Programming Language and Translator

Project Proposal

Language for Linear Algebra (LFLA)

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Motivation

Linear algebra is one of the most important subject for undergraduate STEM students. Currently, the undergraduate linear algebra courses are matrix oriented, as opposed to theory oriented. However, due to the abstractness of linear algebra, many students still find it too difficult to catch its essence. We believe that computer science should play a crucial role in math education - especially helping students to learn tough subjects like linear algebra. Yet, unfortunately, the common available math software involving linear algebra like MATLAB and R only deal with matrix. This only leave students with the impression that linear algebra is just matrix. Moreover, in MATLAB the vectors are treated as the (1 x m) or (n x 1) matrix, and this is not conceptually correct. It often leads to confusion and problems in coding and debugging. Therefore, these math oriented language cannot truly help the students understand the fundamentals and the beauty of the linear algebra. Given this situation, we would like to create a domain language to help students and teachers in learning and teaching linear algebra. The Language for Linear Algebra (LFLA) is mainly designed for educational purpose.

Features

- In LFLA we have several primitive types direct corresponding to the concept in linear algebra. Beside the matrix, we also have vector, vector space, affine spaces and inner product space. It will tell the students that linear algebra is more than just matrix.

- Besides the normal matrix calculation, our language emphasizes the relation between vector space, vectors and matrix considered as a map. We believe that understanding their relation is important in learning the linear algebra and students can get insight by using the language to exploring the linear algebra world.

- We take the best to make the language syntax similar to the math notation, so that learning the language will not be a large overhead.
Built-in Types

Types

**var** // An integer or a float as scalar

**vector** // An element of the real coordinate space $\mathbb{R}^n$.
  // An n-vector is a list of n numbers.

**vecspace** // Vector space. It’s a collection of vectors.

**matrix** // A rectangular array of numbers, arranged in
  // rows and columns.

**inspace** // Inner product space. It’s a vector space
  // with an inner product.

**affspace** // Affine space.

Type Declaration and Initialization

**VECTOR**

// Declare a vector v
vector v;

// Declare and initialize a vector v
vector v = [a_1, a_2, …, a_N]; // Separate by comma

// Examples
vector v;
vector v = [1,2,3];

**MATRIX**

// Declare a matrix m
matrix m;

// Declare and initialize a matrix m
matrix m = [
  a_{11}, a_{12}, …, a_{1M};
  a_{21}, a_{22}, …, a_{2M};
]
\[ a_{N1}, a_{N2}, \ldots, a_{NM} \]

// Each element in vector is separated by comma
// Each vector is separated by semicolon

// Examples
matrix m;
matrix m = [1,2; 3,4];

// Note matrix cannot interchange with vector
vector v1 = [1,3];
vector v2 = [3,4];
matrix m = [v1; v2]; // This is not allowed

VECSMPL

// A vector space is defined as a function of
// multiple vectors
vecspace vs = L(v1, v2, ..., vN);

// Examples
vector v1 = [1,0];
vector v2 = [0,1];
vecspace vs = L(v1, v2);

Operators

LFLA contains all operators in common languages like Java and C++, except for bit manipulation operators. However, there are subtile differences between scalar-scaler, scalar-object and object-object operations. Here objects are vectors, matrix, vector space, affine space and inner product space.
Scalar-Scalar operations

- **Assignment** = Assign a value to a var
- **Addition** + Add two var
- **Subtract** − Subtract one var from another
- **Multiply** * Multiply two var
- **Division** / Divide one var from another
- **Exponential** ^ Exponential computation on var
- **Modular** % Modular computation on var
- **Increment** ++ Add 1 to the value of var
- **Decrement** -- Subtract 1 from the value of var
- **Compare** !=, ==, <, >, <=, >= Normal comparison operations on var

Scalar-Object operations

- **Addition** +. Add a var to each element in vector, matrix
- **Subtract** -. Subtract a var from each element in vector, matrix
- **Multiply** *. Multiply a var to each element in vector, matrix
- **Division** /. Each element in vector, matrix is divided by a var
- **Exponential** ^. Exponential computation on each element in vector, matrix
- **Modular** %.. Modular computation on each element in vector, matrix

Object-Object operations

- **Assignment** = Assign an object, like vector, matrix, vecspace
- **Addition** + Add corresponding elements in two objects
- **Subtract** − Subtract between corresponding elements in two objects
- **Multiply** * Multiply corresponding elements in two objects
- **Transpose** ´ Transpose of a matrix
- **Inner Product** <,> Inner product
- **Lie Bracket** [ ] Lie Bracket
- **Compare** <, >, ==, @ Some specific comparisons

Library Functions

In LFLA language, it provides several library functions.

- `sqrt(var)`
  - Take square root of a scalar
• ceil(var)
  Take ceiling of a scalar

• floor(var)
  Take floor of a scalar

• dim(vector v)
  Give the dimension of a vector

• dim(matrix m)
  Give the dimensions of a matrix

• basis(vecspace vs)
  Give out one a array of basis (vector) of a vector space.

• rank(matrix m)
  Give the rank of a matrix

• trace(matrix m)
  Give the trace of a square matrix

• eigenValue(matrix m)
  Give the eigenvalues of a matrix

• image(matrix m)
  Give the image of a matrix

• orthoBasis(inspace i)
  Give the orthogonal basis of a inner product space

Control Flows and Function

```c
// if else statement
if expr
{
    statements;
}
```
else if
{
    statements;
}
else
{
    statements;
}
// for loop
for expr
{
    statements;
    break;
}
// while loop
while expr
{
    statements;
    continue;
}
// define a function
function name(args)
{
    statements;
    return [value1, value2, ..];
}

Sample Program

// Problem 1: Given an array of vectors of the same
// dimension, judge whether they are linear independent.
// Solution:
function linearIndep(vector[] vectors)
{
if vectors.length == 0
    return true;
if vectors.length > dim(vectors[0])
    return false;

var i;
vecspace vs;
for i=0:vectors.length
{
    if vectors[i]@vs
        return false;
    vs = vs + L(vectors[i]);
}
return true;

// Problem 2: Given a inner product space W = (V,a),
// please orthonormalising a basis of vector space of V
// by the Gram-Schmidt method
// Solution:
othonormalising(vector[] basis)
{
    var i, j;
    for i=0:basis.length
    {
        vector v = basis[i];
        for j=0:i
        {
            w = w-a<v,basis[i], basis[j]>*.basis[j];
        }
        basis[i] = w/.sqrt(a<w, w>);
    }
}