

Fundamentals of Computer Systems

Thinking Digitally

Stephen A. Edwards

Columbia University

Summer 2021

The Subject of this Class

0

The Subjects of this Class

0

1

But let your communication be, Yea, yea; Nay, nay: for whatsoever is more than these cometh of evil.

— Matthew 5:37



SUPERCODER 2000

Air cooled coding keyboard for professional use.





0

1

Space!

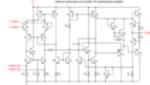
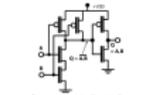
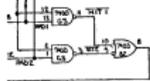
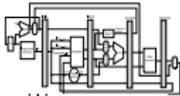
ENTER



Engineering Works Because of Abstraction



```
;; voice 1 wave select
ld a, (#CHI_W_NUM)
and a
ld a, (#CHI_W_SEL)
jr nz, #00b4
ld a, (#CHI_E_TABLE0)
```



Application Software

Operating Systems

Architecture

Micro-Architecture

Logic

Digital Circuits

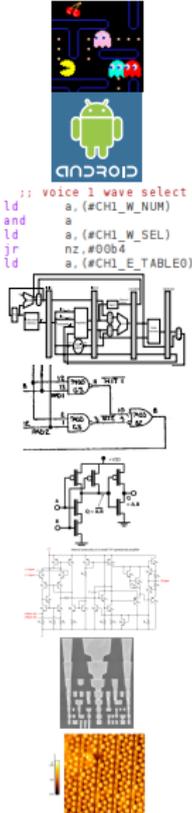
Analog Circuits

Devices

Physics



Engineering Works Because of Abstraction



Application Software COMS 3157, 4156, et al.

Operating Systems COMS W4118

Architecture Second Half of 3827 CS 😊

Micro-Architecture Second Half of 3827 EE 😞

Logic First Half of 3827 EE

Digital Circuits First Half of 3827 ~~CS~~

Analog Circuits ELEN 3331

Devices ELEN 3106

Physics ELEN 3106 et al.

Boring Stuff

<http://www.cs.columbia.edu/~sedwards/classes/2021/3827-summer/>

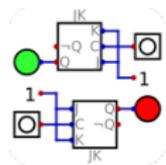
Prof. Stephen A. Edwards
sedwards@cs.columbia.edu

Lectures 4:10 – 6:40 PM, Mondays and Wednesdays
May 3–June 14

Weight	What	When
40%	Homeworks	See Webpage
60%	Final exam	June 18th

Submit homework online via Courseworks

Software You Need



Digital Simulator github.com/hneemann/Digital

Circuit design problems: download (class website) .zip file with .dig files, edit with Digital, upload to Courseworks

SPIM: A MIPS32 Simulator spimsimulator.sourceforge.net

MIPS assembly coding:, download .zip file with .s files, edit in favorite text editor, test and debug in SPIM, upload to Courseworks



INKSCAPE

The Inkscape SVG File Editor inkscape.org

You can do homework by downloading an SVG file from the class website, editing it in Inkscape, and uploading it to Courseworks

Rules and Regulations

Each assignment turned in must be unique; work must ultimately be your own.

Don't cheat: Columbia Students Aren't Cheaters

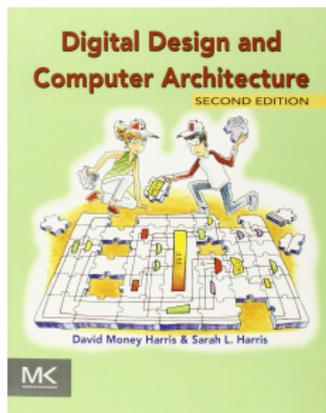
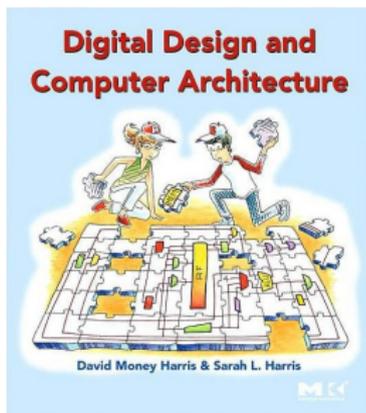
Test will be closed-book; you may use a single sheet of your own notes

Optional Texts: Alternative 1

No required text. One option:

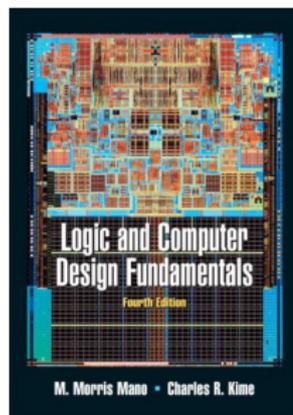
- ▶ David Harris and Sarah Harris. *Digital Design and Computer Architecture*. Either 1st or 2nd ed.

Almost precisely right for the scope of this class: digital logic and computer architecture.

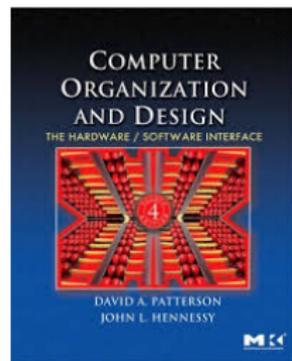


Optional Texts: Alternative 2

- ▶ M. Morris Mano and Charles Kime. *Logic and Computer Design Fundamentals*. 4th ed.



- ▶ David A. Patterson and John L. Hennessy. *Computer Organization and Design, The Hardware/Software Interface*. 4th ed.



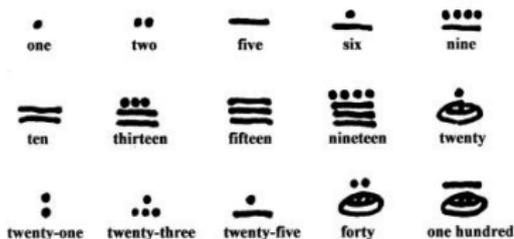
GILDAN
ULTRA
COTTON™

There are only 10 types
of people in the world:
Those who understand binary
and those who don't.

Which Numbering System Should We Use?



Roman: I II III IV V VI VII VIII IX X



Mayan: base 20, Shell = 0

1	∩	11	∩ < ∩	21	∩ ∩ ∩	31	∩ ∩ ∩ ∩	41	∩ ∩ ∩ ∩ ∩	51	∩ ∩ ∩ ∩ ∩ ∩
2	∩ ∩	12	∩ ∩ < ∩	22	∩ ∩ ∩ ∩	32	∩ ∩ ∩ ∩ ∩	42	∩ ∩ ∩ ∩ ∩ ∩	52	∩ ∩ ∩ ∩ ∩ ∩ ∩
3	∩ ∩ ∩	13	∩ ∩ ∩ < ∩	23	∩ ∩ ∩ ∩ ∩	33	∩ ∩ ∩ ∩ ∩ ∩	43	∩ ∩ ∩ ∩ ∩ ∩ ∩	53	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩
4	∩ ∩ ∩ ∩	14	∩ ∩ ∩ ∩ < ∩	24	∩ ∩ ∩ ∩ ∩ ∩	34	∩ ∩ ∩ ∩ ∩ ∩ ∩	44	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	54	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩
5	∩ ∩ ∩ ∩ ∩	15	∩ ∩ ∩ ∩ ∩ < ∩	25	∩ ∩ ∩ ∩ ∩ ∩ ∩	35	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	45	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	55	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩
6	∩ ∩ ∩ ∩ ∩ ∩	16	∩ ∩ ∩ ∩ ∩ ∩ < ∩	26	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	36	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	46	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	56	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩
7	∩ ∩ ∩ ∩ ∩ ∩ ∩	17	∩ ∩ ∩ ∩ ∩ ∩ ∩ < ∩	27	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	37	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	47	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	57	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩
8	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	18	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ < ∩	28	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	38	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	48	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	58	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩
9	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	19	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ < ∩	29	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	39	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	49	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩	59	∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩ ∩
10	∩	20	∩	30	∩ ∩ ∩	40	∩ ∩ ∩ ∩	50	∩ ∩ ∩ ∩ ∩		

Babylonian: base 60

2004 5 6 10
12. m

The Decimal Positional Numbering System



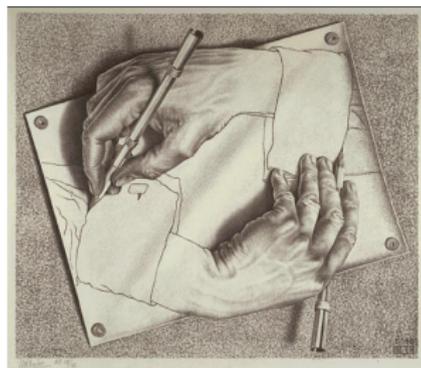
Ten figures: 0 1 2 3 4 5 6 7 8 9

$$730_{10} = 7 \times 10^2 + 3 \times 10^1 + 0 \times 10^0$$

$$990_{10} = 9 \times 10^2 + 9 \times 10^1 + 0 \times 10^0$$

2.5 10, ...

Why base ten?



Hexadecimal, Decimal, Octal, and Binary

Hex	Dec	Oct	Bin
0	0	0	0
1	1	1	1
2	2	2	10
3	3	3	11
4	4	4	100
5	5	5	101
6	6	6	110
7	7	7	111
8	8	10	1000
9	9	11	1001
A	10	12	1010
B	11	13	1011
C	12	14	1100
D	13	15	1101
E	14	16	1110
F	15	17	1111

Base

16

$$2^4 = 16$$

$$20_8 = 16_{10}$$

$$10_{16} =$$

Base

8

Octal

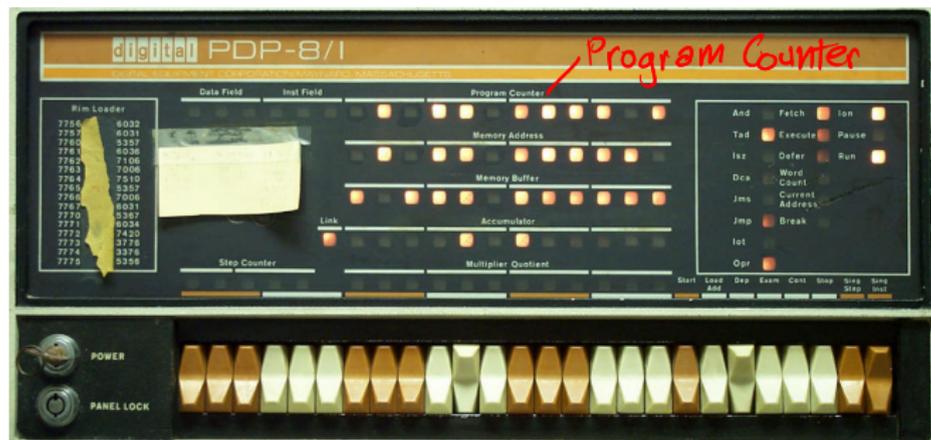
8^4 8^3 8^2
 8^1 8^0 8^0 8^0 8^0

| | | | |

3 4 5 4 3 2 1 0
 | | | | | | | |

second octal digit one octal digit

Binary and Octal: Electronics Likes Powers of Two



DEC PDP-8/I, c. 1968

Oct	Bin
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111

$$\begin{aligned} \text{PC} &= \overbrace{010110111101}_2 \\ &= 0 \times 2^{11} + 1 \times 2^{10} + 0 \times 2^9 + 1 \times 2^8 + 1 \times 2^7 + 0 \times 2^6 + \\ &\quad 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 2675_8 \\ &= 2 \times 8^3 + 6 \times 8^2 + 7 \times 8^1 + 5 \times 8^0 \\ &= 1469_{10} \end{aligned}$$

Hexadecimal Numbers

Base 16: 0 1 2 3 4 5 6 7 8 9 A B C D E F

Instead of groups of 3 bits (octal), Hex uses groups of 4.

zero

$$\begin{aligned} \text{CAFEEF00D}_{16} &= 12 \times 16^7 + 10 \times 16^6 + 15 \times 16^5 + 14 \times 16^4 + \\ &\quad 15 \times 16^3 + 0 \times 16^2 + 0 \times 16^1 + 13 \times 16^0 \\ &= 3,405,705,229_{10} \end{aligned}$$

	C	A	F	E	F	0	0	D		Hex		
<i>0</i>	1100	1010	1111	1110	1111	1000	0000	0110	1	Binary		
	3	1	2	7	7	5	7	0	0	1	5	Octal

Computers Rarely Manipulate True Numbers

Infinite memory still very expensive

00000 }
99999 } 100,000

Finite-precision numbers typical

32-bit processor: naturally manipulates 32-bit numbers

64-bit processor: naturally manipulates 64-bit numbers

How many different numbers can you

represent with 5
binary
octal
decimal
hexadecimal
digits?

100000_2 $2^5 = 32$
 100000_8 $8^5 = 32768$
 $100,000_{10}$ $10^5 =$
 100000_{16} $16^5 =$

one megabyte
of memory ——— 1 M
1,048,576

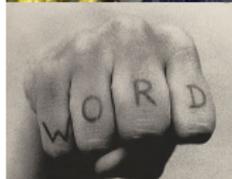
Jargon



Bit Binary digit: 0 or 1



Byte Eight bits



Word Natural number of bits for the processor, e.g., 16, 32, 64



LSD

LSB Least Significant Bit ("rightmost")



MSG

MSB Most Significant Bit ("leftmost")

~~1010~~₂ = 10₁₀

Decimal Addition Algorithm

$$\begin{array}{r} 434 \\ + 628 \\ \hline \end{array}$$

$$4 + 8 = 12$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} 1 \\ 434 \\ + 628 \\ \hline 2 \end{array}$$

$$4 + 8 = 12$$

$$1 + 3 + 2 = 6$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} 1 \\ 434 \\ + 628 \\ \hline 62 \end{array}$$

$$4 + 8 = 12$$

$$1 + 3 + 2 = 6$$

$$4 + 6 = 10$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} 1\ 1 \\ 434 \\ +628 \\ \hline 062 \end{array}$$

$$4 + 8 = 12$$

$$1 + 3 + 2 = 6$$

$$4 + 6 = 10$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Decimal Addition Algorithm

$$\begin{array}{r} 1\ 1 \\ 434 \\ +628 \\ \hline 1062 \end{array}$$

$$4 + 8 = 12$$

$$1 + 3 + 2 = 6$$

$$4 + 6 = 10$$

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18
10	10	11	12	13	14	15	16	17	18	19

Binary Addition Algorithm

$$\begin{array}{r} 10011 \\ +11001 \\ \hline \end{array}$$

$$1 + 1 = 10$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} 1 \\ 10011 \\ +11001 \\ \hline 0 \end{array}$$

$$1 + 1 = 10$$

$$1 + 1 + 0 = 10$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} 11 \\ 10011 \\ +11001 \\ \hline 00 \end{array}$$

$$1 + 1 = 10$$

$$1 + 1 + 0 = 10$$

$$1 + 0 + 0 = 01$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} 011 \\ 10011 \\ +11001 \\ \hline 100 \end{array}$$

$$1 + 1 = 10$$

$$1 + 1 + 0 = 10$$

$$1 + 0 + 0 = 01$$

$$0 + 0 + 1 = 01$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} 0011 \\ 10011 \\ +11001 \\ \hline 1100 \end{array}$$

$$1 + 1 = 10$$

$$1 + 1 + 0 = 10$$

$$1 + 0 + 0 = 01$$

$$0 + 0 + 1 = 01$$

$$0 + 1 + 1 = 10$$

+	0	1
0	00	01
1	01	10
10	10	11

Binary Addition Algorithm

$$\begin{array}{r} 10011 \\ 10011 \\ +11001 \\ \hline 101100 \end{array}$$

$$1 + 1 = 10$$

$$1 + 1 + 0 = 10$$

$$1 + 0 + 0 = 01$$

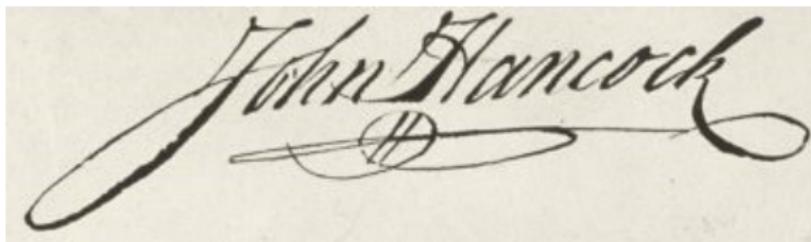
$$0 + 0 + 1 = 01$$

$$0 + 1 + 1 = 10$$

+	0	1
0	00	01
1	01	10
10	10	11

Signed Numbers: Dealing with Negativity

How should we represent negative numbers?

A handwritten signature in cursive script, reading "John Hancock". The signature is written in dark ink on a light-colored, aged paper. The letters are fluid and connected, with a prominent flourish under the "H" and a long, sweeping underline that extends to the right.

Binary Signed Magnitude Numbers



The familiar notation: negative numbers have a leading –

Binary signed-magnitude encoding: leading 1 indicates negative; remaining bits treated as binary.



$$0000_2 = 0$$

$$0010_2 = 2$$

$$1010_2 = -2$$

$$1111_2 = -7 \quad -111_2$$

$$1000_2 = -0?$$

↳ signed magnitude

Can be made to work, but addition is annoying:

If the signs match, add the magnitudes and use the same sign.

If the signs differ, subtract the smaller number from the larger; return the sign of the larger.

One's Complement Numbers

$$\begin{array}{r} 2 \\ + -2 \\ \hline 0 \end{array} \quad \begin{array}{r} 0010 \\ + 1101 \\ \hline 1111 = -0 = 0 \end{array}$$

Like Signed Magnitude, a leading 1 indicates a negative One's Complement number. However, number magnitude is *complement* of remaining bits interpreted as binary.

To negate a number, complement (flip) each bit.

$$0000_2 = 0$$

$$0010_2 = 2$$

$$1101_2 = -2 \quad -010$$

$$1000_2 = -7$$

$$1111_2 = -0?$$

↑
negative 000

Addition is nicer: just add the one's complement numbers as if they were normal binary.

Really annoying having a -0 : two numbers are equal if their bits are the same or if one is 0 and the other is -0 .

one's comp
 $0000 = 1111$

| Note this ↙



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ZEROS
ARE CREATED
EQUAL**

ZERO CALORIES. MAXIMUM PEPSI™ TASTE.



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Two's Complement Numbers



Really neat trick: just make only the most significant bit represent a *negative* number instead of positive; treat the rest as binary.

~~-8 + 4 + 1~~
 $1101_2 = -8 + 4 + 1 = -3$

$$1111_2 = -8 + 4 + 2 + 1 = -1$$

$$0111_2 = 4 + 2 + 1 = 7$$

$$1000_2 = -8$$

$1110 = -2$
 $1111 = -1$
 $\times 1101$
 $1101 = -3$
 $0000_2 = 0$
 $1 + 4 - 8 = -3$

✓ Easy addition: just add in binary and discard any carry.

Negation: complement each bit (as in one's complement) then add 1. *Not as good as 1's complement*

$$\begin{array}{r} 0001 \\ 1110 \\ + 1111 \end{array}$$

✓ Subtraction done with negation and addition.

✓ Very good property: no -0

✓ Two's complement numbers are equal if and only if all their bits are the same.

Number Representations Compared

Code	Binary	Signed Mag.	One's Comp.	Two's Comp.
0000	0	0	0	0
0001	1	1	1	1
⋮				
0111	7	7	7	7
1000	8	-0	-7	-8
1001	9	-1	-6	-7
⋮				
1110	14	-6	-1	-2
1111	15	-7	-0	-1

16

15

15

16

How many we represent

same address all / 4

symmetric

awkward

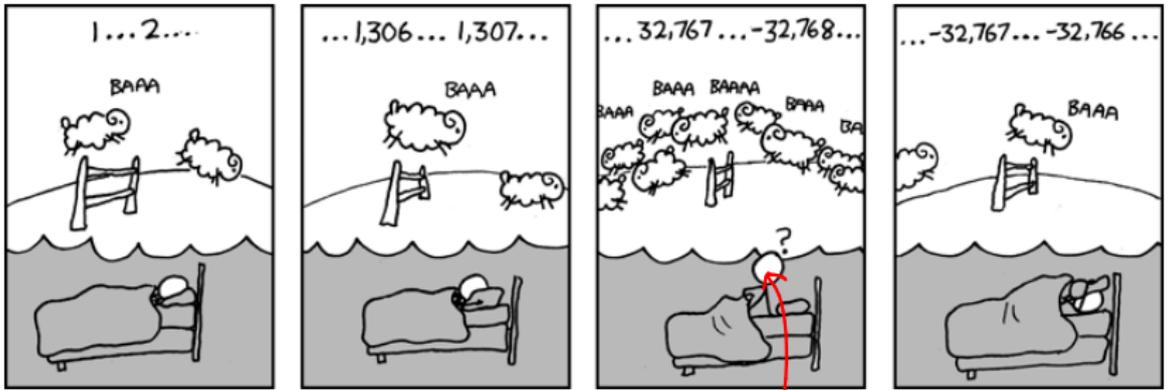
feels nice

Smallest number

Largest number

symmetric

$32,767$, $-32,768$,



<https://xkcd.com/571/>

How many bits in his brain?

How many bits?
16 bits, two's complement

Fixed-point Numbers



How to represent fractional numbers? In decimal, we continue with negative powers of 10:

$$31.4159 = 3 \times 10^1 + 1 \times 10^0 + 4 \times 10^{-1} + 1 \times 10^{-2} + 5 \times 10^{-3} + 9 \times 10^{-4}$$

tenths *hundredths*

Also works in binary:

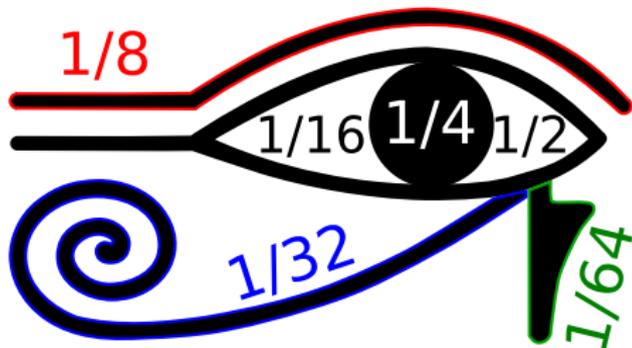
$$\begin{aligned} 1011.0110_2 &= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3} + 0 \times 2^{-4} \\ &= 8 + 2 + 1 + 0.25 + 0.125 \\ &= 11.375 \end{aligned}$$

halves
quarters
eighths

Addition and subtraction algorithms the same.

F F
u a
c
Interesting

The ancient Egyptians used binary fractions:



The Eye of Horus

Binary-Coded Decimal

0-9 in hex/binary

12826:16



Humans prefer reading decimal numbers; computers prefer binary.

BCD is a compromise: every four bits represents a decimal digit.

thinkgeek.com



59



Dec	BCD
0	0000 0000
1	0000 0001
2	0000 0010
⋮	⋮
8	0000 1000
9	0000 1001
10	0001 0000
11	0001 0001
⋮	⋮
18	0001 1000
19	0001 1001
20	0010 0000
⋮	⋮

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

$$\begin{array}{r} 158_{10} \\ +242_{16} \\ \hline \end{array}$$

$$\begin{array}{r} 101011000 \\ +001001000010 \\ \hline 1010 \text{ First group} \end{array}$$

$$\begin{array}{r} 1010 \\ \hline 0000 \end{array}$$

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

$$\begin{array}{r} 158 \\ +242 \\ \hline \end{array}$$

$$\begin{array}{r} 000101011000 \\ +001001000010 \\ \hline 1010 \\ +0110 \\ \hline \end{array}$$

*> 9
so add 6*

First group
Correction

0000

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

$$\begin{array}{r} 1 \\ 158 \\ +242 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1 \\ 000101011000 \\ +001001000010 \\ \hline 1010 \quad \text{First group} \\ +0110 \quad \text{Correction} \\ \hline 10100000 \quad \text{Second group} \\ \hline \\ \hline \end{array}$$

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

$$\begin{array}{r} 1 \\ 158 \\ +242 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 1 \\ 000101011000 \\ +001001000010 \\ \hline 1010 \quad \text{First group} \\ +0110 \quad \text{Correction} \\ \hline 10100000 \quad \text{Second group} \\ +0110 \quad \text{Correction} \\ \hline \hline \end{array}$$

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

$$\begin{array}{r} 11 \\ 158 \\ +242 \\ \hline 00 \end{array}$$

$$\begin{array}{r} 111000 \\ +001001000010 \\ \hline 1010 \quad \text{First group} \\ +0110 \quad \text{Correction} \\ \hline 10100000 \quad \text{Second group} \\ +0110 \quad \text{Correction} \\ \hline 01000000 \quad \text{Third group} \\ \hline \end{array}$$

BCD Addition

Binary addition followed by a possible correction.

Any four-bit group greater than 9 must have 6 added to it.

Example:

$$\begin{array}{r} 11 \\ 158 \\ +242 \\ \hline 400 \end{array}$$

combinations
16 codes
patterns
-10 symbols

10 + 6 = 16
11 + 6 = 17

1 1 }

$$\begin{array}{r} 000101011000 \\ +001001000010 \\ \hline \end{array}$$

1010 First group
+ 0110 Correction

10100000 Second group
+ 0110 Correction

01000000 Third group
(No correction)

010000000000 Result

4 0 0

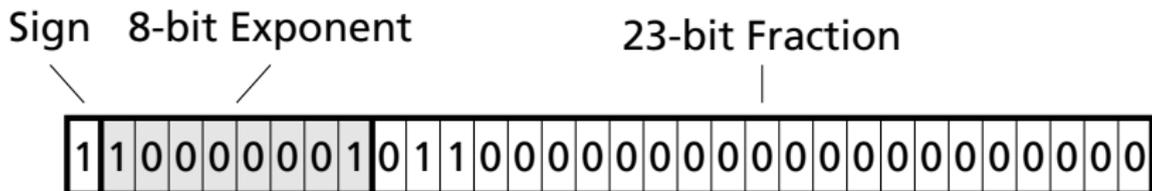
→ correction

Floating-Point Numbers: "Scientific Notation"

Greater dynamic range at the expense of precision
 Excellent for real-world measurements

~~1.375 × 10⁻¹³~~

IEEE 754 Single-Precision (32-bit) *mantissa*



*explicit
sign
bit*

implicit

"excess 127"

$$= - \textcircled{1.0110000}_2 \times 2^{10000001_2 - 127}$$

$$= -1.375 \times 2^2$$

$$= -5.5$$

*1 × 10²
0.35 × 10¹*

*1.000
0.035
1.035 × 10²
0.035 × 10²*

ASCII For Representing Characters and Strings

	0	1	2	3	4	5	6	7
0	NUL	DLE	SP	0	@	P	'	p
1	SOH	DC1	!	1	A	Q	a	q
2	STX	DC2	"	2	B	R	b	r
3	ETX	DC3	#	3	C	S	c	s
4	EOT	DC4	\$	4	D	T	d	t
5	ENQ	NAK	%	5	E	U	e	u
6	ACK	SYN	&	6	F	V	f	v
7	BEL	ETB	'	7	G	W	g	w
8	BS	CAN	(8	H	X	h	x
9	HT	EM)	9	I	Y	i	y
A	LF	SUB	*	:	J	Z	j	z
B	VT	ESC	+	;	K	[k	{
C	FF	FS	,	<	L	\	l	
D	CR	GS	-	=	M]	m	}
E	SO	RS	.	>	N	^	n	~
F	SI	US	/	?	O	_	o	DEL
