CSEE W3827
Fundamentals of Computer Systems
Homework Assignment 1
Solutions

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Columbia University
Due February 6, 2012 at 1:10 PM

Write your name and UNI on your solutions
Show your work for each problem; we are more interested in how you get the answer than whether you get the right answer.
1. (5 pts.) What are the values, in decimal, of the bytes

   \[10011100\]

and

   \[01111000,\]

if they are interpreted as 8-bit

(a) binary numbers;

   \[10011100_2 = 128 + 16 + 8 + 4 = 156;\]
   \[01111000_2 = 64 + 32 + 16 + 8 = 120\]

(b) one’s complement numbers; and

   \[-(1100011_2) = -(64 + 32 + 2 + 1) = -99;\]
   \[01111000_2 = 64 + 32 + 16 + 8 = 120\]

(c) two’s complement numbers?

   \[10011100_2 = -128 + 16 + 8 + 4 = -100 \text{ or}\]
   \[01100011 + 1 = 01100100 = 64 + 32 + 4 = -100;\]
   \[01111000_2 = 64 + 32 + 16 + 8 = 120\]
2. (10 pts.) Show how to compute $6 + -14$ using 5-bit

(a) signed-magnitude numbers;

$00110 + 11110 = -(1110 - 0110) = -(1000) = 11000$

Make sure you strip off the sign bits

(b) one’s complement numbers; and

$00110 + 10001 = 10111 = -(1000)$ (normal binary addition)

(c) two’s complement numbers.

$00110 + 100010 = 11000 = -(1000)$ (normal binary addition)
3. (10 pts.) Show how to compute $45 + 57$ in BCD.

```
0100 0101
+ 0101 0111
--------
1001 1100 The result of normal binary addition
  + 0110 Add 6 since the first digit exceeded 9
--------
1010 0010 Add 6 since the second digit exceeded 9
  + 0110
--------
0001 0000 0010 =102_{10}
```
4. (10 pts.) Complete the truth table for the following Boolean functions:

\[
\begin{align*}
    a &= X \bar{Y} \bar{Z} + \bar{X} \bar{Y} \bar{Z} + \bar{X} \bar{Z} \\
    b &= (X + \bar{Y})(Y + \bar{Z})(X + \bar{Z})
\end{align*}
\]

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<th>X</th>
<th>Y</th>
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The truth table for the Boolean functions is completed as shown above.
5. (10 pts.) Consider the function $F$, whose truth table is below.

<table>
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(a) Write $F$ as a sum of minterms and draw the corresponding circuit.

$$X\bar{Y}Z + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z}$$

(b) Write $F$ as a product of maxterms and draw the corresponding circuit.

$$(X + Y + Z)(X + \bar{Y} + \bar{Z})(X + Y + \bar{Z})$$

(c) Complete the Karnaugh map for $F$ as shown below. You do not have to simplify it.

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5. (a) \[ X \land Y \land Z + \overline{X} \land Y \land Z + X \land \overline{Y} \land \overline{Z} \]
5. (b)

\[ (X + Y + Z)(X + \bar{Y} + \bar{Z})(\bar{X} + Y + \bar{Z}) \]

\[ (\bar{X} + \bar{Y} + Z)(\bar{X} + \bar{Y} + \bar{Z}) \]
6. (10 pts.) Consider the function $F$ whose truth table is shown below

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(a) Write the function $F$ in sum-of-minterms form

$$F = \overline{W}XYZ + \overline{W}X\overline{Y}Z + \overline{W}XYZ + WX\overline{Y}Z + WX\overline{Z} + WX\overline{Y}Z$$

(b) Minimize the sum-of-minterms expression, justifying each step

see next page
\[ F = \overline{W} \overline{X} \overline{Y} Z + \overline{W} X \overline{Y} Z + \overline{W} X Y Z + W X \overline{Y} \overline{Z} + W X \overline{Y} Z \]
\[ = \overline{W}(X Y Z + X \overline{Y} Z + X Y Z) + W X \overline{Y}(\overline{Z} + Z) \text{ (Factoring)} \]
\[ = \overline{W}((\overline{X} + X)Y Z + X \overline{Y} Z) + W X \overline{Y} \text{ (Factoring, } A + \overline{A} = 1) \]
\[ = W Y Z + W X Y Z + W X \overline{Y} \]

or

\[ F = \overline{W} \overline{X} \overline{Y} Z + \overline{W} X \overline{Y} Z + \overline{W} X Y Z + W X \overline{Y} \overline{Z} + W X \overline{Y} Z \]
\[ = \overline{W}(X Y Z + X \overline{Y} Z + X Y Z) + W X \overline{Y}(\overline{Z} + Z) \text{ (Factoring)} \]
\[ = \overline{W}((\overline{Y} + Y)X Z + \overline{X} Y Z) + W X \overline{Y} \text{ (Factoring, } A + \overline{A} = 1) \]
\[ = W X Z + W X Y Z + W X \overline{Y} \]

or

\[ F = \overline{W} \overline{X} \overline{Y} Z + \overline{W} X \overline{Y} Z + \overline{W} X Y Z + W X \overline{Y} \overline{Z} + W X \overline{Y} Z \]
\[ = \overline{W}(X Y Z + X \overline{Y} Z + X Y Z + X Y Z) + W X \overline{Y}(\overline{Z} + Z) \text{ (Factoring, } A + A = 1) \]
\[ = \overline{W}(Y Z(X + \overline{X}) + X Z(\overline{Y} + Y)) + W X \overline{Y} \text{ (factoring)} \]
\[ = W Y Z + W X Z + W X \overline{Y} \text{ (} A + \overline{A} = 1) \]
7. (10 pts.) Consider the function $F$ from problem 6.

(a) Fill in and minimize the following Karnaugh map for $F$

(b) Express your minimized Karnaugh map as a Boolean expression

\[ \overline{WYZ} + \overline{WXZ} + WXY \] or \[ \overline{WYZ} + X\overline{YZ} + WX\overline{Y} \]

(c) Are your minimized expressions in problem 6 and problem 7 the same? Why or why not?

Not necessarily; it’s easy to get to a “local minimum” where to simplify the expression further, you have to make it messier first.
8. (20 pts.) Design a circuit that takes two two-bit binary numbers 
\((A_1 \text{ and } A_0, B_1 \text{ and } B_0)\) and produces a true output when, in 
binary, \(A\) is greater than or equal to \(B\).

(a) Fill in the truth table

(b) Fill in the Karnaugh map and use it to minimize

\[
\begin{array}{cccc|c}
A_1 & A_0 & B_1 & B_0 & A \geq B \\
0  & 0  & 0  & 0  & 1 \\
0  & 0  & 0  & 1  & 0 \\
0  & 0  & 1  & 0  & 0 \\
0  & 0  & 1  & 1  & 0 \\
0  & 1  & 0  & 0  & 1 \\
0  & 1  & 0  & 1  & 1 \\
0  & 1  & 1  & 0  & 0 \\
0  & 1  & 1  & 1  & 0 \\
1  & 0  & 0  & 0  & 1 \\
1  & 0  & 0  & 1  & 1 \\
1  & 0  & 1  & 0  & 1 \\
1  & 0  & 1  & 1  & 0 \\
1  & 1  & 0  & 0  & 1 \\
1  & 1  & 0  & 1  & 1 \\
1  & 1  & 1  & 0  & 1 \\
1  & 1  & 1  & 1  & 1 \\
\end{array}
\]

\[
\overline{B_1} \overline{B_0} + A_1 A_0 + A_1 \overline{B_0} + A_0 B_1 + A_1 B_1
\]

(c) Draw the corresponding circuit.
\( \overline{B_1 B_0} + A_1 A_0 + A_1 \overline{B_0} + A_0 \overline{B_1} + A_1 \overline{B_1} \)
9. (15 pts.)

(a) Minimize the Karnaugh map for the complement of the $A \geq B$ function from problem 8.

(b) Use this to draw a circuit for $A \geq B$ (i.e., not the complement).
\[ \overline{A_1B_1} + \overline{A_1A_0B_0} + \overline{A_0B_1B_0} \]