Parallel SAT Solver
COMS4995

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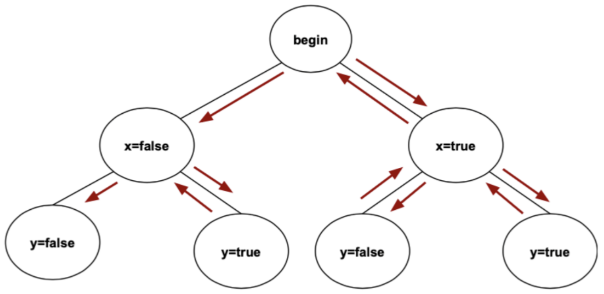
Introduction

SAT is the problem of answering the question whether or not a propositional formula is satisfiable. Despite the worst-case exponential run time of all known algorithms, satisfiability solvers are increasingly leaving their mark as a general- purpose tool in areas as diverse as software and hardware verification. More concretely, as the existing sequential algorithm used the Davis-Putnam-Logemann-Loveland (DPLL) algorithm which decides satisfiability of propositional formula in clause normal form, by using unit propagation and case distinction. This project's goal is parallelizing the DPLL algorithm in the functional programming language Haskell. I would like to find the performance increments between sequential DPLL algorithm and DPLL algorithm after parallelization under 1-4 core VM.

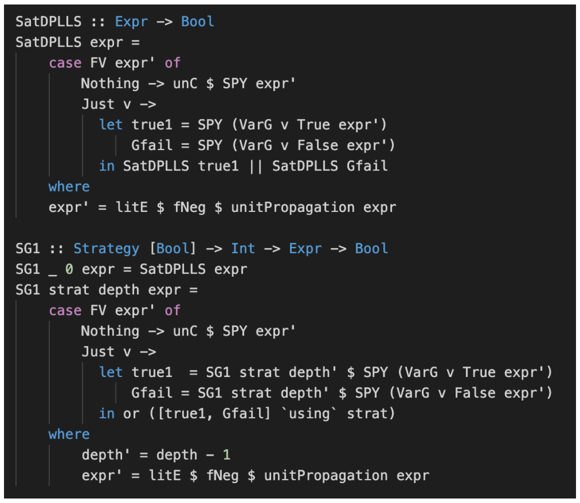
DPLL

DPLL is essentially a depth-first search, which alternates between three strategies. At any stage of the search, there are partial assignments (that is, assigning values to a certain subset of variables) and a set of undecided clauses (that is, unsatisfied clauses).
(1) The first strategy is pure literal elimination: if an unassigned variable x only appears in a set of unsubscribing sentences in its positive form (that is, the literal ~x does not appear anywhere), then we can use it in our assignment Add x = true and satisfy all clauses containing the literal x (again, if x only appears in the negative form, ~x, we can add x = false to our assignment).
(2) The second strategy is unit propagation: if all literals except one literal in the undecided sentence are false, the remaining literals must be true. If the remaining text is x, we add x = true to our assignment; if the remaining text is ~x, we add x = false to our assignment. This distribution can bring more opportunities for unit communication.
(3) The third strategy is to simply select an unassigned variable x and branch the search: try x = true on one side and x = false on the other side.



Parallelizing DPLL

I try to parallelize the Boolean guessing process. Every time a free variable is guessed, the guess is divided into two branches, a true branch and a false branch. Each branch requires additional work by guessing additional variables, and for each recursive call to the process, the work is parallelized to one thread. Although the sequential DPLL algorithm usually traverses the decision tree through depth-first search, the idea of parallelization is to traverse all paths in parallel.



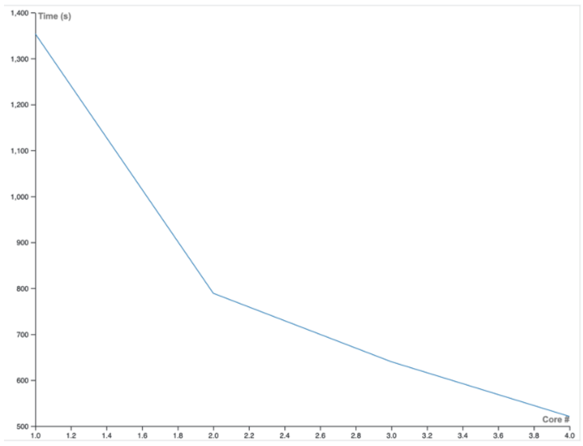
```haskell
SatDPLLS :: Expr -> Bool
SatDPLLS expr =
    case FV expr' of
        Nothing -> unC $ SPY expr'
        Just v ->
          let true1 = SPY (VarG v True expr')
              Gfail = SPY (VarG v False expr')
          in SatDPLLS true1 || SatDPLLS Gfail
    where
    expr' = litE $ fNeg $ unitPropagation expr

SG1 :: Strategy [Bool] -> Int -> Expr -> Bool
SG1 _ 0 expr = SatDPLLS expr
SG1 strat depth expr =
    case FV expr' of
        Nothing -> unC $ SPY expr'
        Just v ->
          let true1  = SG1 strat depth' $ SPY (VarG v True expr')
              Gfail = SG1 strat depth' $ SPY (VarG v False expr')
          in or ([true1, Gfail] `using` strat)
    where
        depth' = depth - 1
        expr' = litE $ fNeg $ unitPropagation expr
```

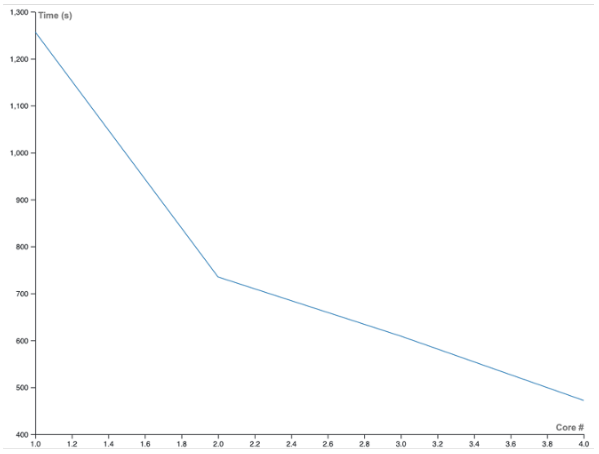
Using divide-and-conquer idea, which incrementally divides the search space into subspaces, successively allocated to sequential CDCL workers. Workers cooperate through some load-balancing strategy, which performs the dynamic transfer of subspaces to idle workers, and through the exchange of conflict clauses.

Result

I used Virtual Machining online using a 2.3 GHz Intel Xeon CPU, processor with 4 cores to generate DIMACS Benchmark Instances online settings, but found that there is no obvious and surprising difference in the running time of sequential and parallel algorithms. This is unexpected.



Sequential Algorithm



Parallel Algorithm

	2 cores	3 cores	4 cores
Performance increase(%)	171%	206%	266%

Runtime increasement for DPLL Parallel Algorithm Under different cores

Future Plan
I think I should try to parallelize sequential algorithms from different aspects to get better performance. (For example, simplification of expressions:Parallelizing the expression simplification led to far too great of spark creation with the vast majority being garbage collected, unit propagation: too much synchronization between threads is required.)
Also, try the algorithm remotely on different machine for a fair amount of time.

Reference

1. M. B¨ohm and E. Speckenmeyer. A Fast Parallel SAT-Solver - Efficient Workload Balancing. Ann. Math. Artif. Intell., 17(3-4):381–400, 1996.
2. https://www.cs.ubc.ca/~hoos/SATLIB/benchm.html
3. https://en.wikipedia.org/wiki/DPLL algorithm
4. https://hackage. haskell.org/package/parse-dimacs/.

Code

ParallelSATSolver.hs
```
module ParallelSATSolver where

import Library

import Control.Parallel.Strategies(Strategy, using, rpar)

import Control.DeepSeq(NFData)


PPS :: (NFData a) => Strategy [a]

PPS [a,b] = do

   a' <- rpar a

   b' <- rpar b

   return [a', b']

PPS _ = undefined


Depthfix :: Int

Depthfix = 50



Sat :: Expr -> Bool
Sat (Const b) = b
Sat orExpr@(Or _ _) = SatDPLL orExpr
Sat (And x y) = and ([SatDPLL x, SatDPLL y]
                       `using` PPS)
Sat _ = undefined

SatDPLL :: Expr -> Bool
SatDPLL = SG1 PPS Depthfix

SatDPLLS :: Expr -> Bool
SatDPLLS expr =
   case FV expr' of
       Nothing -> unC $ SPY expr'
       Just v ->
         let true1 = SPY (VarG v True expr')
             Gfail = SPY (VarG v False expr')
         in SatDPLLS true1 || SatDPLLS Gfail
   where
   expr' = litE $ fNeg $ unitPropagation expr

SG1 :: Strategy [Bool] -> Int -> Expr -> Bool
SG1 _ 0 expr = SatDPLLS expr
SG1 strat depth expr =
   case FV expr' of
       Nothing -> unC $ SPY expr'
       Just v ->
         let true1  = SG1 strat depth' $ SPY (VarG v True expr')
             Gfail = SG1 strat depth' $ SPY (VarG v False expr')
         in or ([true1, Gfail] `using` strat)
   where
       depth' = depth - 1
       expr' = litE $ fNeg $ unitPropagation expr
```

Library.hs

```
module Library where
```

```haskell
import Control.Applicative ((<|>))
import Data.Set (Set)
import qualified Data.Set as Set
import Data.Maybe (mapMaybe, catMaybes)
data Expr = Var String
  | And Expr Expr
  | Or Expr Expr
  | Not Expr
  | Const Bool
 deriving (Show, Eq)

litE :: Expr -> Expr
litE e =
   let ls = Set.toList (lits e)
       ps = map (litP e) ls
       exP :: String -> Maybe Polarity -> Maybe (String, Bool)
       exP v (Just Positive) = Just (v, True)
       exP v (Just Negative) = Just (v, False)
       exP _ _ = Nothing
       works :: [(String, Bool)]
       works = catMaybes $ zipWith exP ls ps
       substitutes :: [Expr -> Expr]
       substitutes = map (uncurry VarG) works
       suball :: Expr -> Expr
       suball = foldl (.) id substitutes
   in suball e

SPY :: Expr -> Expr
SPY (Const b) = Const b
SPY (Var v) = Var v
SPY (Not expr) =
 case SPY expr of
    Const b -> Const (not b)
    expr' -> Not expr'
SPY (Or x y) =
  let es = filter (/= Const False) [SPY x, SPY y] in
       if Const True `elem` es
       then Const True
       else
```

```
         case es of
           []       -> Const False
           [e]      -> e
           [e1, e2] -> Or e1 e2
           _        -> error "Should never happen."
SPY (And x y) =
  let es = filter (/= Const True) [SPY x, SPY y] in
       if Const False `elem` es
       then Const False
       else
        case es of
           [] -> Const True
           [e] ->e
           [e1, e2] -> And e1 e2
           _ -> error "Should never happen."


unC :: Expr -> Bool
unC (Const b) = b
unC _ = error "Not Const"

FV :: Expr -> Maybe String
FV (Const _) = Nothing
FV (Var v) = Just v
FV (Not e) = FV e
FV (Or x y) = FV x <|> FV y
FV (And x y) = FV x <|> FV y

VarG :: String -> Bool -> Expr -> Expr
VarG var val expr =
  case expr of
     Var v -> if v == var
             then Const val
             else Var v
     Not expr' -> Not (guess expr')
     Or x y -> Or (guess x) (guess y)
     And x y -> And (guess x) (guess y)
     Const b -> Const b
  where guess = VarG var val
```

```haskell
lits :: Expr -> Set String
lits (Var v) = Set.singleton v
lits (Not e) = lits e
lits (And x y) = Set.union (lits x) (lits y)
lits (Or x y) = Set.union (lits x) (lits y)
lits _ = Set.empty
data Polarity = Positive | Negative | Mixed deriving (Show, Eq)

litP :: Expr -> String -> Maybe Polarity
litP (Var v) v'
   | v == v' = Just Positive
   | otherwise = Nothing

   | v == v' = Just Negative
   | otherwise = Nothing

fNeg :: Expr -> Expr
fNeg expr =
  case expr of
     Not (Not x) -> fNeg x
     Not (And x y) -> Or (fNeg $ Not x) (fNeg $ Not y)
     Not (Or x y) -> And (fNeg $ Not x) (fNeg $ Not y)
     Not (Const b) -> Const (not b)
     Not x -> Not (fNeg x)
     And x y -> And (fNeg x) (fNeg y)
     Or x y -> Or (fNeg x) (fNeg y)
     x -> x
litP expr v =
  case expr of
     And x y -> comP [x, y]
     Or x y  -> comP [x, y]
     Not x   -> error $ "Not in CNF: negation of a non-literal: " ++ show x
     Const _ -> Nothing
     _       -> error "Should never happen."
  where
```

```
     comP es =
       let POL = mapMaybe (flip litP v) es
       in case POL of
         [] -> Nothing
         ps -> if all (== Positive) ps
               then Just Positive
               else if all (== Negative) ps
                    then Just Negative
                    else Just Mixed



CLA :: Expr -> [Expr]
CLA (And x y) = CLA x ++ CLA y
CLA expr = [expr]

oneC :: Expr -> Maybe (String, Bool)
oneC (Var v) = Just (v, True)
oneC (Not (Var v)) = Just (v, False)
oneC _ = Nothing

oneP:: Expr -> Expr
oneP expr = suball expr
  where
     works :: [(String, Bool)]
     works = allCLA expr
     suball :: Expr -> Expr
     suball = foldl (.) id (map (uncurry VarG) works)



allCLA :: Expr -> [(String, Bool)]
allCLA = mapMaybe oneC . CLA
```

ParseIO.hs

```
module ParseIO where
import Library(Expr(..))
import Data.Array.Unboxed
```

```haskell
import Language.CNF.Parse.ParseDIMACS
import Language.Haskell.Exts
CtoE :: CNF -> Expr
CtoE cnf = Cla list1
 where
   list1   = map uAtoE $ CLA cnf
   Cla :: [Expr] -> Expr
   Cla [] = Const True
   Cla [x] = x
   Cla [x,y,z] = And (And x y) z
   Cla xs = And (Cla ft) (Cla bk)
     where (ft, bk) = splitAt ((length xs + 1) `div` 2) xs

LFF :: FilePath -> IO (Either String Expr)
LFF fileName = do
 cnf <- parseFile fileName
 case cnf of
   LT _ -> return $ LT "Parse Error"
   RT cnf -> return $ RT $ CtoE cnf

uAtoE :: UArray Int Int -> Expr
uAtoE = Lit . (map ItoE) . elems
 where
   ItoE :: Int -> Expr
   ItoE n | n > 0 = Var $ show n
   ItoE n =Not $ Var $ show $ abs n

   Lit :: [Expr] -> Expr
   Lit = foldr1 (\x acc -> Or x acc)
```

Main.hs

```haskell
module Main where
import ParallelSATSolver
import ParseIO
import System.Exit(die)
import System.Environment(getArgs, getProgName)
main :: IO ()
main = do
     param <- getArgs
     case param of
           [file] -> do
                 cnf<- loadFile file
                 case cnf of
                       LT err   -> putStrLn err
                       RT cnf' -> case SatDPLL cnf' of
                             True -> putStrLn $ file ++
                                     " is Satisfiable"
                             _  -> putStrLn $ file ++
                                     " is UnSatisfiable"
           _ -> do
                 pn <- getProgName
                 die $ "Usage: " ++ pn ++ " <filename>"
```