LAME
(Linear Algebra Made Easy)

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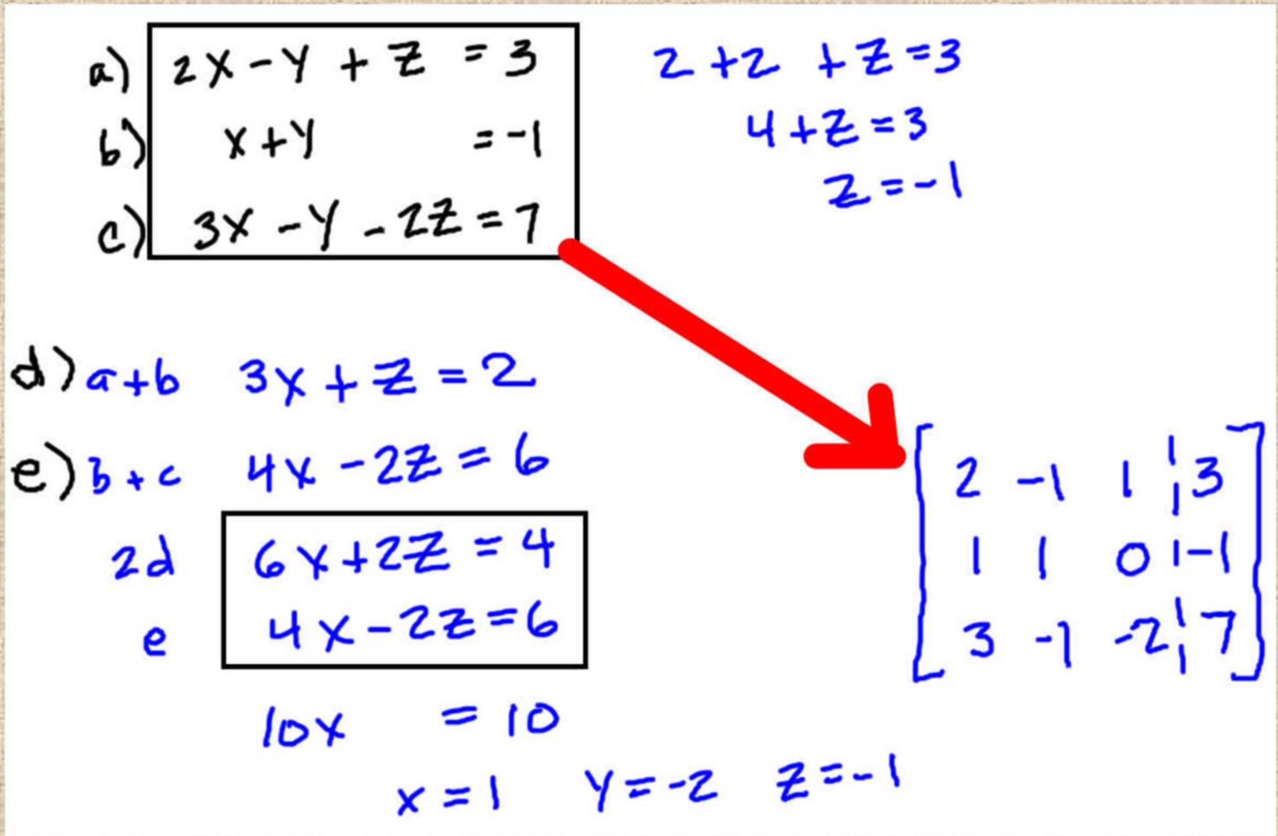
MOTIVATION

Performing calculations and operations by hand is tedious, time consuming and error prone.



However, computers love this stuff!

Why LAME isn't (lame)

- LAME allows for basic control flow operations (if, while)
- Performs matrix/vector operations (resizing, transpose, multiplication, exponentiation) so you don't have to
- Imperative language with C and MATLAB like syntax

The Basics

Basic Types

- Boolean
- String
- Scalar
  - 64 bit signed double precision float
- Matrix
  - Dynamically sized 2-D array
  - Elements can be scalars or other matrices

Basic Operations

- Addition
- Negation
- Multiplication
- Division
- Exponentiation

Basic Operators

- `print`
- `if`
- `while`
- `dim`
- Relational Operators
- Logical Operators
- Transpose

Example time!

Let us implement an algorithm using LAME to solve a system of simultaneous linear equations. The equations are:

$$3x_1 + x_2 = 3$$

$$9x_1 + 4x_2 = 6$$

We use the following algorithm to solve this problem.

$$A = \begin{bmatrix} 3 & 1 \\ 9 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = A^{-1}B$$

$$A^{-1} = \frac{1}{|A|}\mathrm{Adj}(A) = \frac{1}{A_{0,0}A_{1,1} - A_{0,1}A_{1,0}} \begin{bmatrix} A_{1,1} & -A_{0,1} \\ -A_{1,0} & A_{0,0} \end{bmatrix}$$

Doing This (fingers crossed)

```
matrix A = { 3, 1; 9, 4 };
matrix B = { 3; 6 };
matrix X;

print "\nSolving system of simultaneous linear
equations:\n";
print A[0,0] + " x1 + " + A[0,1] + " x2 = " + B[0] +
"\n";
print A[1,0] + " x1 + " + A[1,1] + " x2 = " + B[1] +
"\n";
print "\nA = \n" + A + "\n";
print "\nB = \n" + B + "\n";

scalar det_of_A = A[0,0]*A[1,1] - A[0,1]*A[1,0];
print "\nDeterminant(A) = " + det_of_A + "\n";
```

Outputs This

Solving system of simultaneous linear
equations:

3 x1 + 1 x2 = 3

9 x1 + 4 x2 = 6

A =

3 1

9 4

B =

3

6

Determinant(A) = 3

Doing This

```
if(det_of_A != 0) {
…  //see next slide
} else {
print "A is singular and its inverse doesn't
exist.\n";
}
```

Doing This (continued)

```
matrix inv_of_A;

inv_of_A [0,0] = A[1,1];
inv_of_A [0,1] = -1*A[0,1];
inv_of_A [1,0] = -1*A[1,0];
inv_of_A [1,1] = A[0,0];
inv_of_A = inv_of_A / det_of_A;

X = inv_of_A * B;

print "\nInverse(A) = \n" + inv_of_A + "\n";
print "X = Inverse(A) * B = \n" + X + "\n";
print "Solution:\n";
print "x1 = " + X[0] + "\n";
print "x2 = " + X[1] + "\n";
```

Outputs This (Yay!)

```
Inverse(A) =
1.33333 -0.333333
-3 1

X = Inverse(A) * B =
2
-3

Solution:
x1 = 2
x2 = -3
4
```

The Whole Shebang

```
matrix A = { 3, 1; 9, 4 };
matrix B = { 3; 6 };
matrix X;
print "\nSolving system of simultaneous linear equations:\n";
print A[0,0] + " x1 + " + A[0,1] + " x2 = " + B[0] + "\n";
print A[1,0] + " x1 + " + A[1,1] + " x2 = " + B[1] + "\n";
print "\nA = \n" + A + "\n";
print "\nB = \n" + B + "\n";
scalar det_of_A = A[0,0]*A[1,1] - A[0,1]*A[1,0];
print "\nDeterminant(A) = " + det_of_A + "\n";
if(det_of_A != 0) {
matrix inv_of_A;
inv_of_A [0,0] = A[1,1];
inv_of_A [0,1] = -1*A[0,1];
inv_of_A [1,0] = -1*A[1,0];
inv_of_A [1,1] = A[0,0];
inv_of_A = inv_of_A / det_of_A;
X = inv_of_A * B;
print "\nInverse(A) = \n" + inv_of_A + "\n";
print "X = Inverse(A) * B = \n" + X + "\n";
print "Solution:\n";
print "x1 = " + X[0] + "\n";
print "x2 = " + X[1] + "\n";
} else {
print "A is singular and its inverse doesn't exist.\n";
}
```

All Together Now

Solving system of simultaneous linear equations:

3 x1 + 1 x2 = 3

9 x1 + 4 x2 = 6

A =

3 1

9 4

B =

3

6

Determinant(A) = 3

Inverse(A) =

1.33333 -0.333333

-3 1

X = Inverse(A) * B =

2

-3

Solution:

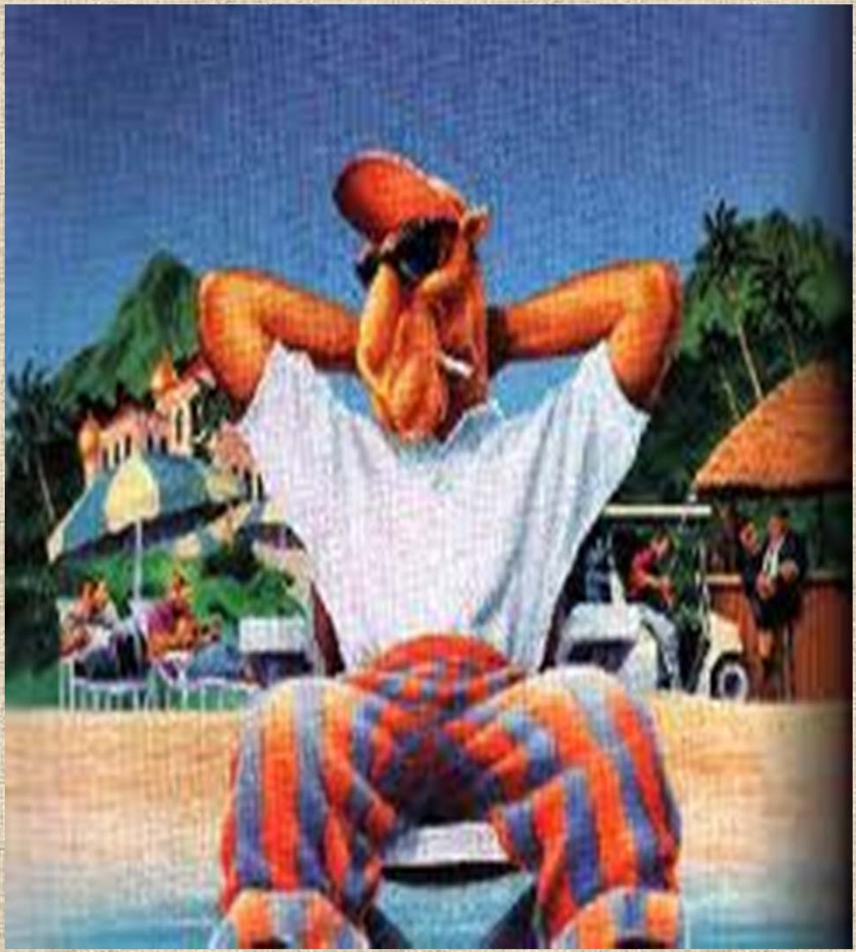
x1 = 2

x2 = -3

4

Implementation

- O'Caml takes care of the hard work
- iLAME 3-op code is converted into C ++
- C++ is then used to perform matrix operations and output the result



Lessons Learned

- Get started early!
- Make sure the problem is well defined – math really is your friend
- Set and keep to deadlines
- Be open to revision
- Formal interfaces - document/code style, module interaction
- AWK is better than O'Caml (kidding)

Questions?