BananaSolve: a Parallel Haskell Bananagrams Solver

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1 Overview

Context Bananagrams is an entertaining scabble-like game in which the players are all given a set number of letter tiles and asked to construct a totally connected scrabble-like board using all of their tiles. Whoever finishes their board fastest wins. (In the real game, after a player completes a board, all players draw a new tile and have to incorporate it into their boards, until all tiles are gone, but for our project we will be focusing on the initial board creation stage of the game.) See Figure 1.

Project Goal Our goal is to write a Haskell algorithm that, given a set number of tiles, will construct a valid Bananagrams board using those tiles. We will then speed up the program using parallel strategies.

Challenges Banagrams is a relatively new game. It has been around since 2006. For this reason it lacks well-known sequential solving algorithms. An algorithm written by college students in python has been of some help to us, but we have veered significantly from their specific implementation (linked here).



Figure 1: A Bananagrams Ad

2 Algorithm

A Sequential Algorithm Our sequential algorithm plays one word onto the board at a time until all tiles have been used. The best word to play is assessed using a heuristic based on letter frequencies and word length. In general, longer words that use more uncommon letters are given priority. A pseudo-code algorithm for our technique can be described as follows:

- 1. Given a dictionary D and a hand of letters h:
- 2. Group D into sub-dictionaries (d : ds) each containing all words of a particular wordsize. Let s be the wordsize of d, the sub-dictionary with the longest words.
- 3. Start BFS, using the empty board as a first BFS-node.
- 4. For each BFS-node, take a BFS step:
 - (a) Find each available space on the board that can be used to add a new word.
 - (b) For each available space, add the highest scoring word in d that can be played with letters in h at that space that does not invalidate the board in a new BFS-strand. Subtract the letters of that word from h and reset s to the length of the longest word in d. If no words could be placed on the board, decrement s and go back to (a) with the head of ds.
- 5. If any new BFS-nodes contain a completed board (i.e., h is empty), return that and halt.
- 6. Sort all the new BFS nodes by score. Score is calculated based on total tiles played and letter frequency values of tiles played, with more tiles and less frequently used letters scoring more. Pick the highest n scoring BFS nodes to run a new BFS step on, where n can be determined by the user.
- 7. Return to step 4. with the new BFS-nodes chosen in 6., and repeat until some limit is reached or a solution is found.

3 Some Implementation Details

3.1 Important Custom Types

BWord BWord is made up of the word's string, starting coordinates, and direction (i.e. horizontal or vertical).

data BWord = BWord String Location Direction

OMatrix OMatrix is made up of the matrix's origin and a CharMatrix. Keeping track of the 'virtual' origin's location abstracts the underlying matrix and allows for easier management of the matrix.

data OMatrix = OMatrix Location CharMatrix

-															7
													'a'		
			1.1	1.1	1.1	1.1	1.1	1.1		1.1	1.1	1.1	'v'		1 1
	1.1	1.1	1.1	1.1	1.1		1.1	1.1	1.1	т т	1.1		'o'	'a'	'k'
	т т	1.1	1.1	т т	1 1	т т	т т	т т	т. т.	т т	т т	т т	'w'	т. т.	1.1
	т т	т. т	т. т	т т	т т	т т	т т	т т	т. т.	т т	т т	т т	'a'	т. т.	1 I I
'i'	'n'	't'	'e'	'r'	'c'	'h'	'a'	'n'	'q'	'e'	'a'	'b'	'1'	'v'	
	т. т	т. т	1.1	т. т.	1 1	'a'	т т	т т	Ň	т т	т т	т т	т т	τ, i	1.1
	т т	т т	т. т.	т. т	т т	'r'	т т	т т	т. т.	т т	т т	т т	т т	т т	
	т т	1.1	т. т	т. т.	т. т	'm'	т т	т т	т. т.	т т	т т	т т	т т	т. т.	1.1
	т т	1.1	1.1	т. т	1 1	'o'	т т	1.1	т. т.	т т	т т	т т	т т	т. т.	
	т т	т т	т т	т т	т т	'n'	т т	т т	т. т.	т т	т т	т т	т т	т т	1.1
	т т	т т	т т	т т	т т	'i'	т т	т т	т. т.	т т	т т	т т	т т	т т	1.1
	т т	т. т	т. т.	т. т	т. т	'o'	т т	т т	т. т.	т т	т т	т т	т т	т. т	
	т. т	т т	т. т	т. т	т. т	'u'	т т	т т	т. т.	т т	т т	т т	т. т	т. т.	1.1
'u'	т т	т т	т. т.	т. т	т. т	's'	т т	т т	т. т.	т т	т т	т т	т т	т. т	
's'	'q'	'u'	'a'	'b'	'b'	'1'	'e'	т т	т. т.	т т	т т	т т	т т	т т	н н (
1.1	1.1	1.1	1.1	1.1	1.1	'v'	1.1	т т	т. т.	1.1	т т	т т	т т	т. т	
						-									

Figure 2: Our algorithm's solution for input "howareyousounbelievablyquickatbananagrams"

Board Board stores an OMatrix with the list of BWords it uses.

data Board = Board [BWord] OMatrix

State State stores a *Board* with a *Hand* of letters yet to be played where *Hand* is a Hashmap of letters with their respective counts in the player's hand.

```
type State = (Hand, Board)
```

3.2 Choosing Next Word

Choosing the next word uses the algorithms in src/WordChooser.hs to pick the highest scoring word that can be played with Hand h at a specific tile on the board. The scoring is done based on wordlength and letter frequency. We created a Map.Map Char Int letterPoints to assign points to each letter based on that letter's frequency in the English language, where the least frequent letter scores the most points. In our letterPoints, based on autogenerated values from chatGPT,'z' scores 14926, while 'e' scores 2298.

The scoring algorithm is as follows:

```
scoreWord :: String -> Int
scoreWord w = 15000 * length w
        + sum (map scoreChar w)
scoreChar :: Char -> Int
scoreChar ' ' = 0
scoreChar c = fromJust $ Map.lookup c letterPoints
```

3.3 Other Details

The code as a whole can be found in the appendix, as well as on github.



Figure 3: Threadscope output for parallel step calculations.

4 Parallel

4.1 A Test Case For Parallelization.

For our parallelization journey, we began testing with the test input "howareyousounbelievablyquickatbananagrams". This input produced a board under the sequential algorithm in around 7.2s.¹ The board produced is pictured in Figure 2

4.2 Parallel BFS-Step Computation

Our first idea for parallelization was to calculate each BFS-node's children in parallel. For example, at BFS step x with 100 nodes, create a spark to calculate each node's children.

At first we tried this with the strategy parPar, but this did not effect speedup. The reason was that parPar only goes to WHN form, and for each BFS-node, we wanted to calculate a list of its children, and WHN form for this would only actually evaluate the first child, and leave the rest as a thunk.

Therefore we switched to using the rdeepseq strategy, and added implementations for instance NFData for all our custom datatypes. This was successful. The new implementation for going from one bfs step to the next required minimal changes. We implemented it in src/Bfs.hs by modifying bfsNext to bfsNextPar, which ran as such:

The new runtime was around 2.2s, around a 3.3x speedup from the sequential algorithm on this input. Threadscope logs are shown in Figure 3

¹When run the first time, it can take up to 13s, but if run again its much faster, averaging around 7.2s. I assume this is because some of the dictionary is kept in the cache after the first run.



Figure 4: Threadscope output for parallel within-step calculations.

There was still room for improvement. Looking at the threadscope logs, we could see serious gaps of inactivity on each core. This could be caused by uneven load balancing. For example, if some of the BFS-nodes were much harder than others (for example, boards with more tiles already played require more possible new plays and more computations), one core could be given all of the harder nodes to complete.

4.3 Parallel Within-Step Computation

We decided to try to optimize our parallel speedup further by introducing parallelism to within BFS-node computations. Each BFS-node begins with a State containing a Hand h and a Board b, and needs to find all open tiles on the b, and calculate the best possible word play with h at each spot. So we can parallelize this by calculating the best possible play for each open tile in parallel.

To implement this, we changed our playTurn function in src/Bfs.hs to playTurnPar, where:

```
playTurnPar :: DictPair -> State -> [State]
playTurnPar dictpair state@(_, board) =
    catMaybes $ parMap rdeepseq (playBestWordAt dictpair state)
        $ getOpenTiles board
```

The runtime for this improved algorithm was between 2.0 and 2.1s on average, which calculates to around a 3.5x speedup from the sequential algorithm. Threadscope logs, shown in Figure 4, show how core inactivities were significantly reduced in this algorithm.

One caveat is that we noticed our new algorithm resulted in a lot of fizzled sparks. Fizzled sparks are supposed to happen with a spark finds that the thunk it was supposed to evaluate was already evaluated by another part of the program. As of the due date for this project we have not been able to discover the reason for this problem in our code, as we do not think we have multiple sparks running on the same thunks.

4.4 Speed Up Results For Other Test-Cases

Input	Sequential Time	Parallel Time	Speed-up
icandothisonequickly	1.7s	0.8s	2.26x
howareyousogoodatpuzzlesitstrulyaston ishing	11.65s	3.52s	3.31x
whydoesntanyonewanttoplaybananagram swithmeiguessbeingthisgoodisntsofu nafterall	59.69s	19.90s	2.99x
acomputerwoulddeservetobecalledintelli gentifitcoulddeceiveahumanintobelieving thatitwashuman	76.99s	27.62s	2.79x

5 Appendix - Code

5.1 Main.hs

```
module Main (main) where
import Bfs (bfsPar, bfsSeq)
import Data.Set (fromList)
import Types (splitDict)
import System.Environment (getArgs, getProgName)
import System.Exit(die)
import Data.Maybe (isNothing, fromJust)
import Data.Char (isAlpha, toLower)
usage :: IO ()
usage = do
    pn <- getProgName</pre>
    die $ "Usage: stack exec " ++ pn ++ " -- +RTS -ls -N4 -- <algo> <tiles
   >\n" ++
          "<algo> must be 's' for sequential or 'p' for parallel. Tiles
   must be letters only."
main :: <mark>IO</mark> ()
main = do
    args <- getArgs
    case args of
        [_, algo, tiles] -> do
                let algoType = case algo of "s" -> Just bfsSeq
                                              "p" -> Just bfsPar
                                              _ -> Nothing
                 if isNothing algoType then usage
                 else do
                    if any (not . isAlpha) tiles then usage
                     else do
                         let formattedTiles = map toLower tiles
                         fcontents <- readFile "words.txt"</pre>
                         let ws = lines fcontents
```

```
dictlist = splitDict ws
dictset = Data.Set.fromList ws
putStrLn $ "Prompt: " ++ formattedTiles
let lim = 20
stepsize = 20
res = fromJust algoType formattedTiles lim
stepsize (dictset, dictlist)
case res of
Nothing -> putStrLn $ "no solution in " ++
show lim
Just state -> print state
_ -> usage
```

5.2 Types.hs

```
module Types (
    StringSet,
    StringLists,
    splitDict,
    CharMatrix,
    Hand,
    Direction (H, V),
    flipD,
    Location (..),
    BWord (..),
    OMatrix (...),
    setElemOMatrix,
    Board (..),
    boardID,
    State,
    stateID,
    DictPair
) where
import Data.Set (Set)
import Data.HashMap.Strict (HashMap)
import Data.Matrix (Matrix, toList, setElem)
import Control.DeepSeq ( NFData(..) )
import Data.List (groupBy, sortBy)
type StringSet = Set String
type StringLists = [[String]]
splitDict :: [String] -> StringLists
splitDict dict = groupBy lengthEq $ sortBy lengthCmp dict
    where lengthCmp x y = length y 'compare' length x
          lengthEq x y = length x == length y
type Hand = HashMap Char Int
data Direction = H|V deriving (Eq, Show) -- horizontal or vertical
flipD :: Direction -> Direction
```

```
flipD H = V
flipD V = H
instance NFData Direction where
    rnf d = d 'seq' ()
data Location = Location Int Int deriving (Eq)
instance Show Location where
    show (Location y x) = show (y, x)
instance NFData Location where
    rnf (Location y x) = rnf y 'seq' rnf x
type CharMatrix = Matrix Char
-- OMatrix stores the 'virtual' origin with a CharMatrix to allow
-- for easier usage of the CharMatrix.
data OMatrix = OMatrix Location CharMatrix
setElemOMatrix :: Char -> Location -> OMatrix -> OMatrix
setElemOMatrix c (Location y x) (OMatrix p m) = OMatrix p new_m
    where new_m = setElem c (y,x) m
instance Show OMatrix where
    show (OMatrix p m) = show m ++ "\n" ++ show p
instance NFData OMatrix where
    rnf om = om 'seq' ()
data BWord = BWord String Location Direction
             deriving (Eq, Show)
instance NFData BWord where
    rnf (BWord word p d) = rnf word 'seq' rnf p 'seq' rnf d
data Board = Board [BWord] OMatrix
instance Show Board where
    show (Board bwords om) =
         "bwords: " ++ show bwords ++ "\n"
         ++ show om
boardID :: Board -> String
boardID (Board _ (OMatrix _ m)) = toList m
instance NFData Board where
    rnf (Board bwords om) = rnf bwords 'seq' rnf om
type State = (Hand, Board)
stateID :: State -> String
stateID (_, board) = boardID board
type DictPair = (StringSet, StringLists)
```

5.3 BananaBoard.hs

```
module BananaBoard (
    singleton,
    getSpaceAt,
    joinWordAt,
    isValidBoard
```

```
) where
import Types (
    Direction (..),
    flipD,
    Location (...),
    OMatrix (..),
    setElemOMatrix,
    BWord (..),
    Board (..),
    StringSet)
import Data.Set (member)
import Data.Maybe (fromMaybe)
import Data.Matrix
    ( (<->), (<|>), fromLists, matrix,
      safeGet, Matrix(..), toLists, transpose )
empty :: Int -> Int -> Matrix Char
empty y x = matrix y x (\(_, _) \rightarrow ')
singleton :: String -> Board
singleton word = Board [BWord word (Location 1 1) H] (OMatrix (Location 1
   1) (fromLists [word]))
-- add origin offset to coords
add01 :: Num a => a -> a -> a
add01 c c0 = c+c0-1
addO :: Location -> Location -> Location
add0 (Location y x) (Location y0 x0) = Location (add01 y y0) (add01 x x0)
placeWord :: String -> Location -> Direction -> OMatrix -> OMatrix
placeWord word p@(Location y x) d om
    | d == H = placeWordH word (Location y x) sizedOM
    otherwise = placeWordV word (Location y x) sizedOM
    where
          endP = if d == H then Location y (x+length word-1)
                           else Location (y+length word-1) x
          sizedOM = resizeTo endP (resizeTo p om)
          placeWordH :: String -> Location -> OMatrix -> OMatrix
          placeWordH [] _ m = m
          placeWordH (w:ws) _p@(Location _y _x) _om@(OMatrix og _) =
             placeWordH ws (Location _y (_x+1)) $ setElemOMatrix w (add0
   _p og) _om
          placeWordV :: String -> Location -> OMatrix -> OMatrix
          placeWordV [] _ m = m
          placeWordV (w:ws) _p@(Location _y _x) _om@(OMatrix og _) =
            placeWordV ws (Location (_y+1) _x) $ setElemOMatrix w (add0 _p
    og) _om
          resizeTo :: Location -> OMatrix -> OMatrix
          resizeTo _p@(Location _y _x) _om@(OMatrix og@(Location y0 x0) m)
            | yo < 1 = let yoff = 1 + abs yo in
```

```
resizeTo (Location 1 _x) $ OMatrix (Location (y0 + yoff)
   x0)
                    $ empty yoff (ncols m) <-> m
            | xo < 1 = let xoff = 1 + abs xo in
                resizeTo (Location _y 1) $ OMatrix (Location y0 (x0 + xoff
   ))
                    $ empty (nrows m) xoff <|> m
            | yo > nrows m = resizeTo _p
                $ OMatrix og $ m <-> empty (yo - nrows m) (ncols m)
            xo > ncols m = resizeTo _p
                $ OMatrix og $ m <|> empty (nrows m) (xo-ncols m)
            otherwise = _om
            where (Location yo xo) = add0 _p og
getSpaceAt :: Location -> Int -> Direction -> OMatrix -> String
getSpaceAt p len d (OMatrix og m)
    d == H = map getElemX $ take len [xo..]
    otherwise = map getElemY $ take len [yo..]
    where
        (Location yo xo) = add0 p og
        getElemX :: Int -> Char
        getElemX x = fromMaybe ' ' $ safeGet yo x m
        getElemY :: Int -> Char
        getElemY y = fromMaybe ' ' $ safeGet y xo m
-- on top of a wordspace on a board, can we play this word?
validPlay :: String -> String -> Bool
validPlay [] _ = True
validPlay _ [] = False
validPlay (space:ss) (word:ws)
    | space == ' ' = validPlay ss ws
    otherwise = space == word && validPlay ss ws
-- is this play both valid and also meaningful?
goodPlay :: String -> String -> Bool
goodPlay wordspace word =
    , , 'elem' wordspace && validPlay wordspace word
isValidBoard :: StringSet -> Board -> Bool
isValidBoard dict (Board _ (OMatrix _ m)) =
    areValidRows (toLists m)
    && areValidRows (toLists $ transpose m)
    where
        isValidRow :: String -> Bool
        isValidRow row = all ('member' dict) $
            filter (\w -> length w /= 1) (words row)
        areValidRows :: [String] -> Bool
        areValidRows = all isValidRow
joinWordAt :: StringSet -> String -> Int -> BWord -> Int -> Board -> Maybe
(Board, String)
```

5.4 Hand.hs

```
module Hand (
   toHand,
    joinHands,
    playTile,
    addTile
) where
import Types (Hand)
import Data.HashMap.Strict (fromList, unionWith, update, alter)
import Data.List (group, sort)
toHand :: String -> Hand
toHand hand = fromList $ map (\s -> (head s, length s))
    $ (group . sort) hand
joinHands :: Hand -> Hand -> Hand
joinHands = unionWith (+)
playTile :: Char -> Hand -> Hand
playTile = update dec
    where dec :: Int -> Maybe Int
          dec 1 = Nothing
          dec n = Just (n-1)
addTile :: Char -> Hand -> Hand
addTile = alter inc
    where inc :: Maybe Int -> Maybe Int
          inc Nothing = Just 1
          inc (Just n) = Just (n+1)
```

5.5 WordChooser.hs

```
module WordChooser (
    Hand,
    buildWords,
```

```
scoreCmp,
    sortWHPairs,
    wordsWithChar,
    scoreWord
) where
import Data.List (sortBy)
import Data.Maybe (fromJust, mapMaybe)
import Data.HashMap.Strict (member)
import qualified Data.Map as Map
import Types (Hand)
import Hand (playTile)
-- Define a map between letters and their inverted usage frequencies
   generated by ChatGPT.
letterPoints :: Map.Map Char Int
letterPoints = Map.fromList
    [('a',6833),('b',13508),('c',12218),('d',10747),('e',2298),('f',12772)
     ('g',12985),('h',8906),('i',8034),('j',14847),('k',14228),('l',10975)
    ('m',12594),('n',8251),('o',7493),('p',13071),('q',14905),('r',9013),
     ('s',8673),('t',5944),('u',12242),('v',14022),('w',12640),('x',14850)
     ('y',13026),('z',14926)]
buildWord :: String -> Hand -> Maybe Hand
buildWord [] hand = Just hand
buildWord (w:ws) hand
    | null hand || not (member w hand) = Nothing
    otherwise = buildWord ws $ playTile w hand
buildWords :: Hand -> [String] -> [(String, Hand)]
buildWords hand = mapMaybe bw_pair
    where
        bw_pair [_] = Nothing
        bw_pair word = case buildWord word hand of
            Nothing -> Nothing
            Just _hand -> Just (word, _hand)
scoreCmp :: String -> String -> Ordering
-- flip x and y so sort puts highest scorers first
scoreCmp x y = scoreWord y 'compare' scoreWord x
scoreWord :: String -> Int
scoreWord w = 15000 * length w
    + sum (map scoreChar w)
scoreChar :: Char -> Int
scoreChar ' = 0
scoreChar c = fromJust $ Map.lookup c letterPoints
sortWHPairs :: [(String, Hand)] -> [(String, Hand)]
sortWHPairs = sortBy (\x y -> scoreCmp (fst x) (fst y))
```

```
wordsWithChar :: Char -> [String] -> [String]
wordsWithChar c = filter (elem c)
```

5.6 Bfs.hs

```
module Bfs (
    playFirstTurn,
    bfsSeq,
    bfsPar
) where
import Data.Char (isAlpha)
import Data.Maybe (fromJust, isJust, catMaybes, mapMaybe)
import Data.List (elemIndex, nubBy, sortBy)
import Control.Parallel.Strategies (parMap, rdeepseq)
import BananaBoard
    ( singleton,
      joinWordAt)
import Hand (
    joinHands,
    playTile,
    addTile,
    toHand)
import WordChooser(
    buildWords,
    scoreCmp,
    sortWHPairs,
    wordsWithChar)
import Types (
    Hand,
    StringLists,
    DictPair,
    Board (..),
    BWord (..),
    State,
    stateID)
playFirstTurn :: Hand -> StringLists -> [State]
playFirstTurn _ [] = []
playFirstTurn hand (d:ds)
    | null bests = playFirstTurn hand ds
    otherwise =
        map (\(w, h) -> (h, singleton w)) bests
    where
          bests = sortWHPairs $ buildWords hand d
playBestWordAt :: DictPair -> State -> (BWord, Int) -> Maybe State
playBestWordAt (_, []) _ _ = Nothing
playBestWordAt (dictset, d:ds) s@(hand, board) (bword@(BWord word _ _), i)
 | <mark>isJust</mark> best = best
```

```
otherwise = playBestWordAt (dictset, ds) s (bword, i)
    where
        c = word !! i
        bests = sortWHPairs $ buildWords (addTile c hand) $ wordsWithChar
   c d
        best = joinBestWord bests
        joinBestWord :: [(String, Hand)] -> Maybe State
        joinBestWord [] = Nothing
        joinBestWord ((w, h): xs)
            | isJust res = res
            otherwise = joinBestWord xs
            where
                w_ind = fromJust (elemIndex c w)
                joinRes = joinWordAt dictset w w_ind bword i board
                res = do
                    (newboard, playedOverSpace) <- joinRes</pre>
                    let newhand = playTile c $ joinHands h $
                         (toHand . filter isAlpha) playedOverSpace
                    return (newhand, newboard)
getOpenTiles :: Board -> [(BWord, Int)]
getOpenTiles (Board bwords _) =
    [(word, i) | word@(BWord s _ ) <- bwords, i <- [0..length s - 1]]
playTurnSeq :: DictPair -> State -> [State]
playTurnSeq dictpair state@(_, board) =
    mapMaybe (playBestWordAt dictpair state) openTiles
        where openTiles = getOpenTiles board
playTurnPar :: DictPair -> State -> [State]
playTurnPar dictpair state@(_, board) =
    catMaybes $ parMap rdeepseq (playBestWordAt dictpair state)
        $ getOpenTiles board
uniqueStates :: [State] -> [State]
uniqueStates = nubBy (\x y -> stateID x == stateID y)
bestStates :: Int -> [State] -> [State]
bestStates stepsize states = take stepsize $
    sortBy scoreCmpState states
    where scoreCmpState :: State -> State -> Ordering
          scoreCmpState x y = scoreCmp (lettersOf x) (lettersOf y)
          lettersOf :: State -> String
          lettersOf state = filter isAlpha $ stateID state
bfsNextSeq :: DictPair -> [State] -> [State]
bfsNextSeq dictpair states = do
    state <- states</pre>
    playTurnSeq dictpair state
bfsNextPar :: DictPair -> [State] -> [State]
```

```
bfsNextPar dictpair states = concat $ parMap rdeepseq (playTurnPar
   dictpair) states
bfsLoop :: (DictPair -> [State] -> [State]) -> Int -> Int -> DictPair -> [
   State] -> Maybe State
bfsLoop _ 0 _ _ _ = Nothing
bfsLoop _ _ _ [] = Nothing
bfsLoop bfsNexter lim stepsize dictpair beginStates
    | isJust solved = solved
    | otherwise = next
    where
        solved = completeFrom beginStates
        completeFrom :: [State] -> Maybe State
        completeFrom [] = Nothing
        completeFrom (s@(hand, _):ss)
           | null hand = Just s
            otherwise = completeFrom ss
        next = bfsLoop bfsNexter (lim-1) stepsize dictpair
            $ (bestStates stepsize . uniqueStates . bfsNexter dictpair)
                beginStates
runBfs :: (DictPair -> [State] -> [State]) -> String -> Int -> Int ->
   DictPair -> Maybe State
runBfs f handstring lim stepsize d@(_, dictlist) =
    bfsLoop f lim stepsize d $ playFirstTurn (toHand handstring) dictlist
bfsSeq :: String -> Int -> Int -> DictPair -> Maybe State
bfsSeq = runBfs bfsNextSeq
bfsPar :: String -> Int -> Int -> DictPair -> Maybe State
bfsPar = runBfs bfsNextPar
```