LOGICAL REASONING

- How do we know what we know?

- Knowledge representation in AI
  - How to represent knowledge in a specific domain
  - Reason and make decisions about this knowledge

- Logic
  - well studied area which is a formal language to describe facts (syntax and symantics) and the tools to perform reasoning about those facts.
WHY LOGIC?

- Example of some facts:
  - Someone throws a rock through your window
  - You get more hate mail than usual
  - Your telephone is always ringing

- Use logic to draw a conclusion
- Use logic to decide on how to move forward
Top down approach
- Deductive reasoning: take general rules/axioms and apply to logical conclusions

Bottom up approach
- Inductive reasoning: moving from specifics to general
DEDUCTION EXAMPLE

1. If it is midterm time, Students feel they are being treated unfairly.
DEDUCTION EXAMPLE

1. If it is midterm time, Students feel they are being treated unfairly.

2. If Students feel they are being treated unfairly, they hate their Professor.
**Deduction Example**

1. If it is midterm time, Students feel they are being treated unfairly.

2. If Students feel they are being treated unfairly, they hate their Professor.

3. If Students hate their Professor, the Professor is unhappy.
DEDUCTION EXAMPLE

1. If it is midterm time, Students feel they are being treated unfairly.

2. If Students feel they are being treated unfairly, they hate their Professor.

3. If Students hate their Professor, the Professor is unhappy.

Question:
Is the Professor unhappy??
PROPOSITION LOGIC

- Variables/Symbols:
  - P, Q, R

- Connectors
  - ~: negation
  - \&: conjunction
  - ∨: disjunction
  - ⇒: implication
  - ⇔: biconditional

- Sentences
  - wffs
Logic

- Syntax:
  - Legal symbols we can use

- Sentences
  - WFFs
    - Well formed formulas
  - True and False are sentences
  - Legal symbols are sentences
  - Connectors + Symbols are sentences

- Meaning of sentence will be T/F
- **Interpretation / Evaluation:**
  - Specific set of t/f assignments to the set of atoms

- **Model:**
  - Specific set of assignments to make the sentence true

- **Valid:**
  - A valid wff is true under all interpretations
    - It is raining

- **Inconsistent / unsatisfiable**
  - False under all interpretations
    - Raining AND ~Raining
Deduction Theory

G is a logical consequence of statements \(F_1, \ldots, F_n\) if a model of the statements is also a model of G

i.e.

\[ A = (F_1 \land F_2 \land F_3 \land \ldots F_n) \supset G \]

How to prove this?
LOGICAL CONSEQUENCE

- G is a logical consequence of wff’s F1..Fn iff for any model of (F1 \land F2 \land .. Fn) \implies G is valid

- Plain english: if all wff are true, the conclusion must be consistent.
Deductive Theorem:
- A follows from a logical consequence the premises $F_1, \ldots, F_n$ iff $(F_1 \land \ldots \land F_n) \supset S$

Interpretation
- Assignment of T/F to each proposition

Satisfiability
- Finding the model where conclusion is true
Example 2

- P = Hot
- Q = Humid
- R = Raining
- Given Facts:
  - (P ^ Q) => R
    - if its hot and humid its raining
  - (Q => P)
    - if its humid then its hot
- Q
  - It is humid
- Question: IS IT RAINING?
REFUTATION

- Sometimes can also prove the opposite
- Proof by contradiction
- Attempt to show ~S is inconsistent

\[ \sim S = F_1 \land F_2 \land \ldots \land F_n \land \sim G \]
MILLION DOLLAR QUESTION

- Given $F_1, F_2, \ldots, F_n$ can we conclude $G$??

- Mechanical way:
  - $(F_1 \land F_2 \land \ldots \land F_n) \Rightarrow G$
    - Establish it is valid: no matter what it evaluates to TRUE
      $G$ is a logical consequence of $F_1 \land F_2 \land \ldots \land F_n$
**Example 3**

- $P = \text{“it is midterm season”}$
- $Q = \text{“Students feel treated unfairly”}$
- $S = \text{“Hate Prof”}$
- $U = \text{“Prof unhappy”}$

**Facts:**

- $P \supset Q$
- $Q \supset S$
- $S \supset U$
- $P$
- $??U??$
**Example 3**

- \(( (P \supset Q) \land (Q \supset S) \land (S \supset U) \land (P)) \Rightarrow (U)\)

- Most mechanical way:
- Truth Tables!

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- 2\textsuperscript{d} decidable
- At worst would need to step through \(2^n\) if enumerate every state
Example 2

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<th>Q</th>
<th>R</th>
<th>(P ^ Q) =&gt; R</th>
<th>Q =&gt; P</th>
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Is propositional logic decidable?
A BETTER METHOD

- Instead of listing the truth table
- Can use inference to deduce the truth
  - Called natural deduction
NATURAL DEDUCTION TOOLS

- Modus Ponens (i.e. forward chaining)
  - If A, then B
    - A is true
    - Therefore B

- Unit resolution
  - A or B is true
  - ~B given, therefore A

- And Elimination
  - (A and B) are true
  - Therefore A is true

- Implication elimination
  - If A then B equivalent ~A ∨ B
USEFUL TOOLS

- Double negation
  - $\neg(\neg A)$ equivalent $A$

- De Morgan’s Rule
  - $\neg(A \land B)$ equivalent to $(\neg A \lor \neg B)$
  - $\neg(A \lor B) \equiv (\neg A \land \neg B)$

- Distribution
  - $F \lor (G \land H) \equiv (F \lor G) \land (F \lor H)$
PROVING

- Proof is a sequence of wffs each given or derived

1. Q (premise)
2. Q => P (premise)
3. P (modus p)
4. (P ^ Q) => R (premise)
5. P ^ Q (and introduction)
6. R (conclusion)
NORMAL FORMS

- To expand the known facts, we can move to another logically equivalent form

- Biconditional:
  - $A \iff B$
    - $(A \rightarrow B) \land (B \rightarrow A)$
    - $(\neg A \lor B) \land (A \lor \neg B)$
SATISFIABILITY

- Many problems can be framed as a list of constraints

- Some students want the final early
- Some students can’t take it before 11am
- Some can’t stay more than $X$ hours except Tuesday

- Usually written as CNF
  - $(A \lor B) \land (\lnot B \lor C) \land ..$
\[(A \lor B) \land (\neg B \lor C) \land \ldots\]

- \((A \lor B)\) is a clause
- \(A, B\) are literals
- Every sentence in Propositional Logic can be written as CNF

**Converting:**
- Get rid of implications and conditionals
- Fold in negations using De Morgan’s Law
- Or’s insider, ands outside
- Will end up growing the sentence
- Used for input for resolution
**Example**

- \((A \lor B) \rightarrow (C \rightarrow D)\)

- Can you convert this to CNF ??
EXAMPLE

- \((A \lor B) \rightarrow (C \rightarrow D)\)
- \(C \rightarrow D\)
  - \(\sim C \lor D\)

- \(\sim(A \lor B) \lor (\sim C \lor D)\)
- \((\sim A \land \sim B) \lor (\sim C \lor D)\)
- \((\sim A \lor \sim C \lor D) \land (\sim B \lor \sim C \lor D)\)
RESOLUTION

- $A \lor B$
- $\neg B \lor C$

Conclude:
- $A \lor C$

Algorithm:
- Convert everything to CNF
- Negate the desired state
- Apply resolution until get False or can’t go on
**Example**

- \( A \lor B \)
- \( A \Rightarrow C \)
- \( B \Rightarrow C \)

- Is \( C \) true?
1. $A \lor B$ (known)
2. $\neg A \lor C$ (known)
3. $\neg B \lor C$ (known)
4. $\neg C$ (negate target)
5. $B \lor C$ (combine first 2)
6. $\neg A$ (2,4)
7. $\neg B$ (3,4)
8. $C$ (5,7)
9. (4,8)
Horn Clause

- Disjunction of literals with exactly one positive
  \( \neg F_1 \lor \neg F_2 \lor \neg F_3 \ldots \lor \neg F_n \lor A \)

- \( P \rightarrow Q \)
- \( L \land M \rightarrow P \)
- \( B \land L \rightarrow M \)
- \( A \land P \rightarrow L \)
- \( A \land B \rightarrow L \)
- \( A \)
- \( B \)
LIMITATIONS

- Proposition logic limits

- FOPL