Theorem 1. *Let $Y_1, \ldots, Y_m$ be m independent random variables that take on values in $[0,1]$, where $\mathbb{E}[Y_i] = p_i$, and $\sum_{i=1}^m p_i = P$. For any $\gamma \in (0,1]$ we have*

$$\text{(additive bound)} \quad \Pr\left[\sum_{i=1}^m Y_i > P + \gamma m\right],\ \Pr\left[\sum_{i=1}^m Y_i < P - \gamma m\right] \le \exp(-2\gamma^2 m) \tag{1}$$

$$\text{(multiplicative bound)} \quad \Pr\left[\sum_{i=1}^m Y_i > (1+\gamma)P\right] < \exp(-\gamma^2 P/3) \tag{2}$$

and

$$\text{(multiplicative bound)} \quad \Pr\left[\sum_{i=1}^m Y_i < (1-\gamma)P\right] < \exp(-\gamma^2 P/2). \tag{3}$$

The bound in Equation (2) is derived from the following more general bound, which holds from any $\gamma > 0$:

$$\Pr\left[\sum_{i=1}^m Y_i > (1+\gamma)P\right] \le \left(\frac{e^\gamma}{(1+\gamma)^{1+\gamma}}\right)^P, \tag{4}$$

and which also implies that for any $B > 2eP$,

$$\Pr\left[\sum_{i=1}^m Y_i > B\right] \le 2^{-B}. \tag{5}$$

Remark. Additive bound is better when $p \stackrel{\text{def}}{=} \frac{P}{m} = \Omega(1)$:

p	Multiplicative *(Chernoff)*	Additive *(Hoeffding)*
$< \frac{1}{6}$	$\surd$	
$\simeq \frac{1}{6}$	$\surd$	$\surd$
$> \frac{1}{6}$		$\surd$

The following extension of the multiplicative bound is useful when we only have upper and/or lower bounds on P

Corollary 2. In the setting of Theorem 1 suppose that $P_L \le P \le P_H$. Then for any $\gamma \in (0,1]$, we have

$$\Pr\left[\sum_{i=1}^m Y_i > (1+\gamma)P_H\right] < \exp(-\gamma^2 P_H/3) \tag{6}$$

$$\Pr\left[\sum_{i=1}^m Y_i < (1-\gamma)P_L\right] < \exp(-\gamma^2 P_L/2) \tag{7}$$

We will also use the following corollary of Theorem 1:

Corollary 3. Let $0 \le w_1, \ldots, w_m \in \mathbb{R}$ be such that $w_i \le \kappa$ for all $i \in [m]$ where $\kappa \in (0,1]$. Let $X_1, \ldots, X_m$ be i.i.d. Bernoulli random variables with $\Pr[X_i = 1] = 1/2$ for all i, and let $X = \sum_{i=1}^m w_i X_i$ and $W = \sum_{i=1}^m w_i$. For any $\gamma \in (0,1]$,

$$\Pr\left[X > (1+\gamma)\frac{W}{2}\right] < \exp\left(-\gamma^2 \frac{W}{6\kappa}\right) \quad \text{and} \quad \Pr\left[X < (1-\gamma)\frac{W}{2}\right] < \exp\left(-\gamma^2 \frac{W}{4\kappa}\right),$$

and for any $B > e \cdot W$,

$$\Pr[X > B] < 2^{-B/\kappa}.$$