Tue May 13 01:01:02 2025 Build 842 10/21/2021 SJ Lite Edition ler wrapper	EntitysInstance yclone V: 5CSEMA5F31C6 • multiplier_wrapper 2		Table of Contents	Slow 1100mV 85C	Model Fmax Sum	nary
sful - Tue May 13 01:01:02 2025 Build 842 10/21/2021 SJ Lite Edition ler wrapper	yclone V: 5CSEMA5F31C6 ▶ multiplier_wrapper ਾੱb		Flow Summary Flow Settings Flow Non-Default Global Settings	< <filter>> Fmax</filter>		
sful - Tue May 13 01:01:02 2025 Build 842 10/21/2021 SJ Lite Edition ier wrapper	▶ multiplier_wrapper rb		Flow Settings Flow Non-Default Global Settings	Fmax		
Build 842 10/21/2021 SJ Lite Edition ier wrapper			and the second second	1 400.0 MHz	Restricted Fmax	Cl
ier wrapper			Flow Elapsed Time Flow QS Summary			
			Flow Log Analysis & Synthesis			
ier_wrapper			Fitter Flow Messages			
e V			Flow Suppressed Messages Assembler			
A5F31C6			Timing Analyzer Summary			
			Parallel Compilation			
,070 (< 1 %)			Clocks			
		•	Finax Summary			
7 (9%)	Compilation	- I 0 0 X	Setup Summary			
· · · · · · · · · · · · · · · · · · ·	Task	Time 00:00:42	Recovery Summary			
55,280 (0 %)	Analysis & Synthesis Fitter (Place & Route)	00:00:09	Minimum Pulse Width Sun			
(0%)	Assembler (Generate programming files)	00:00:07	 Slow 1100mV 0C Model Emax Summary 			
1%)	EDA Netlist Writer	00.00.07	E Setup Summary			
	Edit Settings Program Device (Open Programmer)		Recovery Summary			
			Minimum Pulse Width Sun			
			Fast 1100mV 85C Model Fast 1100mV 0C Model			
			Multicorner Timing Analysis S			
)%)			Clock Transfers Report TCCS	This panel reports	EMAX for every cir	wk in
)%)			Report RSKM	that the duty cycle	(in terms of a perc	entag
	A5F31C6 070 (< 1 %) 7 (9 %) 5,280 (0 %) (0 %) 1 %)	ASF31C6 070 (< 1 %) 7 (9 %) 5,280 (0 %) (0 %) 1 %) %) %)	ASF31C6 070 (< 1 %) 7 (9 %) 5,280 (0 %) (0 %) 1 %) %)	A5F31C6 070 (< 1 %)	ASF31C6 070 (< 1 %) 7 (9 %) 5,280 (0 %) (0 %) 1 %) %) %) %) %) %) %) %)	ASF31C6 070 (< 1 %) 7 (9 %) 5,280 (0 %) (0 %) 1 %) %) %) %) %) %) %) %)

Figure 15: Fitter Analysis

Figure 16: Fmax Analysis



Figure 17: Timing Diagram

This simulation verifies the behavior of a pipelined multiplier module (tb_multiplier) receiving sequential inputs X and Y. Key signals demonstrate staged pipeline computation and throughput efficiency.

- Inputs: X and Y increment per clock cycle, latched on rising edges.
- Pipelining: Internal signals (dut1, dut2, data_reg) confirm multi-stage buffering.
- Output: mult_out delivers $X \cdot Y$ with ~3-cycle latency. Examples:
 - * $X = 4, Y = 7 \Rightarrow \texttt{mult_out} = 28$
 - * $X = 25, Y = 42 \Rightarrow \texttt{mult_out} = 1050$
 - * $X = 38, Y = 34.6 \Rightarrow \texttt{mult_out} = 1316$
- **Conclusion:** Accurate pipelined multiplication with consistent throughput and result stability.

Signed Adder/Subtractor Simulation Analysis

The simulation waveform in Figure . illustrates the behavior of the tb_signed_adder _subtractor module, which performs signed arithmetic based on a control signal. The inputs dataa and datab are 16-bit signed values, and the add_sub signal determines whether an addition (add_sub = 0) or subtraction (add_sub = 1) is performed. The result output reflects the correct operation for each input pair and control signal setting.

For example, when dataa = 62756, datab = 40577, and add_sub = 1, the module correctly computes the result 22179 (dataa - datab). The internal DUT signals (dut/dataa, dut/datab, and dut/result) match the top-level signals, verifying proper signal propagation. Additionally, the expected output matches result across all cycles, confirming functional correctness.

All signal transitions are aligned with the rising clock edge, demonstrating proper synchronous design behavior. The pipeline exhibits no glitches or misalignments, validating the timing integrity of the module. This confirms that the signed adder/subtractor operates reliably under pipelined conditions with accurate arithmetic and control logic.



Figure 18: Fitter Analysis



Figure 20: Timing Diagram

Single-Port RAM Timing Analysis

The timing diagram in Fig. shows the behavior of the tb_single_port_ram module during sequential read operations. The address line increments with each rising clock

edge, and the corresponding data appears at the output q after a one-cycle delay, confirming typical synchronous single-port RAM behavior. The output values align with the initialized memory contents, validating functional correctness.

- 1-cycle latency: Output data appears exactly one clock after the address input.
- Synchronous operation: All transitions are clock-aligned.
- Full address sweep: Addresses range from 0 to 511, covering the entire memory space.
- Correct outputs: Data values match memory initialization, proving proper read access.
- Clean waveform: No glitches or misaligned transitions are observed.

Fitter Summary				
< <filter>></filter>		🗋 🗖 🖶 🤟 👘 🔿	C single_port_ram_with_init +	∠ � ☆ ☆ @ ▶ ≯ ₺ ♀ ✿ ≛ ऄॗॾ▌ ♥
Fitter Status	Successful - Tue May 13 00:24:14 2025	Project Navigator 🔥 Hierarchy 🔹 🔍 🗏 🛞 📾	Compilation Report - single_port_rar	n_with_init ×
Our stars Deleve Mender		Entitycinstance	Table of Contents	Slow 1100mV 85C Model Fmax Summary
Quartus Prime version	21.1.0 Build 842 10/21/2021 SJ Lite Edition	 memory_wrapper db 	Flow Summary Flow Settings	Fmax Restricted Fmax Clock Name Note
Revision Name	single_port_ram_with_init		Flow Non-Default Global Settings	1 483.33 MHz 315.06 MHz clk limin]
Top-level Entity Name	memory_wrapper		Flow OS Summary Flow Log	
Family	Cyclone V		Analysis & Synthesis Fitter	
Device	5CSEMA5F31C6		Flow Messages Flow Suppressed Messages	
Timing Models	Final		Assembler Timing Analyzer B Summany	
Logic utilization (in ALMs)	5 / 32,070 (< 1 %)		Parallel Compilation SDC File List	
Total registers	12		Clocks F Slow 1100m/V 85C Model	
Total pins	11 / 457 (2 %)	4 P	Fmax Summary Timing Closure Recommer	
Total virtual pins	0	Tasks Compliation * II (#) (8) (8) Task	Hold Summary	
Total block memory bits	256 / 4,065,280 (< 1 %)	Compile Design Analysis & Synthesis	Removal Summary Minimum Pulse Width Sum	
Total RAM Blocks	1/397(<1%)	Fitter (Place & Route) Assembler (Generate programming	Metastability Summary Slow 1100mV 0C Model	
Total DSP Blocks	0/87(0%)	EDA Netlist Writer	Setup Summary Hold Summary	
Total HSSI RX PCSs	0	Edit Settings Program Device (Open Programmer)	Recovery Summary	
Total HSSI PMA RX Deserializers	0		Minimum Pulse Width Sun Metastability Summary	
Total HSSI TX PCSs	0		Fast 1100mV 85C Model Fast 1100mV 0C Model Multicorear Timing technic 5	
Total HSSI PMA TX Serializers	0		Advanced I/O Timing Clock Transfers	
Total PLLs	0/6(0%)		Report TCCS	This panel reports FMAX for every clock in the design, regardless of the user-specified clock periods. FMAX [In terms of a percentage] is maintained. Altera recommends that you always use clock constraints and other
Total DLLs	0/4(0%)	6	Imconstrained Paths	
		Fig	rure 22: I	Fmax Analysis
				1110011 1 111001 / 010

Figure 21: Fitter Analysis



Figure 23: Timing Diagram of Sequential Read Access

Unsigned Adder Timing Analysis

The waveform in Fig. validates the behavior of the tb_unsigned_adder module under continuous stimulus. The input operands dataa and datab increment on each rising

clock edge, and the output result reflects their unsigned sum. The output is synchronized to the clock and verified against expected values, ensuring correctness. With the carry-in cin signal held at zero throughout, this test validates simple unsigned addition.

- Operand Sweep: Inputs range from 0–155, covering a wide span of test cases.
- Correct Output: result matches the expected sum at every clock cycle.
- Clock Sync: All transitions are aligned with rising edges of clk.
- No Carry-In Effect: With cin = 0, the test checks pure two-input addition.
- DUT Integrity: dut/result and top-level result signals match exactly.

Fitter Summary					
< <filter>></filter>		: 🗋 📷 🖬 🤟 🗇	n c adder_wra	pper 🔹 🖌 🎸 🎸	♦ 💿 ► 🔶 🛇 & ♥ 🚮 🦻
Fitter Status Quartus Prime Version Revision Name Top-level Entity Name Family Device	Successful - Tue May 13 00:42:43 2025 21.1.0 Build 842 10/21/2021 SJ Lite Edition adder_wrapper adder_wrapper Cyclone V 5CSEMA5F31C6	Projet Kakupita Entity Induces Grant V 2020AV3110 > adder wrapper fs	AHerarchy - 0.00 %	Compliation Report - adder, wanger Table of contents Compliation Report - adder, wanger The or sammary The or sammary The or Sammary The of Sammary	X Stor Strong PC Head Fand Sommy Control Final Control Final Control Final Addition Final Reproced Final Control Final
Timing Models Logic utilization (in ALMs) Total registers Total pins Total virtual pins Total block memory bits	Final 18 / 32,070 (< 1 %) 44 20 / 457 (4 %) 0 0 / 065,280 (0 %)	Tabls 0	mp[ation	Printing Analyzer Printing Analyzer Printing Completion Printing Pr	
Total DSP Blocks Total HSSI RX PCSs Total HSSI PMA RX Deserializers Total HSSI TX PCSs Total HSSI PMA TX Serializers Total PLI s	0/39/(0%) 0/87(0%) 0 0 0 0 0/6(0%)		programming files) 00:00:59 00:00:10 rogramme?	See 110am/02 Model Trac Summary See Jummary Hold Summary Hold Summary Hold Summary Hold Summary Hold Summary Headsaldly Summary	
Total DLLs	0/4(0%)	٩		Report TCCS Report RSKM	This panel reports FMAX for every clock in the design, regardless of the user- that the duty cycle (in terms of a percentage) is maintained. Altera recommen

Figure 24: Fitter Analysis

Figure 25: Fmax Analysis



Figure 26: Waveform of unsigned adder showing sum result and DUT agreement

18-bit Combinational Adder Timing Analysis

The comb_adder_18b module implements a fully combinational 18-bit adder tree, accepting sixteen unsigned operands (op0 to opF) as inputs. These are grouped pairwise and summed hierarchically in multiple levels—first forming intermediate sums such as op01, op23, and so on—until a total result is produced. The final output signal matches both expected_sum and captured_result, confirming functional correctness.

Since the output is computed within the same cycle as the adder_start signal, the design demonstrates true combinational behavior with zero latency. This structure is particularly beneficial in datapaths where low-latency parallel accumulation is required.

Since the output is computed within the same cycle as the adder_start signal, the design demonstrates true combinational behavior with zero latency. This structure is particularly beneficial in datapaths where low-latency parallel accumulation is required.

- Zero-Latency Result: Output is computed in the same cycle as the start signal.
- Adder Tree Hierarchy: Pairwise additions build intermediate results up to a 240-sum.
- Waveform Matching: All output signals match expected and captured values.
- Synthesis Success: Design meets timing and resource constraints based on fitter and Fmax analysis.

Fitter Summary	
< <filter>></filter>	
Fitter Status	Successful - Tue May 13 03:01:05 2025
Quartus Prime Version	21.1.0 Build 842 10/21/2021 SJ Lite Edition
Revision Name	comb_adder_18b_wrapper
Top-level Entity Name	comb_adder_18b_wrapper
Family	Cyclone V
Device	5CSEMA5F31C6
Timing Models	Final
Logic utilization (in ALMs)	112 / 32,070 (< 1 %)
Total registers	231
Total pins	22 / 457 (5 %)
Total virtual pins	0
Total block memory bits	0 / 4,065,280 (0 %)
Total RAM Blocks	0/397(0%)
Total DSP Blocks	0/87(0%)
Total HSSI RX PCSs	0
Total HSSI PMA RX Deserializers	0
Total HSSI TX PCSs	0
Total HSSI PMA TX Serializers	0
Total PLLs	0/6(0%)
Total DLLs	0 / 4 (0 %)

Figure 27: Fitter Analysis



Figure 28: Fmax Analysis

4 .	Msgs					
/tb comb adder 18b/clk	1'd0					
/tb comb adder 18b/reset	1'd0					
/tb comb adder 18b/adder start	1'd0					
+ /tb_comb_adder_18b/result	22'd240	0			240	
+ /tb comb adder 18b/expected sum	22'd240		240			
+ 🔶 /tb_comb_adder_18b/captured_result	22'd240				24	0
+ 🔶 /tb_comb_adder_18b/i	32'd16	16				
/tb_comb_adder_18b/dut/clk	1'd0					1
/tb_comb_adder_18b/dut/reset	1'd0					
/tb_comb_adder_18b/dut/adder_start	1'd0					
🕀 🌧 /tb_comb_adder_18b/dut/op0	18'd0	0				
🛨 🌧 /tb_comb_adder_18b/dut/op1	18'd2	0	2			
🛨 🌧 /tb_comb_adder_18b/dut/op2	18'd4	0	4			
🗄 🧼 /tb_comb_adder_18b/dut/op3	18'd6	0	<u>(</u> 6			
🕒 🌧 /tb_comb_adder_18b/dut/op4	18'd8	0	<u> (8</u>			
Itb_comb_adder_18b/dut/op5	18'd10	0	<u>(10</u>			
🛨 🌧 /tb_comb_adder_18b/dut/op6	18'd12	0	12			
	18'd14	0	14			
🛨 🧼 /tb_comb_adder_18b/dut/op8	18'd16	0	<u>, 16</u>			
	18'd18	0	18			
	18'd20	0	120			
	18'd22	0	122			
	18'd24	0	<u>, 24</u>			
/tb_comb_adder_18b/dut/opD	18'd26	0	126			-
	18'd28	0	<u>, 28</u>			
/tb_comb_adder_18b/dut/opF	18'd30	0	<u>į</u> 30)(-
/tb_comb_adder_18b/dut/result	18'd240	0	1/		<u>į</u> 240	-
+ /tb_comb_adder_18b/dut/op01	18'd2	0	12			=
tb_comb_adder_18b/dut/op23	18'd10	0	10			=
the comb_adder_18b/dut/op45	18/018	0	<u>, 18</u>			=
to comb_adder_18b/dut/op67	18026		126 V 24			=
+ VID_comb_adder_18b/dut/op89	18034		<u>134</u> 142			=
th comb_adder_18b/dut/opCD	18042	10 10	142			=
th comb_adder_18b/dut/opEE	10'050	10	<u>150</u>			=
th comb_adder_18b/dut/local_op01	10'00	6	<u>, 100</u>			=
th comb adder 18b/dut/local op23	18'd10	6	<u>, 72</u> 110			=
th comb adder 18b/dut/local op25	18'd18	0	118 V 18			=
th comb_adder_18b/dut/local_op43	18'd26	0	126			
+ + /th comb adder 18b/dut/local on89	18'd34	0	134			=
+ + /tb comb adder 18b/dut/local opAB	18'd42	0	142			=
+ + /tb comb adder 18b/dut/local opCD	18'd50	0	1 50			=
+ > /tb comb adder 18b/dut/local opEF	18'd58	0	ľ 58			
+ > /tb comb adder 18b/dut/op0123	18'd12	0	¥ 12			F
+ > /tb comb adder 18b/dut/op4567	18'd44	0	<u>44</u>			
+ /tb comb adder 18b/dut/op89AB	18'd76	0	76			
★ /tb comb adder 18b/dut/opCDEF	18'd108	0	108	3		
→ /tb comb adder 18b/dut/local op0123	18'd12	0		12		
+ 🔶 /tb_comb_adder_18b/dut/local_op4567	18'd44	0		44		
	18'd76	0		76		
How the comb_adder_18b/dut/local_opCDEF	18'd108	0		108		
	18'd56	0		(56		
How the comb adder 18b/dut/op89ABCDEF	18'd184	0		184		
	18'd56	0		(56		
→ /tb_comb_adder_18b/dut/local_op89A	18'd184	0		<u>,</u> 18	4	
the same addae 10b/dublical sum	101-010	<u> </u>		1 1		
Now	85000 ps)S		50000 ps		
Cursor 1	0 ne	0 ns				

Figure 29: Timing diagram of 18-bit combinational adder tree showing intermediate and final sums

Verilog Code

$comb_adder_18b.sv$

```
`include "lr_acc.svh"
module comb_adder_18b (
    //Port Structure
    input clk,
    input reset,
    input logic adder_start,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            op0,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            op1,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            op2,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            op3,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            op4,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            op5,
                                            op6,
    input logic [`ADDER_IN_SIZE_18b-1:0]
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            op7,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            op8,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            op9,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            opA,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            opB,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            opC,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            opD,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            opE,
    input logic [`ADDER_IN_SIZE_18b-1:0]
                                            opF,
    output logic [`ADDER_OUT_SIZE_18b-1:0] result
);
    logic [`ADDER_IN_SIZE_18b-1:0]
                                      op01;
    logic [`ADDER_IN_SIZE_18b-1:0]
                                      op23;
    logic [`ADDER_IN_SIZE_18b-1:0]
                                      op45;
    logic [`ADDER_IN_SIZE_18b-1:0]
                                      op67;
    logic [`ADDER_IN_SIZE_18b-1:0]
                                      op89;
    logic [`ADDER_IN_SIZE_18b-1:0]
                                      opAB;
    logic [`ADDER_IN_SIZE_18b-1:0]
                                      opCD;
    logic [`ADDER_IN_SIZE_18b-1:0]
                                      opEF;
    unsigned_adder #(.WIDTH(18)) adder_inst0 (
        .dataa(op0),
        .datab(op1),
        .cin('0),
        .result(op01)
    );
    unsigned_adder #(.WIDTH(18)) adder_inst1 (
        .dataa(op2),
```

```
.datab(op3),
    .cin('0),
    .result(op23)
);
unsigned_adder #(.WIDTH(18)) adder_inst2 (
    .dataa(op4),
    .datab(op5),
    .cin('0),
    .result(op45)
);
unsigned_adder #(.WIDTH(18)) adder_inst3 (
    .dataa(op6),
    .datab(op7),
    .cin('0),
    .result(op67)
);
unsigned_adder #(.WIDTH(18)) adder_inst4 (
    .dataa(op8),
    .datab(op9),
    .cin('0),
    .result(op89)
);
unsigned_adder #(.WIDTH(18)) adder_inst5 (
    .dataa(opA),
    .datab(opB),
    .cin('0),
    .result(opAB)
);
unsigned_adder #(.WIDTH(18)) adder_inst6 (
    .dataa(opC),
    .datab(opD),
    .cin('0),
    .result(opCD)
);
unsigned_adder #(.WIDTH(18)) adder_inst7 (
    .dataa(opE),
    .datab(opF),
    .cin('0),
    .result(opEF)
);
logic [`ADDER_IN_SIZE_18b:0]
                               local_op01;
logic [`ADDER_IN_SIZE_18b-1:0]
                                 local_op23;
logic [`ADDER_IN_SIZE_18b-1:0]
                                 local_op45;
logic [`ADDER_IN_SIZE_18b-1:0]
                                 local_op67;
```

```
logic [`ADDER_IN_SIZE_18b-1:0]
                                    local_op89;
logic [`ADDER_IN_SIZE_18b-1:0] local_opAB;
logic [`ADDER_IN_SIZE_18b-1:0]
                                    local_opCD;
logic [`ADDER_IN_SIZE_18b-1:0]
                                    local_opEF;
 always_ff @(posedge clk or posedge reset) begin
    if (reset) begin
         local_op01 <= '0;</pre>
         local_op23 <= '0;</pre>
         local_op45 <= '0;</pre>
         local_op67 <= '0;</pre>
         local_op89 <= '0;</pre>
         local_opAB <= '0;</pre>
         local_opCD <= '0;</pre>
         local_opEF <= '0;</pre>
    end else begin
        local_op01 <= op01;</pre>
         local_op23 <= op23;</pre>
         local_op45 <= op45;</pre>
         local_op67 <= op67;</pre>
         local_op89 <= op89;</pre>
         local_opAB <= opAB;</pre>
         local_opCD <= opCD;</pre>
         local_opEF <= opEF;</pre>
    end
end
logic [`ADDER_IN_SIZE_18b-1:0]
                                    op0123;
logic [`ADDER_IN_SIZE_18b-1:0]
                                    op4567;
logic [`ADDER_IN_SIZE_18b-1:0]
                                    op89AB;
logic [`ADDER_IN_SIZE_18b-1:0]
                                    opCDEF;
unsigned_adder #(.WIDTH(18)) adder_inst00 (
    .dataa(local_op01),
    .datab(local_op23),
    .cin('0),
    .result(op0123)
);
unsigned_adder #(.WIDTH(18)) adder_inst01 (
    .dataa(local_op45),
    .datab(local_op67),
    .cin('0),
    .result(op4567)
);
unsigned_adder #(.WIDTH(18)) adder_inst02 (
    .dataa(local_op89),
    .datab(local_opAB),
    .cin('0),
    .result(op89AB)
```

```
);
unsigned_adder #(.WIDTH(18)) adder_inst03 (
    .dataa(local_opCD),
    .datab(local_opEF),
    .cin('0),
    .result(opCDEF)
);
logic [`ADDER_IN_SIZE_18b-1:0]
                                   local_op0123;
logic [`ADDER_IN_SIZE_18b-1:0]
                                   local_op4567;
logic [`ADDER_IN_SIZE_18b-1:0]
                                   local_op89AB;
logic [`ADDER_IN_SIZE_18b-1:0] local_opCDEF;
always_ff @(posedge clk or posedge reset) begin
    if (reset) begin
        local_op0123 <= '0;</pre>
        local_op4567 <= '0;</pre>
        local_op89AB <= '0;</pre>
        local_opCDEF <= '0;</pre>
    end else begin
        local_op0123 <= op0123;</pre>
        local_op4567 <= op4567;</pre>
        local_op89AB <= op89AB;</pre>
        local_opCDEF <= opCDEF;</pre>
    end
end
logic [`ADDER_IN_SIZE_18b-1:0]
                                   op01234567;
logic [`ADDER_IN_SIZE_18b-1:0]
                                   op89ABCDEF;
unsigned_adder #(.WIDTH(18)) adder_inst000 (
    .dataa(local_op0123),
    .datab(local_op4567),
    .cin('0),
    .result(op01234567)
);
unsigned_adder #(.WIDTH(18)) adder_inst001 (
    .dataa(local_op89AB),
    .datab(local_opCDEF),
    .cin('0),
    .result(op89ABCDEF)
);
logic [`ADDER_IN_SIZE_18b-1:0] local_op01234567;
logic [`ADDER_IN_SIZE_18b-1:0] local_op89ABCDEF;
always_ff @(posedge clk or posedge reset) begin
    if (reset) begin
        local_op01234567 <= '0;</pre>
```

```
local_op89ABCDEF <= '0;</pre>
        end else begin
            local_op01234567 <= op01234567;</pre>
            local_op89ABCDEF <= op89ABCDEF;</pre>
        end
    end
    logic [`ADDER_IN_SIZE_18b-1:0] final_sum;
    // Final addition
    unsigned_adder #(.WIDTH(18)) adder_final0000 (
        .dataa(local_op01234567),
        .datab(local_op89ABCDEF),
        .cin('0),
        .result(final_sum)
    );
    // Registered final result
    logic [`ADDER_IN_SIZE_18b-1:0] final_sum_reg;
    always_ff @(posedge clk or posedge reset) begin
        if (reset)
            final_sum_reg <= '0;</pre>
        else
            final_sum_reg <= final_sum;</pre>
    end
    assign result = final_sum_reg;
endmodule
```

comb_mult_4b.sv

```
`include "lr_acc.svh"
module comb_mult_4b (
    input [`MULT_INPUT_SIZE_4b-1:0] opa,
    input [`MULT_INPUT_SIZE_4b-1:0] opb,
    output [`MULT_OUTPUT_SIZE_8b-1:0] result
);
    wire [`MULT_OUTPUT_SIZE_8b-1:0] p0 = opb[0] ? {4'b0, opa } : 8'b0;
    wire [`MULT_OUTPUT_SIZE_8b-1:0] p1 = opb[1] ? {3'b0, opa, 1'b0} : 8'b0;
    wire [`MULT_OUTPUT_SIZE_8b-1:0] p2 = opb[2] ? {2'b0, opa, 2'b0} : 8'b0;
    wire [`MULT_OUTPUT_SIZE_8b-1:0] p3 = opb[3] ? {1'b0, opa, 3'b0} : 8'b0;
    assign result = p0 + p1 + p2 + p3;
endmodule
```

lr_acc_8b.sv

```
`include "lr_acc.svh"
module lr_acc_8b (
    input logic
                                            clk,
    input logic
                                            reset,
    input logic
                                            start,
    input logic [`SPLIT_DATA_LEN-1:0]
                                            x,
    input logic [`SPLIT_DATA_LEN-1:0]
                                            y,
    output logic [`MULT_DATA_IN_LEN-1:0]
                                            sum_x,
    output logic [`MULT_DATA_IN_LEN-1:0]
                                            sum_y,
    output logic [`MULT_DATA_IN_LEN-1:0]
                                            sum_x_squared,
    output logic [`MULT_DATA_IN_LEN-1:0]
                                            sum_xy,
    output logic
                                             done
);
    // ----- Pipeline Stage 1: Register Inputs ------
    logic [`SPLIT_DATA_LEN-1:0] local_x, local_y;
                                local_start;
    logic
    always_ff @(posedge clk or posedge reset) begin
        if (reset) begin
            local_x <= '0;</pre>
            local_y <= '0;</pre>
            local_start <= 0;</pre>
        end else if (start) begin
            local_x <= x;</pre>
            local_y <= y;</pre>
            local_start <= 1;</pre>
        end else begin
            local_start <= 0;</pre>
        end
    end
    // ----- Pipeline Stage 2: Multiplier ------
    logic [`MULT_OUTPUT_SIZE_8b-1:0] local_x_squared, local_x_times_y;
    logic [`SPLIT_DATA_LEN-1:0] inp1, inp2;
    multiplier #(.WIDTH(4)) mult_x_x (
        .clk(clk),
        .X(local_x),
        .Y(local_x),
        .I1(),
        .I2(),
        .P(local_x_squared)
    );
```

```
multiplier #(.WIDTH(4)) mult_x_y (
   .clk(clk),
   .X(local_x),
   .Y(local_y),
   .I1(inp1),
   .I2(inp2),
    .P(local_x_times_y)
);
// ----- Delay Stage to Capture Multiplier Outputs -----
                              delay_reg;
logic
logic
                              local_local_start;
always_ff @(posedge clk or posedge reset) begin
    if (reset) begin
                          <= 0;
       delay_reg
       local_local_start <= 0;</pre>
    end else begin
       delay_reg
                     <= local_start;
       local_local_start <= delay_reg;</pre>
   end
end
// ----- Pipeline Register After Multipliers -----
logic [`MULT_OUTPUT_SIZE_8b-1:0] x_squared, x_times_y;
logic [`SPLIT_DATA_LEN-1:0]
                                inp_x, inp_y;
logic
                                start_next_stage;
logic [`SPLIT_DATA_LEN-1:0] x_pipe, y_pipe;
always_ff @(posedge clk or posedge reset) begin
   if (reset) begin
                      <= '0;
       inp_x
       inp_y
                      <= '0;
                     <= '0;
       x_squared
                      <= '0;
       x_times_y
                       <= '0;
       x_pipe
                      <= '0;
       y_pipe
       start_next_stage <= 0;</pre>
   end else if (local_local_start) begin
       x_pipe
                      <= local_x;
                      <= local_y;
       y_pipe
       x_squared
                     <= local_x_squared;
                     <= local_x_times_y;
       x_times_y
                      <= inp1; // corrected: use proper pipelined x
       inp_x
                      <= inp2;
                                   // corrected: use proper pipelined y
       inp_y
       start_next_stage <= 1;</pre>
    end else begin
       start_next_stage <= 0;</pre>
    end
end
```

```
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```

```
// ----- Pipeline Stage 3: Accumulator ------
    logic [`MULT_DATA_IN_LEN-1:0] local_sum_x, local_sum_y;
    logic [`MULT_DATA_IN_LEN-1:0] local_sum_xx, local_sum_xy;
    logic
                                   sum_done;
    always_ff @(posedge clk or posedge reset) begin
        if (reset) begin
            local_sum_x <= '0;</pre>
            local_sum_y <= '0;</pre>
            local_sum_xx <= '0;</pre>
            local_sum_xy <= '0;</pre>
            sum_done <= 0;</pre>
        end else if (start_next_stage) begin
            local_sum_x <= local_sum_x + inp_x;</pre>
            local_sum_y <= local_sum_y + inp_y;</pre>
            local_sum_xx <= local_sum_xx + x_squared;</pre>
            local_sum_xy <= local_sum_xy + x_times_y;</pre>
            sum_done
                      <= 1;
        end else begin
            sum_done <= 0;</pre>
        end
    end
    // ----- Output Assignment -----
   assign sum_x = local_sum_x;
assign sum_y = local_sum_y;
    assign sum_x_squared = local_sum_xx;
   assign sum_xy = local_sum_xy;
    assign done
                         = sum_done;
endmodule
```

```
lr_{acc_512_8b.sv}
```

```
`include "lr_acc.svh"
module lr_acc_512_8b (
   //Port Structure
    input logic
                                             clk,
    input logic
                                             reset,
    input logic [31:0]
                                             writedata,
    input logic
                                             write,
    input logic
                                             read,
    input
                                             chipselect,
    input logic [9:0]
                                             address,
    output logic [31:0]
                                             readdata
```

```
);
logic master_done;
memory [`DATA_SET_SIZE-1:0] mem_bank;
pc_tracker [`PIPELINE_SETS-1:0] pc_status_bank;
logic go;
logic [`ADDRESS_BITS_SIZE-1:0] pc;
logic done;
logic done_trigger;
logic [`MULT_DATA_IN_LEN-1:0] local_s_1;
logic [`MULT_DATA_IN_LEN-1:0] global_s_1, global_s_2, global_s_3, global_s_4,
\rightarrow global_s_5;
logic [`OUTPUT_DATA_LEN-1:0] s1s4_minus_s2s2;
logic [`OUTPUT_DATA_LEN-1:0] s3s4_minus_s2s5;
logic [`OUTPUT_DATA_LEN-1:0] s1s5_minus_s2s3;
always_ff @(posedge clk)
begin
           //Get data from SW
  if (reset)
  begin
    go <= 1'b0;
    master_done <= '0;</pre>
    done <= '0;
    done_trigger <= '0;</pre>
    local_s_1 <= '0;
    // global_s_1 <= '0;
    // global_s_2 <= '0;
    // global_s_3 <= '0;
    // global_s_4 <= '0;
    // global_s_5 <= '0;
    // s1s4_minus_s2s2 <= '0;
    // s3s4_minus_s2s5 <= '0;
    // s1s5_minus_s2s3 <= '0;
    for (int i = 0; i < `DATA_SET_SIZE; i++)</pre>
    begin
      mem_bank[i].x <= '0;</pre>
      mem_bank[i].y <= '0;</pre>
    end
  \operatorname{end}
  else if (chipselect && write)
  begin
              //Read data only when past operation is complete
    case (address[`ADDRESS_BITS_SIZE])
      1'h0:
               begin
                 if (!go)
                 begin
                   mem_bank[address[8:0]].x <= writedata[3:0];</pre>
                   mem_bank[address[8:0]].y <= writedata[7:4];</pre>
```

```
end
             end
    1'h1:
             begin
               if (address[8:0] == 0)
               begin
                 go <= 1'b0;
                 master_done <= '0;</pre>
                 done <= '0;
                 done_trigger <= '0;</pre>
                 local_s_1 <= '0;
    // global_s_1 <= '0;
    // global_s_2 <= '0;
    // global_s_3 <= '0;
    // global_s_4 <= '0;
    // global_s_5 <= '0;
    // s1s4_minus_s2s2 <= '0;
    // s3s4_minus_s2s5 <= '0;
    // s1s5_minus_s2s3 <= '0;
                 for (int i = 0; i < `DATA_SET_SIZE; i++)</pre>
                 begin
                   mem_bank[i].x <= '0;</pre>
                   mem_bank[i].y <= '0;</pre>
                 end
               end
               else
               begin
                 go <= 1'b1;
                 local_s_1 <= address[8:0];</pre>
               end
             end
  endcase
end
else if (chipselect && read)
            //Read data only when past operation is complete
begin
  case (address)
    10'h0:
    begin
      readdata <= master_done;</pre>
    end
    10'h1:
    begin
      readdata <= s1s4_minus_s2s2;</pre>
    end
    10'h2:
    begin
      readdata <= s3s4_minus_s2s5;</pre>
    end
    10'h3:
    begin
```

```
readdata <= s1s5_minus_s2s3;</pre>
```

```
end
      10'h4:
      begin
        readdata <= global_s_1;</pre>
      end
      10'h5:
      begin
        readdata <= global_s_2;</pre>
      end
      10'h6:
      begin
        readdata <= global_s_3;</pre>
      end
      10'h7:
      begin
        readdata <= global_s_4;</pre>
      end
      10'h8:
      begin
        readdata <= global_s_5;</pre>
      end
    endcase
  end
  if (done)
  begin
    done_trigger <= '1;</pre>
    done <= '0;
  end
  else if (done_trigger)
  begin
    master_done <= '1;</pre>
    done_trigger <= '0;</pre>
  end
  else if (pc == 496)
  begin
    go <= '0;
    done <= '1;</pre>
  end
end
logic [`PIPELINE_SETS-1:0] start ;
logic [`SPLIT_DATA_LEN-1:0] x [`PIPELINE_SETS-1:0];
logic [`SPLIT_DATA_LEN-1:0] y [`PIPELINE_SETS-1:0];
logic [`MULT_DATA_IN_LEN-1:0] sum_x [`PIPELINE_SETS-1:0];
logic [`MULT_DATA_IN_LEN-1:0] sum_y [`PIPELINE_SETS-1:0];
logic [`MULT_DATA_IN_LEN-1:0] sum_x_squared [`PIPELINE_SETS-1:0];
logic [`MULT_DATA_IN_LEN-1:0] sum_xy [`PIPELINE_SETS-1:0];
logic [`PIPELINE_SETS-1:0] local_done ;
```

```
logic [`PC_BITS_SIZE-1:0] local_pc [`PIPELINE_SETS-1:0];
logic [`PC_BITS_SIZE-1:0] dut_num;
logic master_start;
logic [`ADDRESS_BITS_SIZE-1:0] addr;
genvar i;
generate
  for (i = 0; i < `PIPELINE_SETS; i = i + 1)</pre>
  begin : lr_acc_gen
    lr_acc_8b lr_acc_8b_0 (
                 .clk(clk),
                 .reset(reset),
                 .start(start[i]),
                 .x(x[i]),
                 .y(y[i]),
                 .sum_x(sum_x[i]),
                 .sum_y(sum_y[i]),
                 .sum_x_squared(sum_x_squared[i]),
                 .sum_xy(sum_xy[i]),
                 .done(local_done[i])
               );
  end
endgenerate
always_ff @( posedge clk )
begin
  if (reset)
  begin
    pc <= '0;
    master_start <= '0;</pre>
    for (int i = 0; i < `PIPELINE_SETS; i++)</pre>
    begin
      start[i] <= '0;</pre>
      x[i] <= '0;
      y[i] <= '0;
    end
    addr <= '0;
    // master_done <= '0;</pre>
  // end
  // else if (!master_start & go)
  // begin
  // for (int i = 0; i < `PIPELINE_SETS; i++)</pre>
  11
      begin
  11
         start[i] <= '0;
  11
       end
  11
      master_start <= '1;</pre>
  // end
  end else if (go)
  begin
    for (int i = 0; i < `PIPELINE_SETS; i++)</pre>
```

```
begin
       addr <= pc + i;
       x[i] <= mem_bank[pc + i].x;</pre>
       y[i] <= mem_bank[pc + i].y;</pre>
       start[i] <= '1;</pre>
       pc_status_bank[i].pc <= pc + i;</pre>
       pc_status_bank[i].dut_num <= pc + i;</pre>
       pc_status_bank[i].done <= '0;</pre>
     end
    if(pc < 512)
    begin
       pc <= pc + 16;
    end
    else
    begin
      pc <= '0;
    end
    master_start <= '0;</pre>
  end
  else if (!go)
  begin
    for (int i = 0; i < `PIPELINE_SETS; i++)</pre>
    begin
       start[i] <= '0;</pre>
    \operatorname{end}
  end
end
logic global_start_adders;
always_ff @( posedge clk )
begin
  if(reset)
  begin
    for (int i = 0; i < `PIPELINE_SETS; i++)</pre>
    begin
       pc_status_bank[i].pc <= '0;</pre>
       pc_status_bank[i].dut_num <= '0;</pre>
       pc_status_bank[i].done <= '0;</pre>
       pc_status_bank[i].local_s_2 <= '0;</pre>
       pc_status_bank[i].local_s_3 <= '0;</pre>
      pc_status_bank[i].local_s_4 <= '0;</pre>
       pc_status_bank[i].local_s_5 <= '0;</pre>
    end
    global_start_adders <= '0;</pre>
  end
  else if (master_done)
  begin
    for (int i = 0; i < `PIPELINE_SETS; i++)</pre>
    begin
```

```
pc_status_bank[i].done <= '1;</pre>
     pc_status_bank[i].local_s_2 <= sum_x[i];</pre>
      pc_status_bank[i].local_s_3 <= sum_y[i];</pre>
      pc_status_bank[i].local_s_4 <= sum_x_squared[i];</pre>
     pc_status_bank[i].local_s_5 <= sum_xy[i];</pre>
    end
    global_s_1 <= local_s_1;</pre>
    global_start_adders <= '1;</pre>
  end
end
comb_adder_18b sum_s2_calc (
                .clk(clk),
                .reset(reset),
                 .adder_start(global_start_adders),
                 .op0 (pc_status_bank[ 0].local_s_2),
                 .op1 (pc_status_bank[ 1].local_s_2),
                 .op2 (pc_status_bank[ 2].local_s_2),
                  .op3 (pc_status_bank[ 3].local_s_2),
                  .op4 (pc_status_bank[ 4].local_s_2),
                  .op5 (pc_status_bank[ 5].local_s_2),
                  .op6 (pc_status_bank[ 6].local_s_2),
                 .op7 (pc_status_bank[ 7].local_s_2),
                  .op8 (pc_status_bank[ 8].local_s_2),
                  .op9 (pc_status_bank[ 9].local_s_2),
                  .opA (pc_status_bank[10].local_s_2),
                       (pc_status_bank[11].local_s_2),
                 .opB
                 .opC (pc_status_bank[12].local_s_2),
                  .opD (pc_status_bank[13].local_s_2),
                 .opE (pc_status_bank[14].local_s_2),
                 .opF (pc_status_bank[15].local_s_2),
                  .result(global_s_2)
               );
comb_adder_18b sum_s3_calc (
                .clk(clk),
                .reset(reset),
                 .adder_start(global_start_adders),
                 .op0 (pc_status_bank[ 0].local_s_3),
                  .op1 (pc_status_bank[ 1].local_s_3),
                 .op2 (pc_status_bank[ 2].local_s_3),
                  .op3 (pc_status_bank[ 3].local_s_3),
                 .op4 (pc_status_bank[ 4].local_s_3),
                 .op5 (pc_status_bank[ 5].local_s_3),
                 .op6 (pc_status_bank[ 6].local_s_3),
                 .op7 (pc_status_bank[ 7].local_s_3),
```

```
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```

```
.op8
                      (pc_status_bank[ 8].local_s_3),
                 .op9
                      (pc_status_bank[ 9].local_s_3),
                 .opA (pc_status_bank[10].local_s_3),
                 .opB
                      (pc_status_bank[11].local_s_3),
                 .opC
                      (pc_status_bank[12].local_s_3),
                      (pc_status_bank[13].local_s_3),
                 .opD
                 .opE (pc_status_bank[14].local_s_3),
                 .opF (pc_status_bank[15].local_s_3),
                 .result(global_s_3)
               );
comb_adder_18b sum_s4_calc (
                .clk(clk),
                .reset(reset),
                 .adder_start(global_start_adders),
                 .op0 (pc_status_bank[ 0].local_s_4),
                 .op1 (pc_status_bank[ 1].local_s_4),
                 .op2 (pc_status_bank[ 2].local_s_4),
                 .op3 (pc_status_bank[ 3].local_s_4),
                      (pc_status_bank[ 4].local_s_4),
                 .op4
                 .op5
                      (pc_status_bank[ 5].local_s_4),
                 .op6 (pc_status_bank[ 6].local_s_4),
                      (pc_status_bank[ 7].local_s_4),
                 .op7
                 .op8 (pc_status_bank[ 8].local_s_4),
                 .op9
                      (pc_status_bank[ 9].local_s_4),
                      (pc_status_bank[10].local_s_4),
                 .opA
                      (pc_status_bank[11].local_s_4),
                 .opB
                      (pc_status_bank[12].local_s_4),
                 .opC
                 .opD (pc_status_bank[13].local_s_4),
                 .opE (pc_status_bank[14].local_s_4),
                 .opF (pc_status_bank[15].local_s_4),
                 .result(global_s_4)
               );
comb_adder_18b sum_s5_calc (
                .clk(clk),
                .reset(reset),
                 .adder_start(global_start_adders),
                 .op0 (pc_status_bank[ 0].local_s_5),
                 .op1 (pc_status_bank[ 1].local_s_5),
                 .op2 (pc_status_bank[ 2].local_s_5),
                 .op3 (pc_status_bank[ 3].local_s_5),
                 .op4 (pc_status_bank[ 4].local_s_5),
                 .op5 (pc_status_bank[ 5].local_s_5),
                 .op6 (pc_status_bank[ 6].local_s_5),
                 .op7 (pc_status_bank[ 7].local_s_5),
```

```
.op8 (pc_status_bank[ 8].local_s_5),
                  .op9 (pc_status_bank[ 9].local_s_5),
                  .opA (pc_status_bank[10].local_s_5),
                  .opB (pc_status_bank[11].local_s_5),
                  .opC (pc_status_bank[12].local_s_5),
                  .opD (pc_status_bank[13].local_s_5),
                  .opE (pc_status_bank[14].local_s_5),
                  .opF (pc_status_bank[15].local_s_5),
                  .result(global_s_5)
                );
logic [`MULT_DATA_IN_LEN-1:0] local_global_s_1, local_global_s_2,
→ local_global_s_3, local_global_s_4, local_global_s_5;
always_ff @( posedge clk )
begin
  if(reset)
  begin
    local_global_s_1 <= '0;</pre>
    local_global_s_2 <= '0;</pre>
    local_global_s_3 <= '0;</pre>
    local_global_s_4 <= '0;</pre>
    local_global_s_5 <= '0;</pre>
  end
  else
  begin
    local_global_s_1 <= global_s_1;</pre>
    local_global_s_2 <= global_s_2;</pre>
    local_global_s_3 <= global_s_3;</pre>
    local_global_s_4 <= global_s_4;</pre>
    local_global_s_5 <= global_s_5;</pre>
  end
end
//assign s1_times_s4 = global_s_1 * global_s_4;
//assign s2_squared = qlobal_s_2 * qlobal_s_2;
//assign s3_times_s4 = global_s_3 * global_s_4;
//assign s2_times_s5 = global_s_2 * global_s_5;
//assign s1_times_s5 = global_s_1 * global_s_5;
//assign s2_times_s3 = global_s_2 * global_s_3;
logic [`DSP_MULT_OUT_SIZE-1:0] local_s1_times_s4;
logic [`DSP_MULT_OUT_SIZE-1:0] local_s2_squared;
logic [`DSP_MULT_OUT_SIZE-1:0] local_s3_times_s4;
logic [`DSP_MULT_OUT_SIZE-1:0] local_s2_times_s5;
logic [`DSP_MULT_OUT_SIZE-1:0] local_s1_times_s5;
logic [`DSP_MULT_OUT_SIZE-1:0] local_s2_times_s3;
```

```
multiplier m1 (
             .clk(clk),
                 (local_global_s_1),
             .Х
                  (local_global_s_4),
             .Ү
             .I1(),
             .I2(),
             .P (local_s1_times_s4)
           );
multiplier m2 (
             .clk(clk),
                  (local_global_s_2),
             .X
             .Ү
                  (local_global_s_2),
             .I1(),
             .I2(),
                (local_s2_squared)
             .P
           );
multiplier m3 (
             .clk(clk),
                  (local_global_s_3),
             .X
                  (local_global_s_4),
             .Υ
             .I1(),
             .I2(),
             .P (local_s3_times_s4)
           );
multiplier m4 (
             .clk(clk),
             .X (local_global_s_2),
                 (local_global_s_5),
             .Υ
             .I1(),
             .12(),
             .P (local_s2_times_s5)
           );
multiplier m5 (
             .clk(clk),
                 (local_global_s_1),
             .X
             .Υ
                  (local_global_s_5),
             .I1(),
             .12(),
             .P (local_s1_times_s5)
           );
multiplier m6 (
             .clk(clk),
                (local_global_s_2),
             .X
             .Ү
                 (local_global_s_3),
             .I1(),
             .I2(),
```

```
.Р
                      (local_s2_times_s3)
              );
  logic [`DSP_MULT_OUT_SIZE-1:0] s1_times_s4;
  logic [`DSP_MULT_OUT_SIZE-1:0] s2_squared;
  logic [`DSP_MULT_OUT_SIZE-1:0] s3_times_s4;
  logic [`DSP_MULT_OUT_SIZE-1:0] s2_times_s5;
  logic [`DSP_MULT_OUT_SIZE-1:0] s1_times_s5;
  logic [`DSP_MULT_OUT_SIZE-1:0] s2_times_s3;
  always_ff @( posedge clk )
  begin
    if(reset)
    begin
      s1_times_s4 <= '0;</pre>
      s2\_squared <= '0;
      s3_times_s4 <= '0;
      s2_times_s5 <= '0;</pre>
      s1_times_s5 <= '0;</pre>
      s2_times_s3 <= '0;</pre>
    end
    else
    begin
      s1_times_s4 <= local_s1_times_s4;</pre>
      s2_squared <= local_s2_squared;</pre>
      s3_times_s4 <= local_s3_times_s4;</pre>
      s2_times_s5 <= local_s2_times_s5;</pre>
      s1_times_s5 <= local_s1_times_s5;</pre>
      s2_times_s3 <= local_s2_times_s3;</pre>
    end
  \quad \text{end} \quad
  assign s1s4_minus_s2s2 = s1_times_s4 - s2_squared;
  assign s3s4_minus_s2s5 = s3_times_s4 - s2_times_s5;
  assign s1s5_minus_s2s3 = s1_times_s5 - s2_times_s3;
endmodule
```

multiplier.sv

```
);
        // Declare input and output registers
        reg [WIDTH-1:0] dataa_reg;
        reg [WIDTH-1:0] datab_reg;
        wire [2*WIDTH-1:0] mult_out;
        wire [2*WIDTH-1:0] in1;
        wire [2*WIDTH-1:0] in2;
        // Store the result of the multiply
        assign mult_out = dataa_reg * datab_reg;
        assign in1 = dataa_reg;
        assign in2 = datab_reg;
        // Update data
        always @ (posedge clk)
        begin
                 dataa_reg <= X;</pre>
                 datab_reg <= Y;</pre>
                 P <= mult_out;</pre>
                 I1 <= in1;
                 I2 <= in2;
        end
endmodule
```

$unsigned_adder.sv$

$lr_{-acc.svh}$

```
`ifndef __LR_ACC_SVH__
`define __LR_ACC_SVH__
```

```
`timescale 1ns/100ps
`ifndef CLOCK_PERIOD
`define CLOCK_PERIOD 10
`endif
`define SPLIT_DATA_LEN 4
`define INPUT_DATA_LEN 8
`define MULT_DATA_IN_LEN 18
`define OUTPUT_DATA_LEN 32
`define MEM_DATA_WIDTH 8
`define MEM_ADDR_WIDTH 9
`define MULT_4B_WIDTH 4
`define DATA_SET_SIZE 512
`define ADDRESS_BITS_SIZE £cloq2(`DATA_SET_SIZE)
`define PIPELINE_SETS 16
`define PC_BITS_SIZE £clog2(`PIPELINE_SETS)
`define ADDER_IN_SIZE_18b 18
`define ADDER_OUT_SIZE_18b 18
`define ADDER_INPUT_SIZE_4b 4
`define ADDER_OUTPUT_SIZE_5b 5
`define MULT_INPUT_SIZE_4b 4
`define MULT_OUTPUT_SIZE_8b 8
`define DSP_MULT_IN_SIZE 18
`define DSP_MULT_OUT_SIZE 36
typedef struct packed {
    logic [`SPLIT_DATA_LEN-1:0] x;
    logic [`SPLIT_DATA_LEN-1:0] y;
} memory;
typedef struct packed {
    logic [`ADDRESS_BITS_SIZE-1:0] pc;
    logic [`PC_BITS_SIZE-1:0] dut_num;
    logic done;
    logic [`MULT_DATA_IN_LEN-1:0] local_s_2;
    logic [`MULT_DATA_IN_LEN-1:0] local_s_3;
    logic [`MULT_DATA_IN_LEN-1:0] local_s_4;
```

```
logic [`MULT_DATA_IN_LEN-1:0] local_s_5;
} pc_tracker;
```

```
`endif
```

Software Code

 $lr_{-acc.c}$

```
#include <linux/module.h>
#include <linux/init.h>
#include <linux/errno.h>
#include <linux/version.h>
#include <linux/kernel.h>
#include <linux/platform_device.h>
#include <linux/miscdevice.h>
#include <linux/slab.h>
#include <linux/io.h>
#include <linux/of.h>
#include <linux/of_address.h>
#include <linux/fs.h>
#include <linux/uaccess.h>
#include "lr_acc.h"
#define DRIVER_NAME "lr_acc"
/* Device registers */
#define BG_RED(x) (x)
#define BG_GREEN(x) ((x) + 1)
#define BG_BLUE(x) ((x) + 2)
#define VGA_BACKGROUND_OFFSET 0
/*
* Information about our device
*/
struct lr_acc_dev
{
       struct resource res;
                                    /* Resource: our registers */
       void __iomem *virtbase; /* Where registers can be accessed in memory */
       lr_acc_arg_t data;
       lr_acc_read_data_t read_data;
} dev;
/* Write background color */
static void write_data(lr_acc_arg_t *data)
{
       if (data->go)
```

```
{
                iowrite32((u32)1, dev.virtbase + 4 * ((1 << 9) + data->address));
        }
        else
        {
                iowrite32((u32)data->data.data, dev.virtbase + 4 * data->address);
        }
}
static void read_data(lr_acc_read_data_t *data)
{
        int a = ioread32(dev.virtbase + 0);
        int b = ioread32(dev.virtbase + 4);
        int c = ioread32(dev.virtbase + 8);
        int d = ioread32(dev.virtbase + 12);
        int e = ioread32(dev.virtbase + 16);
        int f = ioread32(dev.virtbase + 20);
        int g = ioread32(dev.virtbase + 24);
        int h = ioread32(dev.virtbase + 28);
        int i = ioread32(dev.virtbase + 32);
        data->master_done = a;
        data -> d = b;
        data -> n0 = c;
        data -> n1 = d;
        data -> s1 = e;
        data -> s2 = f;
        data -> s3 = g;
        data -> s4 = h;
        data -> s5 = i;
}
/*
 * Handle ioctl() calls from userspace:
* Read or write the segments on single digits.
* Note extensive error checking of arguments
 */
static long lr_acc_ioctl(struct file *f, unsigned int cmd, unsigned long arg)
{
        lr_acc_arg_t vla;
        lr_acc_read_data_t obj;
        switch (cmd)
        {
        case LR_ACC_WRITE_DATA:
                if (copy_from_user(&vla, (lr_acc_arg_t *)arg,
                                                     sizeof(lr_acc_arg_t)))
                        return -EACCES;
                write_data(&vla);
                break;
```

```
case LR_ACC_READ_DATA:
                read_data(&obj);
                if (copy_to_user((lr_acc_read_data_t *)arg, &obj,
                                                 sizeof(lr_acc_read_data_t)))
                        return -EACCES;
                break;
        default:
                return -EINVAL;
        }
        return 0;
}
/* The operations our device knows how to do */
static const struct file_operations lr_acc_fops = {
       .owner = THIS_MODULE,
        .unlocked_ioctl = lr_acc_ioctl,
};
/* Information about our device for the "misc" framework -- like a char dev */
static struct miscdevice lr_acc_misc_device = {
        .minor = MISC_DYNAMIC_MINOR,
        .name = DRIVER_NAME,
        .fops = &lr_acc_fops,
};
/*
 * Initialization code: get resources (registers) and display
* a welcome message
 */
static int __init lr_acc_probe(struct platform_device *pdev)
ſ
        // lr_acc_color_t beige = { 0xf9, 0xe4, 0xb7 };
        // lr_acc_color_t beige = { Oxff, Oxff, Oxff };
        int ret;
        /* Register ourselves as a misc device: creates /dev/lr_acc */
        ret = misc_register(&lr_acc_misc_device);
        /* Get the address of our registers from the device tree */
        ret = of_address_to_resource(pdev->dev.of_node, 0, &dev.res);
        if (ret)
        {
                ret = -ENOENT;
                goto out_deregister;
        }
```

```
/* Make sure we can use these registers */
        if (request_mem_region(dev.res.start, resource_size(&dev.res),
                                                   DRIVER_NAME) == NULL)
        {
                ret = -EBUSY;
                goto out_deregister;
        }
        /* Arrange access to our registers */
        dev.virtbase = of_iomap(pdev->dev.of_node, 0);
        if (dev.virtbase == NULL)
        ſ
                ret = -ENOMEM;
                goto out_release_mem_region;
        }
        /* Set an initial color */
        // write_background(&beige);
        return 0;
out_release_mem_region:
        release_mem_region(dev.res.start, resource_size(&dev.res));
out_deregister:
        misc_deregister(&lr_acc_misc_device);
        return ret;
}
/* Clean-up code: release resources */
static int lr_acc_remove(struct platform_device *pdev)
{
        iounmap(dev.virtbase);
        release_mem_region(dev.res.start, resource_size(&dev.res));
        misc_deregister(&lr_acc_misc_device);
        return 0;
}
/* Which "compatible" string(s) to search for in the Device Tree */
#ifdef CONFIG_OF
static const struct of_device_id lr_acc_of_match[] = {
        {.compatible = "csee4840,lr_acc-1.0"},
        {},
};
MODULE_DEVICE_TABLE(of, lr_acc_of_match);
#endif
/* Information for registering ourselves as a "platform" driver */
static struct platform_driver lr_acc_driver = {
        .driver = {
                .name = DRIVER_NAME,
```

```
.owner = THIS_MODULE,
                .of_match_table = of_match_ptr(lr_acc_of_match),
        },
        .remove = __exit_p(lr_acc_remove),
};
/* Called when the module is loaded: set things up */
static int __init lr_acc_init(void)
{
        pr_info(DRIVER_NAME ": init\n");
        return platform_driver_probe(&lr_acc_driver, lr_acc_probe);
}
/* Calball when the module is unloaded: release resources */
static void __exit lr_acc_exit(void)
{
        platform_driver_unregister(&lr_acc_driver);
        pr_info(DRIVER_NAME ": exit\n");
}
module_init(lr_acc_init);
module_exit(lr_acc_exit);
MODULE_LICENSE("GPL");
MODULE_AUTHOR("Venkat Suprabath Bitra, Columbia University");
MODULE_DESCRIPTION("LR Accumulator Driver");
```

$lr_{-acc.h}$

```
#ifndef _lr_acc_H
#define _lr_acc_H
#include <linux/ioctl.h>
typedef struct {
   char data;
} lr_acc_data_t;
typedef struct {
   lr_acc_data_t data;
   int address;
   char go;
} lr_acc_arg_t;
typedef struct {
   int master_done;
   int d, n0, n1, s1, s2, s3, s4, s5;
} lr_acc_read_data_t;
```

```
#define LR_ACC_MAGIC 'q'
/* ioctls and their arguments */
#define LR_ACC_WRITE_DATA _IOW(LR_ACC_MAGIC, 1, lr_acc_arg_t)
#define LR_ACC_READ_DATA _IOR(LR_ACC_MAGIC, 2, lr_acc_read_data_t)
```

#endif

main.c

#include <std< th=""><th>lio.h></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th></std<>	lio.h>									
#include < <mark>tim</mark>	ne.h>									
#include "lr_	acc.h"									
#include <sys< td=""><td>/ioctl.h</td><td>1></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></sys<>	/ioctl.h	1>								
#include <sys< td=""><td>/types.h</td><td>1></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></sys<>	/types.h	1>								
#include <sys< td=""><td>s/stat.h></td><td>></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></sys<>	s/stat.h>	>								
#include <fcr< td=""><td>tl.h></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></fcr<>	tl.h>									
#include <str< td=""><td>ing.h></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></str<>	ing.h>									
#include <uni< td=""><td>std.h></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></uni<>	std.h>									
<pre>int lr_acc_fd</pre>	l;									
static const	unsigned	d <mark>int</mark> rat	√_dat	a[][2] =	{				
{1, 15}, { 1	, 14}, -	{1, 15},	{1,	15},	{1,	15},	{ 1,	14},	{1,	15},
{1, 15}, { 1	, 14}, -	{1, 15},	{1,	14},	{1,	15},	{1,	15},	{1,	13},
{1, 15}, { 1	, 14}, -	{1, 15},	{1,	15},	{1,	15},	{ 1,	14},	{1,	14},
{ 1, 15 } , { 1	, 15}, -	{1, 15},	{ 1,	15},	{ 1,	15},	{1,	14},	{1,	15},
{ 1, 15 } , { 1	, 15}, -	{1, 15},	{ 1,	14},	{ 1,	15},	{1,	15},	{1,	15},
{1, 14}, {1	, 14}, -	{1, 15},	{1,	14},	{1,	15},	{1,	15},	{1,	15},
{1, 14}, {1	, 14}, -	{1, 14},	{1,	14},	{1,	14},	{1,	14},	{ 2,	12},
{2, 12}, {2	2, 12}, -	{ 2, 13 } ,	{ 2,	12},	{ 2,	12},	{ 2,	13},	{ 2,	12},
{2, 12}, {2	2, 12}, {	{ 2, 12 } ,	{ 2,	12},	{ 2,	12},	{ 2,	13},	{ 2,	12},
{ 2, 13 } , { 2	2, 13}, {	{2, 13},	{ 2,	13},	{ 2,	13},	{ 2,	13},	{ 2,	13},
{2, 12}, {2	2, 12}, -	{2, 12},	{ 2,	12},	{ 2,	12},	{ 2,	13},	{ 2,	13},
{2, 12}, {2	2, 12}, -	{2, 12},	{ 2,	12},	{ 2,	12},	{ 2,	13},	{ 2,	12},
{2, 12}, {2	2, 12}, {	{ 2, 12 } ,	{ 2,	12},						
{ 2, 12 } , { 2	2, 12}, -	{ 2, 12 } ,	{ 2,	12},	{ 2,	12},	{ 3,	12},	{ 3,	13},
{ 3, 13 } , { 3	3, 12}, -	{ 3, 12 } ,	{ 3,	12},						
{3.12}. {3	3. 12 }	$\{3, 12\}.$	{ 3.	11}.	{ 3.	13}.	{ 3.	12}.	{ 3.	11}.

```
\{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 11\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 13\}, \{3, 12\}, 
                                                                                                                              \{3, 12\}, \{3, 12\}, \{3, 13\}, \{3, 13\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{3, 12\}, \{
                                                  \hookrightarrow
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                                                                                                                              \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 
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                                                                                                                           7}, {7, 6}, {7, 6}, {7, 7}, {7, 7}, {7, 6}, {7, 6}, {7, 6}, {7, 7}, {7, 7},
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                                                                                                                              \{7, 6\}, \{7, 6\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 4\}, \{7, 5\}, \{7, 5\}, \{7, 4\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 6\}, \{7, 
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                                                                                                                              \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{7, 5\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 
                                                  \hookrightarrow
                                                                                                                              1}, {8, 2}, {8, 1}, {8, 1}, {8, 1}, {8, 1}, {8, 1}, {8, 1}, {8, 1}, {8, 2}, {8, 1},
                                                  \hookrightarrow
                                                                                                                           \{8, 4\}, \{8, 4\}, \{8, 2\}, \{8, 0\}, \{8, 2\}, \{8, 2\}, \{8, 1\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 2\}, \{8, 
                                                  \hookrightarrow
                                                                                                                              2, {8, 2}, {8, 1}, {8, 1}, {8, 1}, {8, 1}, {8, 1}, {8, 1}, {8, 1}, {8, 2}, {8, 1}, {8, 1}, {8, 2}, {8, 1},
                                                  \hookrightarrow
                                                                                                                              \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 2\}, \{8, 1\}, \{8, 1\}, \{8, 2\}, \{8, 1\}, \{8, 1\}, \{8, 2\}, \{8, 1\}, \{8, 1\}, \{8, 2\}, \{8, 1\}, \{8, 1\}, \{8, 2\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 2\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 1\}, \{8, 
                                                  \hookrightarrow
                                                                                                                              1, {8, 1}, {8, 1}, {8, 1}, {8, 1}, {8, 2}, {8, 1}, {8, 1}, {9, 0}, {9, 0},
                                                  \hookrightarrow
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                                                  \hookrightarrow
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};
  #define NUM_ENTRIES (sizeof(raw_data) / sizeof(raw_data[0]))
  int read_data_from_array(char **data) {
                                                                                        *data = (char *)malloc(NUM_ENTRIES *
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         sizeof(char));
                                                                                     if (*data == NULL) {
                                                                                                                                                                        perror("Error allocating memory");
                                                                                                                                                                           return -1;
                                                                                  }
                                                                                  for (int i = 0; i < NUM_ENTRIES; i++) {</pre>
                                                                                                                                                                        unsigned int y = raw_data[i][0];
                                                                                                                                                                        unsigned int x = raw_data[i][1];
                                                                                                                                                                           (*data)[i] = (y << 4) | x;
                                                                                  }
```

```
return NUM_ENTRIES;
}
void set_lr_data(const lr_acc_arg_t *d)
ſ
 if (ioctl(lr_acc_fd, LR_ACC_WRITE_DATA, d))
 {
    fprintf(stderr, "ioctl(LR_ACC_SET_DATA) failed");
    return;
 }
}
void read_lr_data(lr_acc_read_data_t *d)
{
 if (ioctl(lr_acc_fd, LR_ACC_READ_DATA, d))
 {
   fprintf(stderr, "ioctl(LR_ACC_GET_DATA) failed");
    return;
 }
}
int main()
{
 lr_acc_arg_t vla;
 lr_acc_read_data_t obj;
 int i;
 static const char filename[] = "/dev/lr_acc";
 if ((lr_acc_fd = open(filename, O_RDWR)) == -1)
 {
    fprintf(stderr, "could not open %s\n", filename);
    return -1;
 }
  char *data = NULL;
  int n = read_data_from_array(&data);
  if (n < 0) {
     fprintf(stderr, "Error reading data\n");
      return -1;
 }
 fprintf(stderr, "Read %d data points\n", n);
 lr_acc_data_t *d = (lr_acc_data_t *)malloc(sizeof(lr_acc_data_t));
 clock_t start, end;
 double cpu_time_used;
 start = clock();
```

```
vla.go = 1;
vla.address = 0;
set_lr_data(&vla);
for (int i = 0; i < n; i++) {
    d->data = data[i];
    vla.data = *d;
    vla.address = i;
    vla.go = 0;
    set_lr_data(&vla);
}
end = clock();
cpu_time_used = ((double)(end - start)) / CLOCKS_PER_SEC;
fprintf(stderr, "Time taken to send data: %f microseconds\n", cpu_time_used *
→ 1e6);
start = clock();
vla.go = 1;
vla.address = n;
set_lr_data(&vla);
while (1)
{
  read_lr_data(&obj);
  if (obj.master_done == 1)
   break;
  usleep(1);
}
end = clock();
cpu_time_used = ((double)(end - start)) / CLOCKS_PER_SEC;
fprintf(stderr, "Time taken to process and read data: %f microseconds\n",
\leftrightarrow cpu_time_used * 1e6);
start = clock();
double w0 = (double)obj.n0 / (double)obj.d;
double w1 = (double)obj.n1 / (double)obj.d;
end = clock();
cpu_time_used = ((double)(end - start)) / CLOCKS_PER_SEC;
```

```
fprintf(stderr, "Time taken to calculate weights: %f microseconds\n",
  \leftrightarrow cpu_time_used * 1e6);
  fprintf(stderr, "Device finished processing\n");
  fprintf(stderr, "Results:\n");
  fprintf(stderr, "d: %d\n", obj.d);
  fprintf(stderr, "n0: %d\n", obj.n0);
  fprintf(stderr, "n1: %d\n", obj.n1);
  fprintf(stderr, "s1: %d\n", obj.s1);
  fprintf(stderr, "s2: %d\n", obj.s2);
 fprintf(stderr, "s3: %d\n", obj.s3);
  fprintf(stderr, "s4: %d\n", obj.s4);
  fprintf(stderr, "s5: %d\n", obj.s5);
  fprintf(stderr, "Weights:\n");
 fprintf(stderr, "w0: %f\n", w0);
  fprintf(stderr, "w1: %f\n", w1);
  fprintf(stderr, "Freeing allocated memory\n");
  free(data);
  free(d);
 close(lr_acc_fd);
 fprintf(stderr, "Closed the device file\n");
 fprintf(stderr, "LR Accumulator Userspace program terminating\n");
 return 0;
}
```

parsing.c

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>
#define MAX_LINES 100000
int n;
int x_vals[MAX_LINES];
int y_vals[MAX_LINES];
void read_data(const char *filename) {
   FILE *fp = fopen(filename, "r");
    if (!fp) {
        perror("File opening failed");
        exit(EXIT_FAILURE);
    }
    if (fscanf(fp, "%d", &n) != 1) {
        fprintf(stderr, "Invalid file format\n");
        fclose(fp);
```

```
exit(EXIT_FAILURE);
    }
    for (int i = 0; i < n; i++) {
        if (fscanf(fp, "%d %d", &y_vals[i], &x_vals[i]) != 2) {
            fprintf(stderr, "Invalid data at line %d\n", i + 2);
            fclose(fp);
            exit(EXIT_FAILURE);
        }
    }
    fclose(fp);
}
void run(double *a, double *b) {
    long long s2 = 0, s3 = 0, s4 = 0, s5 = 0;
    for (int i = 0; i < n; i++) {
       int x = x_vals[i];
       int y = y_vals[i];
       s2 += x;
       s3 += y;
       s4 += (long long)x * x;
       s5 += (long long)x * y;
    }
    long long n0 = s4 * s3 - s2 * s5;
    long long n1 = s5 * n - s2 * s3;
    long long d = n * s4 - s2 * s2;
    *a = (double)n0 / d;
    *b = (double)n1 / d;
}
int main() {
    read_data("preprocessed_data.txt");
    double a = 0, b = 0;
    clock_t start_clock = clock();
    for (int i = 0; i < 1000; i++) {</pre>
        run(&a, &b);
    }
    clock_t end_clock = clock();
    double cpu_time = (double)(end_clock - start_clock) / CLOCKS_PER_SEC;
    double avg_usec = (cpu_time / 1000.0) * 1e6;
```

```
printf("Elapsed time (CPU): %.6f us per run\n", avg_usec);
// printf("a: %.6f, b: %.6f\n", a, b);
return 0;
```

Fmax Timing Summary

}

+ ; Slow 1100mV	85C Model Fmax Su	mmary
; Fmax	; Restricted Fmax	; Clock Name
; 163.75 MHz ; 1184.83 MHz +	; 163.75 MHz ; 717.36 MHz	; clock_50_1 ; soc_system:soc_system0 soc_system_hps_0:hps_0 soc_!

Figure 18: Slow 1100mV 85°C Model Fmax Summary

The table in Figure 18 provides the maximum clock frequencies (**Fmax**) under the slow timing corner (1100 mV, 85° C):

- clock_50_1 has a maximum frequency of 163.75 MHz, which is also the restricted Fmax, indicating no additional constraints or timing bottlenecks were applied to it.
- The HPS system clock path (long hierarchical name: soc_system:soc_system0|...) achieves a significantly higher unrestricted Fmax of 1184.83 MHz. However, the restricted Fmax is lowered to 717.36 MHz, suggesting design or interface constraints limit this clock's usable frequency to avoid setup timing violations or to match external timing requirements.

These Fmax values help determine the safe and efficient operating frequencies for different clock domains within the design.