Unlabeled Dependency Parses

- root is a special root symbol
- Each dependency is a pair \((j, k)\) where \(j\) index of a head word, \(k\) is the index of a modifier word. In the figures, we represent a dependency \((j, k)\) by a directed edge from word \(j\) to word \(k\)
- Dependencies in the above example are \((0, 2)\), \((2, 1)\), \((2, 4)\) and \((4, 3)\). (We take 0 to be the root symbol.)

Conditions on Dependency Structures

- The dependency arcs form a directed tree, with the root symbol at the root of the tree.
- There are no "crossing dependencies". Dependency structures with no crossing dependencies are sometimes referred to as projective structures.

All Dependency Parses for John saw Mary

- Assume \(x\) is a sequence of words \(x_1 \ldots x_m\)
- A dependency structure is a vector \(y\)
- First, define the index set \(I\) to be the set of all possible dependencies. For example, for \(m = 3\),
  \[ I = \{(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)\} \]
- Then \(y\) is a vector of values \(y(j, k)\) for all \((j, k) \in I\).
  \[ y(j, k) = 1 \text{ if the structure contains the dependency } (j, k), \]
  \[ y(j, k) = 0 \text{ otherwise.} \]
- We use \(Y\) to refer to the set of all possible well-formed vectors \(y\).
Feature Vectors for Dependencies

- \( \phi(x, j, k) \) is a feature vector representing dependency \((j, k)\) for sentence \(x\)
- Example features:
  - Identity of the words \(x_j\) and \(x_k\)
  - The part-of-speech tags for words \(x_j\) and \(x_k\)
  - The distance between \(x_j\) and \(x_k\)
  - Words/tags that surround \(x_j\) and \(x_k\)
  - etc. etc.

CRFs for Discriminative Dependency Parsing

- We use \( \Phi(x, y) \in \mathbb{R}^d \) to refer to a feature vector for an entire dependency structure \(y\)
- We then build a log-linear model, very similar to a CRF
  \[
  p(y|x; w) = \frac{\exp \left( w \cdot \Phi(x, y) \right)}{\sum_{y' \in \mathcal{Y}} \exp \left( w \cdot \Phi(x, y') \right)}
  \]
- How do we define \( \Phi(x, y) \)? Answer:
  \[
  \Phi(x, y) = \sum_{(j, k) \in \mathcal{I}} y(j, k) \phi(x, j, k)
  \]
  where \( \phi(x, j, k) \) is the feature vector for dependency \((j, k)\)

Decoding

- The decoding problem: find
  \[
  \arg \max_{y \in \mathcal{Y}} p(y|x; w) = \arg \max_{y \in \mathcal{Y}} \exp \left( w \cdot \Phi(x, y) \right)
  \]
  \[
  = \arg \max_{y \in \mathcal{Y}} \exp \left( w \cdot \Phi(x, y) \right)
  \]
  \[
  = \arg \max_{y \in \mathcal{Y}} w \cdot \Phi(x, y)
  \]
  \[
  = \arg \max_{y \in \mathcal{Y}} \sum_{(j, k) \in \mathcal{I}} y(j, k) \phi(x, j, k)
  \]
  \[
  = \arg \max_{y \in \mathcal{Y}} \sum_{(j, k) \in \mathcal{I}} y(j, k) \left( w \cdot \phi(x, j, k) \right)
  \]
- This problem can be solved using dynamic programming, in \(O(m^3)\) time, where \(m\) is the length of the sentence

Parameter Estimation

- To estimate the parameters, we assume we have a set of \(n\) labeled examples, \(\{(x^i, y^i)\}_{i=1}^n\). Each \(x^i\) is an input sequence \(x^i_1 \ldots x^i_m\), each \(y^i\) is a dependency structure (i.e., \(y^i(j, k) = 1\) if the \(i\)th structure contains a dependency \((j, k)\)).
- We then proceed in exactly the same way as for CRFs
- The \textit{regularized log-likelihood function} is
  \[
  L(w) = \sum_{i=1}^n \log p(y^i|x^i; w) - \frac{\lambda}{2} ||w||^2
  \]
- The \textit{parameter estimates} are
  \[
  w^* = \arg \max_{w \in \mathbb{R}^d} \sum_{i=1}^n \log p(y^i|x^i; w) - \frac{\lambda}{2} ||w||^2
  \]
Finding the Maximum-Likelihood Estimates

- We’ll again use gradient-based optimization methods to find \( w^* \).
- How can we compute the derivatives? As before,

\[
\frac{\partial}{\partial w_l} L(w) = \sum_i \Phi_i(x^i, y^i) - \sum_{y \in Y} p(y|x^i; w) \Phi_i(x^i, y) - \lambda w_l
\]

- The first term is easily computed, because

\[
\sum_i \Phi_i(x^i, y^i) = \sum_{(j,k) \in \mathcal{I}} y(j,k) \phi_l(x^i, j, k)
\]

- The second term involves a sum over \( Y \), and because of this looks nasty...

Calculating Derivatives using Dynamic Programming

- We now consider how to compute the second term:

\[
\sum_{y \in Y} p(y|x^i; w) \Phi_i(x^i, y) = \sum_{y \in Y} p(y|x^i; w) \sum_{(j,k) \in \mathcal{I}} y(j,k) \phi_l(x^i, j, k)
\]

\[
= \sum_{(j,k) \in \mathcal{I}} q^i(j,k) \phi_l(x^i, j, k)
\]

where

\[
q^i(j,k) = \sum_{y \in Y: y(j,k)=1} p(y|x^i; w)
\]

(for the full derivation see the notes)

- For a given \( i \), all \( q^i(j,k) \) terms can be computed simultaneously in \( O(m^3) \) time using dynamic programming.

Non-Projective Dependency Parsing

- We can also consider non-projective dependency parses, where crossing dependencies are allowed

- Define \( Y_{np} \) to be the set of all non-projective dependency parses

- Each dependency parse \( y \in Y_{np} \) is a vector of values \( y(j,k) \) for all \( (j,k) \in \mathcal{I} \). \( y(j,k) = 1 \) if the structure contains the dependency \( (j,k) \), \( y(j,k) = 0 \) otherwise.

An Example from Czech

He is mostly not even interested in the new things and in most cases, he has no money for it either.

(figure taken from McDonald et al, 2005)
CRFs for Non-Projective Structures

- We use $\Phi(x, y) \in \mathbb{R}^d$ to refer to a feature vector for an entire dependency structure $y$
- We then build a log-linear model, very similar to a CRF

$$p(y|w) = \frac{\exp(w \cdot \Phi(x, y))}{\sum_{y' \in \mathcal{Y}_{np}} \exp(w \cdot \Phi(x, y'))}$$

- How do we define $\Phi(x, y)$? Answer:

$$\Phi(x, y) = \sum_{(j,k) \in \mathcal{I}} y(j, k) \phi(x, j, k)$$

where $\phi(x, j, k)$ is the feature vector for dependency $(j, k)$

Only change from projective parsing: we’ve replaced the set of projective parses $\mathcal{Y}$, with the set of non-projective parses, $\mathcal{Y}_{np}$

Decoding in Non-Projective Models

- The decoding problem: find

$$\arg \max_{y \in \mathcal{Y}_{np}} p(y|x; w) = \arg \max_{y \in \mathcal{Y}_{np}} \frac{\exp(w \cdot \Phi(x, y))}{\sum_{y' \in \mathcal{Y}_{np}} \exp(w \cdot \Phi(x, y'))}$$

$$= \arg \max_{y \in \mathcal{Y}_{np}} \exp(w \cdot \Phi(x, y))$$

$$= \arg \max_{y \in \mathcal{Y}_{np}} w \cdot \Phi(x, y)$$

$$= \arg \max_{y \in \mathcal{Y}_{np}} w \cdot \sum_{(j,k) \in \mathcal{I}} y(j, k) \phi(x, j, k)$$

$$= \arg \max_{y \in \mathcal{Y}_{np}} \sum_{(j,k) \in \mathcal{I}} y(j, k) (w \cdot \phi(x, j, k))$$

Only change from projective parsing: we’ve replaced the set of projective parses $\mathcal{Y}$, with the set of non-projective parses, $\mathcal{Y}_{np}$

Decoding in Non-Projective Parsing Models: the Chu-Liu-Edmonds Algorithm

- Goal is to find the highest scoring directed spanning tree

(figure and example from McDonald et al, 2005)

Step 1

- For each word, find the highest scoring incoming edge:

(figure from McDonald et al 2005)

- If the result of this step is a tree, we have the highest scoring spanning tree
- If not, we have at least one cycle. Next step is to pick a cycle, and contract the cycle
The Result of Contracting the Cycle

- We merge John and saw (the words in the cycle) into a single node c
- The weight of the edge from c to Mary is 30 (because the weight from John to Mary is 3, and from saw to Mary is 30: we take the highest score)
- See McDonald et al 2005 (posted on the class website, under lectures) for how the weights from root to c and Mary to c are calculated
- Having created the new graph, we then recurse (return to step 1)

Step 1 (again)

- For each word, find the highest scoring incoming edge:
- If the result of this step is a tree, we have the highest scoring spanning tree
- This time we have a tree, and we’re done (if not, we would repeat step 2 again)
- Retracing the steps taken in contracting the cycle allows us to recover the highest scoring tree:

Efficiency

- A naive implementation takes $O(n^3)$ time ($n$ is the number of nodes in the graph, i.e., the number of words in the input sentence)
- An improved implementation takes $O(n^2)$ time

Estimating the Parameters

- Again, we can choose the parameters that maximize

$$L(w) = \sum_{i=1}^{n} \log p(y^i|x^i; w) - \frac{\lambda}{2} ||w||^2$$

where $\{(x^i, y^i)\}_{i=1}^{n}$ is the training set
- The gradients can again be calculated efficiently (for example, see Koo, Globerson, Carreras, and Collins, EMNLP 2007)