Intro to Logic

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Logical reasoning

- How do we know what we know ?

- Knowledge representation in AI
  - How to represent knowledge in a specific domain
  - Reason and make decisions about this knowledge

- Logic
  - well studied area which is a formal language to describe facts (syntax and symantics) and the tools to perform reasoning about those facts.

Why logic ?

- Example of some facts:
  - Someone throws a rock through your window
  - You get more hate mail than usual
  - Your telephone is always ringing
- Use logic to draw a conclusion
- Use logic to decide on how to move forward

- Top down approach
  - Deductive reasoning: take general rules/axioms and apply to logical conclusions
- Bottom up approach
  - Inductive reasoning: moving from specifics to general

Deduction Example

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Question:

Is the Professor unhappy ??

Proposition Logic

- Variables/Symbols:
  - P, Q, R
- Connectors
  - ~ negation
  - $\wedge$ conjunction
  - $\vee$ disjunction
  - $\Rightarrow$ implication
  - $\Leftrightarrow$ biconditional
- Sentences
  - wffs

Logic

- Syntax:
  - Legal symbols we can use
- Sentences
  - WFFs
    - Well formed formulas
  - True and False are sentences
  - Legal symbols are sentences
  - Connectors + Symbols are sentences
  - Meaning of sentence will be T/F

- Interpretation / Evaluation:
  - Specific set of t/f assignments to the set of atoms
- Model :
  - Specific set of assignments to make the sentence true
- Valid:
  - A valid wff is true under all interpretations
    - It is raining
- Inconsistent / unsatisfiable
  - False under all interpretations
    - Raining AND ~Raining

Deduction Theory

- G is a logical consequence of statements $F_1,..,F_n$ if a model of the statements is also a model of G
  - i.e.

    $A = (F_1 \wedge F_2 \wedge F_3 \wedge \ldots F_n) \supset G$

- How to prove this ?

LOGICAL CONSEQUENCE

- G is a logical consequence of wwf's F1..Fn
  iff
  for any model of (F1 $\wedge$ F2 $\wedge$ .. Fn) $\supset$ G is valid

- Plain english: if all wff are true, the conclusion must be consistent.

- Deductive Theorem:
  - A follows from a logical consequence the premises F1,..,Fn iff $(F1 \wedge .. \wedge Fn) \supset S$
- Interpretation
  - Assignment of T/F to each proposition
- Satisfiability
  - Finding the model where conclusion is true

EXAMPLE 2

- P = Hot
- Q = Humid
- R = Raining
- Given Facts:
- (P ^ Q) => R)
  - if its hot and humid its raining
- (Q => P)
  - if its humid then its hot
- Q
  - It is humid
- Question: IS IT RAINING ?

REFUTATION

- Sometimes can also prove the opposite
- Proof by contradiction
- Attempt to show ~S is inconsistent
- $\sim S = F_1 \wedge F_2 \wedge\ .. \wedge F_n \wedge \sim G$

Million dollar question

- Given F1, F2, .., Fn can we conclude G ??

- Mechanical way:
  - $(F_1 \wedge F_2 \wedge \; .. \wedge F_n) \supset G$
    - Establish it is valid: no matter what it evaluates to TRUE
      G is a logical consequence of $F_1 \wedge F_2 \wedge \; .. \wedge F_n$

Example 3

- P = "it is midterm season"
- Q = "Students feel treated unfairly"
- S = "Hate Prof"
- U = "Prof unhappy"

- Facts:
  - $P \supset Q$
  - $Q \supset S$
  - $S \supset U$
  - P
  - ??U??

Example 3

- $((P \supset Q) \wedge (Q \supset S) \wedge (S \supset U) \wedge (P)) \Rightarrow (U)$

- Most mechanical way:
- Truth Tables!

P	Q	S	U
T	T	T	T
T	T	T	F

- 2^d decidable
- At worst would need to step through 2^n if enumerate every state

EXAMPLE 2

P	Q	R	(P ^ Q) => R	Q => P	Q	KB	R	KB => R
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	F	F	T
T	F	T	T	T	F	F	T	T
T	F	F	T	T	F	F	F	T
F	T	T	T	F	T	F	T	T
F	T	F	T	F	T	F	F	T
F	F	T	T	T	F	F	T	T
F	F	F	T	T	F	F	F	T

Proving

- Is propositional logic decidable ?

A better method

- Instead of listing the truth table
- Can use inference to deduce the truth
  - Called natural deduction

Natural Deduction tools

- Modus Ponens (i.e. forward chaining)
  - If A, then B
    A is true
    Therefore B
- Unit resolution
  - A or B is true
  - ~B given, therefore A
- And Elimination
  - (A and B) are true
  - Therefore A is true
- Implication elimination
  - If A then B equivalent $\sim A \vee B$

USEFUL TOOLS

- Double negation
  - ~(~A) equivalent A
- De Morgan's Rule
  - ~(A ∧ B) equivalent to (~A ∨ ~B)
  - ~(A ∨ B) ≡ (~A ∧ ~B)
- Distribution
  - F ∨ (G ∧ H) ≡ (F ∨ G) ∧ (F ∨ H)

Proving

- Proof is a sequence of wffs each given or derived

1. Q (premise)
2. Q => P (premise)
3. P (modus p)
4. (P ^ Q) => R (premise)
5. P ^ Q (and introduction)
6. R (conclusion)

Normal Forms

- To expand the known facts, we can move to another logically equivalent form

- Biconditional:
- $A \Leftrightarrow B$
  - $(A \Rightarrow B) \wedge (B \Rightarrow A)$
  - $(\sim A \vee B) \wedge (A \vee \sim B)$

SATISFIABILITY

- Many problems can be framed as a list of constraints

- Some students want the final early
- Some students can't take it before 11am
- Some can't stay more than X hours except Tuesday

- Usually written as CNF
  - $(A \vee B) \wedge (\sim B \vee C) \wedge ..$

- $(A \vee B) \wedge (\sim B \vee C) \wedge$ ..
  - $(A \vee B)$ is a clause
  - A, B are literals
  - Every sentence in Propositional Logic can be written as CNF
- Converting:
  - Get rid of implications and conditionals
  - Fold in negations using De Morgans Law
  - Or's insider, ands outside
  - Will end up growing the sentence
  - Used for input for resolution

Example

- $(A \vee B) \text{ -> } (C \text{ -> } D)$

- Can you convert this to CNF ??

Example

- $(A \vee B)$ -> (C -> D)
- C -> D
  - $\sim C \vee D$
- $\sim(A \vee B) \vee (\sim C \vee D)$
- $(\sim A \wedge \sim B) \vee (\sim C \vee D)$
- $(\sim A \vee \sim C \vee D) \wedge (\sim B \vee \sim C \vee D)$

Resolution

- $A \lor B$
- $\sim B \lor C$

- Conclude:
- $A \lor C$

- Algorithm:
  - Convert everything to CNF
  - Negate the desired state
  - Apply resolution until get False or can't go on

Example

- $A \vee B$
- A => C
- B => C

- Is C true ?

1. $A \vee B$ (known)
2. $\sim A \vee C$ (known)
3. $\sim B \vee C$ (known)
4. $\sim C$ (negate target)

5. $B \vee C$ (combine first 2)
6. $\sim A$ (2,4)
7. $\sim B$ (3,4)
8. C (5,7)
9. (4,8)

Horn Clause

- Disjunction of literals with exactly one positive
- $\sim F_1 \vee \sim F_2 \vee \sim F_3 \ldots. \vee \sim F_n \vee A$

- P => Q
- $L \wedge M$ => P
- $B \wedge L$ => M
- $A \wedge P$ => L
- $A \wedge B$ => L
- A
- B

Limitations

- Proposition logic limits
- FOPL