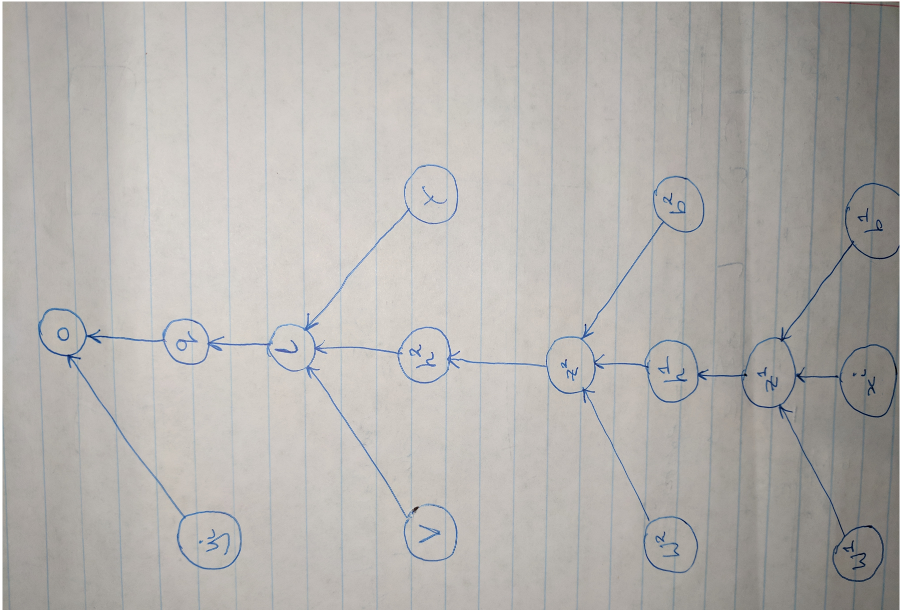
Question 1a



Question 1a

The leaf variables are W^1, b^1, W^2, b^2, V, γ, x^i, y^i

Each leaf in the graph has a single directed path leading to the output o. To calculate the partial derivative, take the product of Jacobians along the path

$$\left.\frac{\partial o}{\partial V}\right|^{\bar{f}^o} = \left.\frac{\partial o}{\partial q}\right|^{f^o} \times \left.\frac{\partial q}{\partial l}\right|^{f^q} \times \left.\frac{\partial l}{\partial V}\right|^{f^l}$$

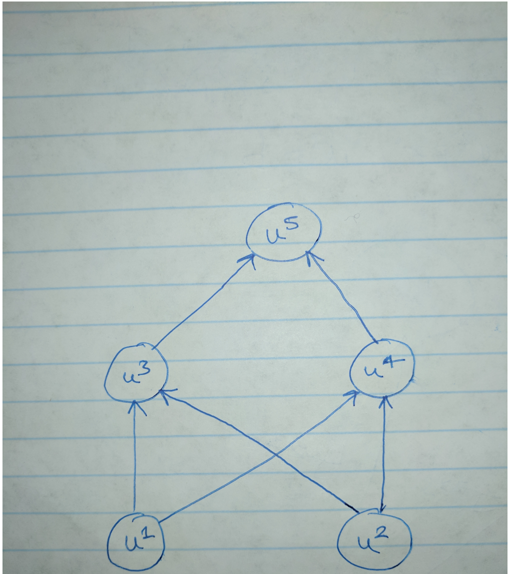
Question 1b

$$\left.\frac{\partial o}{\partial W^1}\right|^{\bar{f}^o} = \left.\frac{\partial o}{\partial q}\right|^{f^o} \times \left.\frac{\partial q}{\partial l}\right|^{f^q} \times \left.\frac{\partial l}{\partial h^2}\right|^{f^l} \times \left.\frac{\partial h^2}{\partial z^2}\right|^{f^{h^2}} \times \left.\frac{\partial z^2}{\partial h^1}\right|^{f^{z^2}} \times \left.\frac{\partial h^1}{\partial z^1}\right|^{f^{h^1}} \times \left.\frac{\partial z^1}{\partial W^1}\right|^{f^{z^1}}$$

Question 1b

$$\frac{\partial o}{\partial W^2}\bigg|^{\bar{f}^o} = \frac{\partial o}{\partial q}\bigg|^{f^o} \times \frac{\partial q}{\partial l}\bigg|^{f^q} \times \frac{\partial l}{\partial h^2}\bigg|^{f^l} \times \frac{\partial h^2}{\partial z^2}\bigg|^{f^{h^2}} \times \frac{\partial z^2}{\partial W^2}\bigg|^{f^{z^2}}$$

Question 2a



Question 2a

We get values

$$u^3 = u^1 \times u^2 = 12$$

$$u^4 = u^1 + u^2 = 7$$

$$u^5 = 2 \times u^3 \times u^4 = 168$$

Question 2b

$$u^n = 2 \times u^3 \times u^4 = 2 \times (u^1 \times u^2) \times (u^1 + u^2) = 2 \times (u^1)^2 \times u^2 + 2 \times u^1 \times (u^2)^2$$

hence

$$\bar{f}^n(u^1, u^2) = 2 \times (u^1)^2 \times u^2 + 2 \times u^1 \times (u^2)^2$$

hence

$$\frac{\partial u^n}{\partial u^1} = 4 \times u^1 \times u^2 + 2 \times (u^2)^2 = 80$$

$$\frac{\partial u^n}{\partial u^2} = 2 \times (u^1)^2 + 4 \times u^1 \times u^2 = 66$$

$$J^{1\to 3}(A^3) = u^2 = 4$$

$$J^{2\to 3}(A^3) = \frac{\partial}{\partial u^2}\left(u^1 \times u^2\right) = u^1 = 3$$

$$J^{1\to 4}(A^4) = \frac{\partial}{\partial u^1}\left(u^1 + u^2\right) = 1$$

$$J^{2\to 4}(A^4) = \frac{\partial}{\partial u^2}\left(u^1 + u^2\right) = 1$$

$$J^{3\to 5}(A^5) = \frac{\partial}{\partial u^3}\left(2 \times u^3 \times u^4\right) = 2 \times u^4 = 14$$

$$J^{4\to 5}(A^5) = \frac{\partial}{\partial u^4}\left(2 \times u^3 \times u^4\right) = 2 \times u^3 = 24$$

$$p^5 = 1$$

$$p^4 = p^5 \times J^{4\to5}(A^5) = 1 \times 24 = 24$$

$$p^3 = p^5 \times J^{3\to5}(A^5) = 1 \times 14 = 14$$

$$p^2 = p^3 \times J^{2\to3}(A^3) + p^4 \times J^{2\to4}(A^4) = 14 \times 3 + 24 \times 1 = 66$$

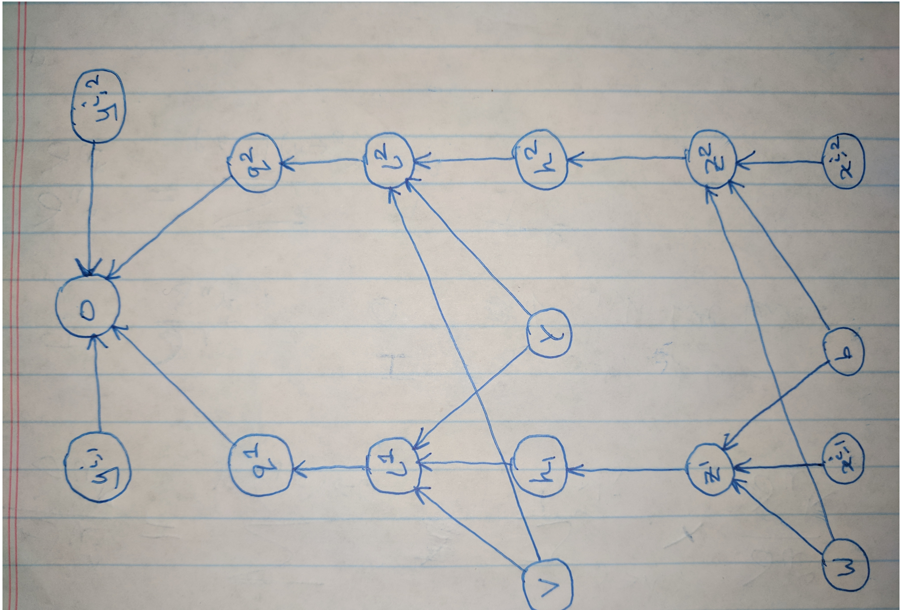
$$p^1 = p^3 \times J^{1\to3}(A^3) + p^4 \times J^{1\to4}(A^4) = 14 \times 4 + 24 \times 1 = 80$$

Question 3a

Equations:

$$
\begin{aligned}
z^1 \in \mathbb{R}^m &= Wx^{i,1} + b \\
h^1 \in \mathbb{R}^m &= g(z^1) \\
l^1 \in \mathbb{R}^K &= Vh^1 + \gamma \\
q^1 \in \mathbb{R}^K &= \mathsf{LS}(l^1) \\
z^2 \in \mathbb{R}^m &= Wx^{i,2} + b \\
h^2 \in \mathbb{R}^m &= g(z^2) \\
l^2 \in \mathbb{R}^K &= Vh^2 + \gamma \\
q^2 \in \mathbb{R}^K &= \mathsf{LS}(l^2) \\
o \in \mathbb{R} &= -q^1_{y^{i,1}} - q^2_{y^{i,2}}
\end{aligned}
$$

Question 3a



Question 3b

There are two directed paths from V to o in the computational graph. To calculate the partial derivative, take a sum of products of Jacobians on these paths

$$
\begin{aligned}
\left.\frac{\partial o}{\partial V}\right|^{\bar{f}^o} \quad = \quad & \left.\frac{\partial o}{\partial q^1}\right|^{f^o} \times \left.\frac{\partial q^1}{\partial l^1}\right|^{f^{q^1}} \times \left.\frac{\partial l^1}{\partial V}\right|^{f^{l^1}} \\
& + \left.\frac{\partial o}{\partial q^2}\right|^{f^o} \times \left.\frac{\partial q^2}{\partial l^2}\right|^{f^{q^2}} \times \left.\frac{\partial l^2}{\partial V}\right|^{f^{l^2}}
\end{aligned}
$$

Question 3c

$$f^z(W, x^{i,1}, x^{i,2}, b) = [Wx^{i,1} + b; Wx^{i,2} + b]$$
$$f^h([z^1; z^2]) = [g(z^1); g(z^2)]$$
$$f^l([h^1; h^2]) = [Vh^1 + \gamma; Vh^2 + \gamma]$$
$$f^q([l^1; l^2]) = [\mathsf{LS}(l^1); \mathsf{LS}(l^2)]$$
$$f^o([q^1; q^2], y^{i,1}, y^{i,2}) = -q^1_{y^{i,1}} - q^2_{y^{i,2}}$$