Question 1a

Set the following translation parameters equal to 1 (all other translation parameters are 0): $t(\text{aate}|\text{ate})$, $t(\text{athe}|\text{the})$, $t(\text{adog}|\text{dog})$, $t(\text{acat}|\text{cat})$, $t(\text{abanana}|\text{banana})$

Set the following alignment parameters equal to 1 (all others are zero):
$q(3|1,3,3)$, $q(2|2,3,3)$, $q(1|3,3,3)$
$q(5|1,5,5)$, $q(4|2,5,5)$, $q(3|3,5,5)$, $q(2|4,5,5)$, $q(1|5,5,5)$

Question 1b

Set the following translation parameters equal to 1 (all other translation parameters are 0): $t($aate$|$ate$)$, $t($athe$|$the$)$, $t($adog$|$dog$)$, $t($acat$|$cat$)$, $t($abanana$|$banana$)$

Set the following alignment parameters equal to 1 (all others are zero):
$q(1|1,3,3)$, $q(2|2,3,3)$, $q(3|3,3,3)$
$q(1|1,5,5)$, $q(2|2,5,5)$, $q(4|3,5,5)$, $q(5|4,5,5)$, $q(3|5,5,5)$

Question 2a

If we set $f_1 = le$, then

$$\sum_{a_1=1}^{2} t(f_1|e_{a_1})q(a_1|1,l,m) = 0.9 \times 0.7 + 0.2 \times 0.3 = 0.69$$

If we set $f_1 = chien$, then

$$\sum_{a_1=1}^{2} t(f_1|e_{a_1})q(a_1|1,l,m) = 0.1 \times 0.7 + 0.8 \times 0.3 = 0.31$$

If we set $f_2 = le$, then

$$\sum_{a_2=1}^{2} t(f_2|e_{a_2})q(a_2|2,l,m) = 0.9 \times 0.4 + 0.2 \times 0.6 = 0.48$$

If we set $f_2 = chien$, then

$$\sum_{a_2=1}^{2} t(f_2|e_{a_2})q(a_2|2,l,m) = 0.1 \times 0.4 + 0.8 \times 0.6 = 0.52$$

Hence we have the probabilities 0.69×0.48 for *le le*, 0.69×0.52 for *le chien*, 0.31×0.48 for *chien le*, 0.31×0.52 for *chien chien*.

Question 2b

$$
\begin{aligned}
p(A_1 = 1|e, f, m = 2) &= \frac{t(f_1|e_1)q(1|1,2,2)}{\sum_{a=1}^{2} t(f_1|e_a)q(a|1,2,2)} \\
&= \frac{0.9 \times 0.7}{0.9 \times 0.7 + 0.2 \times 0.3} = 0.913
\end{aligned}
$$

Question 2c

If we define

$$g(a_1 \dots a_{m-1}) = \prod_{j=1}^{m-1} t(f_j|e_{a_j})q(a_j|j,l,m)$$

$$h(a_m) = t(f_m|e_{a_m})q(a_m|m,l,m)$$

then

$$\begin{aligned}
& \sum_{a_1=0}^{l} \sum_{a_2=0}^{l} \dots \sum_{a_{m-1}=0}^{l} \sum_{a_m=0}^{l} \prod_{j=1}^{m} t(f_j|e_{a_j})q(a_j|j,l,m) \\
= & \sum_{a_1=0}^{l} \sum_{a_2=0}^{l} \dots \sum_{a_{m-1}=0}^{l} \sum_{a_m=0}^{l} g(a_1 \dots a_{m-1}) \times h(a_m) \\
= & \left(\sum_{a_1=0}^{l} \sum_{a_2=0}^{l} \dots \sum_{a_{m-1}=0}^{l} g(a_1 \dots a_{m-1}) \right) \times \left(\sum_{a_m=0}^{l} h(a_m) \right)
\end{aligned}$$

Repeating this process gives the required identity.

Question 2c (continued)

This identity is useful because if we want to calculate

$$p(f_1 \dots f_m | e_1 \dots e_l, m)$$

for some sentence $f_1 \dots f_m$, then under IBM Model 2,

$$\begin{aligned} & p(f_1 \dots f_m | e_1 \dots e_l, m) \\ = & \sum_{a_1=0}^{l} \sum_{a_2=0}^{l} \dots \sum_{a_{m-1}=0}^{l} \sum_{a_m=0}^{l} \prod_{j=1}^{m} t(f_j | e_{a_j}) q(a_j | j, l, m) \end{aligned}$$

In this form, this requires a summation over $(l+1)^m$ possible values for the alignment variables $a_1, a_2, \dots a_m$, taking $O((l+1)^m)$ time. The new expression takes $O((l+1) \times m)$ time, which is much more efficient.

Question 3

$q(j|*) = d(j|0,5,6)$ for $j \in \{1 \ldots 5\}$
$q(k|j) = d(k|j,5,6)$ for $j,k \in \{1 \ldots 5\}$
$e(f|j) = t(f|e_j)$ for $j \in \{1 \ldots 5\}$
For example,

$$e(\text{le}|2) = t(\text{le}|\text{dog})$$

because $e_2 = \text{dog}$.