Problem 1. Give a naive recursive construction of monotone formulas for the Majority function. Show that this results in formulas of size $n^{O(\log n)}$.

Problem 2. This problem is to fill in some of the omitted details in the probabilistic proof of existence of short monotone formulas for the Majority function.

(a). For the quantity $p_t = A(t)(p)$ defined in lecture, show that for $p_0 \geq \alpha + 1/(4n)$ and $t = \log_{1.525} n$, we have $p_t \geq \alpha + 10^{-6}$.

(b). For the quantity $p_t = A(t)(p)$ defined in lecture, show that if $p_t = 1 - c, c < 1/8$, then $p_{t+\log n} \geq 1 - 2^{-(n+1)}$.

Problem 3. Show that Khrapchenko’s lower bound method can never give an $\omega(n^2)$ lower bound on the formula size of an $n$-variable formula.

Problem 4. Show that there is a function $f$ with $\text{DNF}(f) = \text{poly}(n)$ but $\text{DTsize}(f) = 2^{n^{\Omega(1)}}$. 