Dual Decomposition for Parsing with Non-Projective Head Automata

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The Cost of Model Complexity

We are always looking for better ways to model natural language.

Tradeoff: Richer models $\Rightarrow$ Harder decoding

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- Speech Recognition
- Sequence Modeling (e.g. extensions to HMM/CRF)
- Parsing
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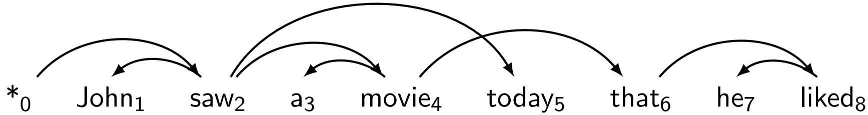
$$y^* = \arg\max_y f(y) \quad \text{Decoding}$$

Non-Projective Dependency Parsing

$*_0$ John$_1$ saw$_2$ a$_3$ movie$_4$ today$_5$ that$_6$ he$_7$ liked$_8$



Important problem in many languages.

Problem is NP-Hard for all but the simplest models.

Dual Decomposition

A classical technique for constructing decoding algorithms.

Solve complicated models

$$y^* = \arg\max_y f(y)$$

by decomposing into smaller problems.

Upshot: Can utilize a toolbox of combinatorial algorithms.

- Dynamic programming
- Minimum spanning tree
- Shortest path
- Min-Cut
- ...

A Dual Decomposition Algorithm for Non-Projective Dependency Parsing

Simple - Uses basic combinatorial algorithms

Efficient - Faster than previously proposed algorithms

Strong Guarantees - Gives a certificate of optimality when exact

Solves 98% of examples exactly, even though the problem is NP-Hard

Widely Applicable - Similar techniques extend to other problems

Roadmap

Algorithm

Experiments

Derivation

Non-Projective Dependency Parsing

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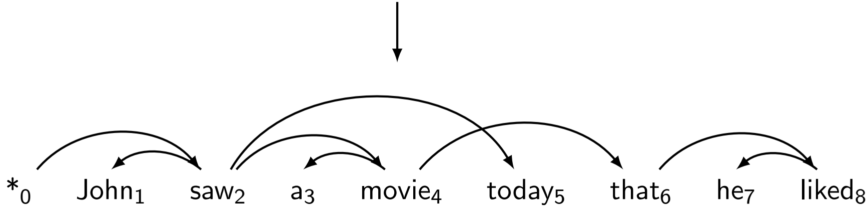
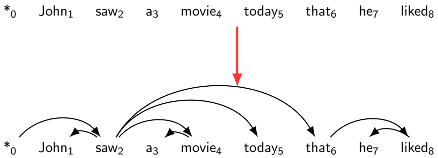


- Starts at the root symbol *
- Each word has a exactly one parent word
- Produces a tree structure (no cycles)
- Dependencies can cross

Algorithm Outline



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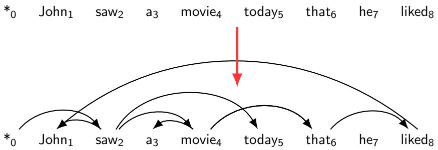
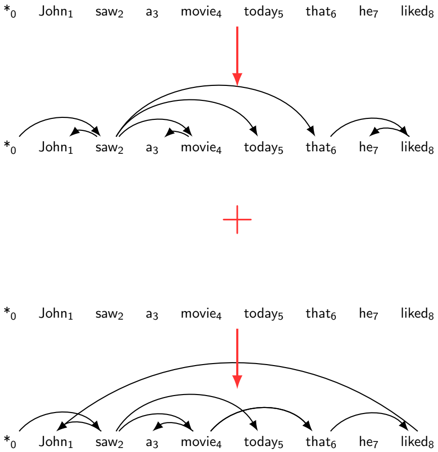


Sibling Model

Algorithm Outline



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Dual Decomposition

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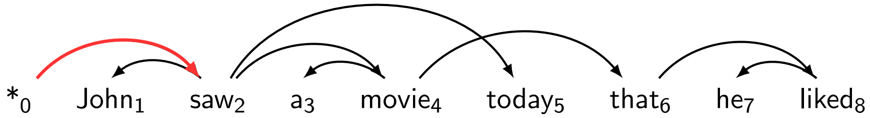

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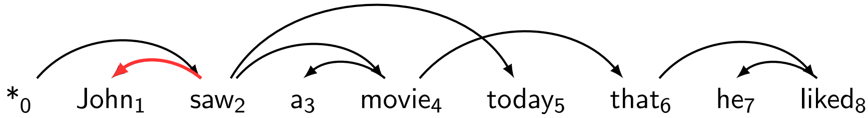

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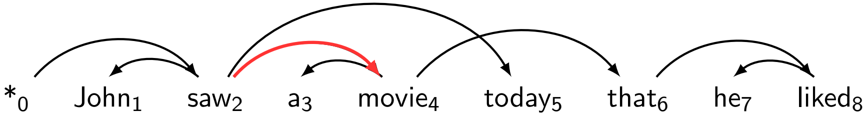

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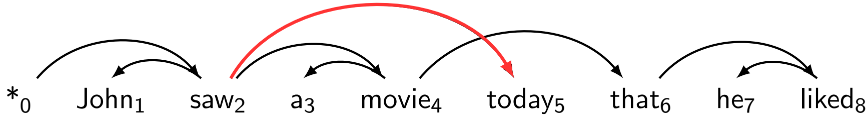

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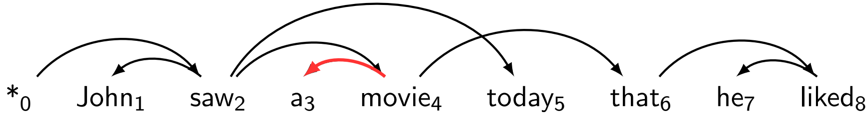

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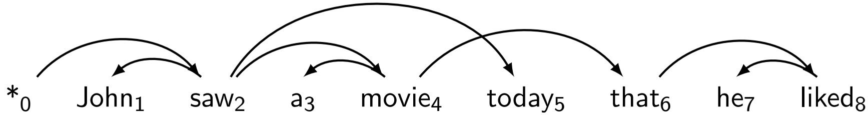

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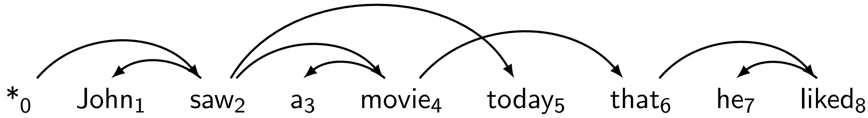

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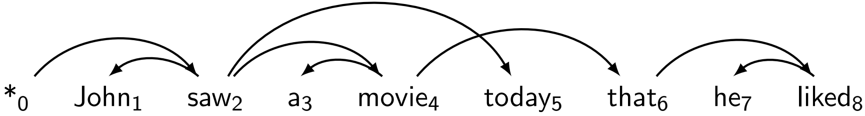

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$$y^* = \arg\max_y f(y) \Leftarrow$$ Minimum Spanning Tree Algorithm

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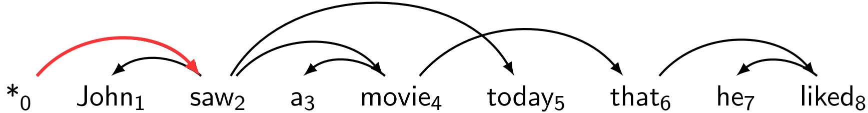

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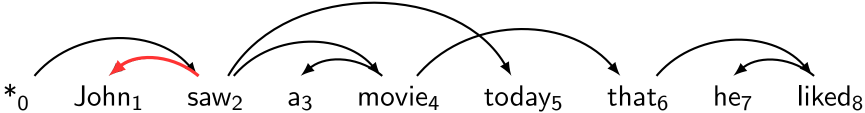

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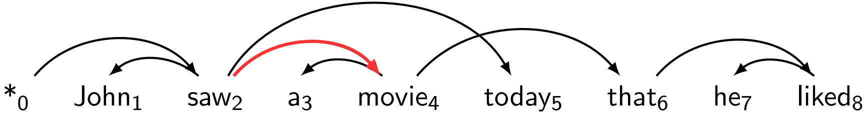

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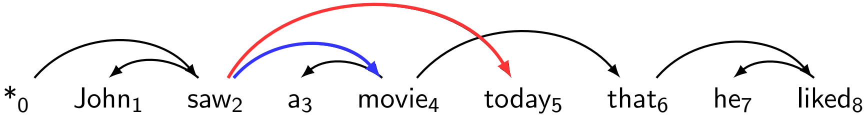

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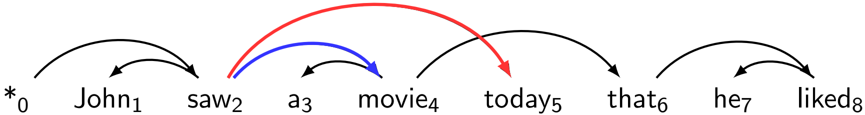

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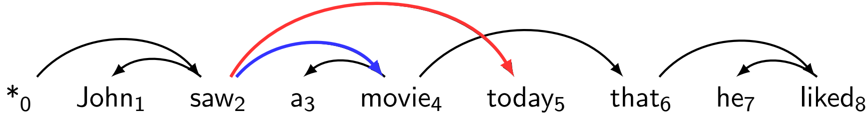

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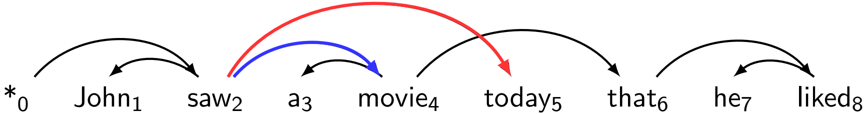

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Under Sibling Model, can solve for each word with Viterbi decoding.

Thought Experiment Continued

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Idea: Do individual decoding for each head word using dynamic programming.

If we're lucky, we'll end up with a valid final tree.

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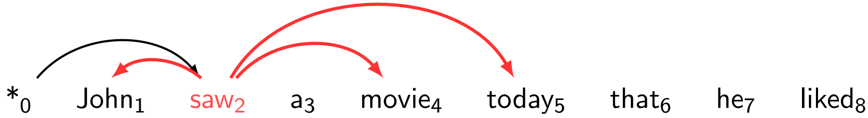

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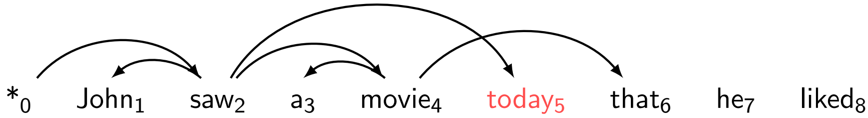

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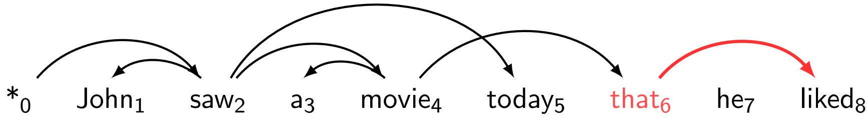

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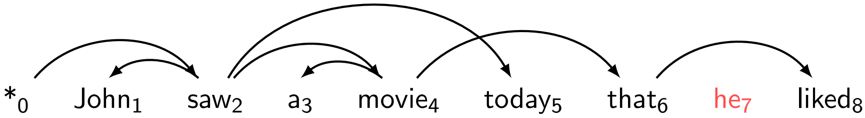

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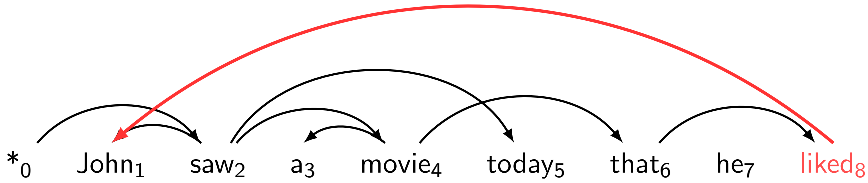

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But we might violate some constraints.

Dual Decomposition Idea

	No Constraints	Tree Constraints
Arc-Factored		Minimum Spanning Tree
Sibling Model	Individual Decoding	

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All Possible **Valid Trees**

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Sibling

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$$\text{such that } \underbrace{z = y}_{\textbf{Constraint}}$$

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Set penalty weights equal to 0 for all edges.

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$y^{(k)} \leftarrow$ Decode $(g(y) - \mathrm{penalty})$ by Minimum Spanning Tree

If $y^{(k)}(i,j) = z^{(k)}(i,j)$ for all i,j **Return** $(y^{(k)}, z^{(k)})$

Algorithm Sketch

Set penalty weights equal to 0 for all edges.

For $k = 1$ **to** K

$z^{(k)} \leftarrow$ Decode $(f(z) + \text{penalty})$ by Individual Decoding

$y^{(k)} \leftarrow$ Decode $(g(y) - \text{penalty})$ by Minimum Spanning Tree

If $y^{(k)}(i,j) = z^{(k)}(i,j)$ for all i,j **Return** $(y^{(k)}, z^{(k)})$

Else Update penalty weights based on $y^{(k)}(i,j) - z^{(k)}(i,j)$

Individual Decoding

Penalties

$u(i,j) = 0$ for all i,j

$*_0$ John$_1$ saw$_2$ a$_3$ movie$_4$ today$_5$ that$_6$ he$_7$ liked$_8$

$$z^* = \arg\max_{z \in \mathcal{Z}} (f(z) + \sum_{i,j} u(i,j) z(i,j))$$

Minimum Spanning Tree

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$$y^* = \arg\max_{y \in \mathcal{Y}} (g(y) - \sum_{i,j} u(i,j) y(i,j))$$

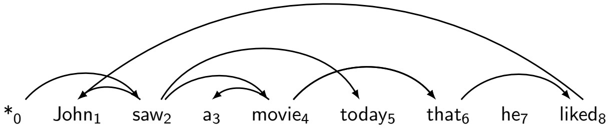
Key

$f(z)$	$\Leftarrow$	Sibling Model	$g(y)$	$\Leftarrow$	Arc-Factored Model
$\mathcal{Z}$	$\Leftarrow$	No Constraints	$\mathcal{Y}$	$\Leftarrow$	Tree Constraints
$y(i,j) = 1$	if	y contains dependency i,j			

Individual Decoding

Penalties

$u(i,j) = 0$ for all i,j



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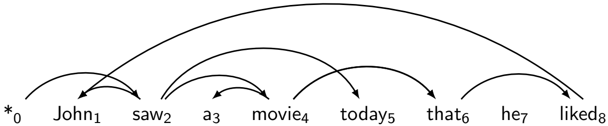

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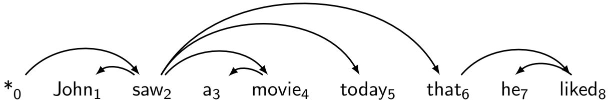

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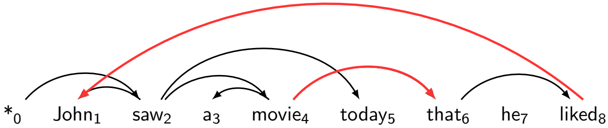

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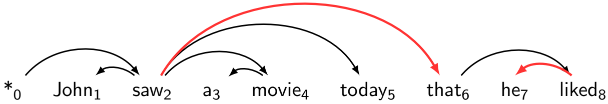

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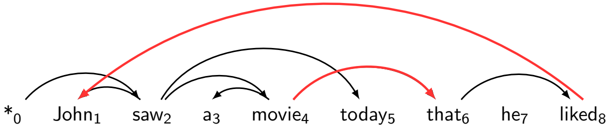

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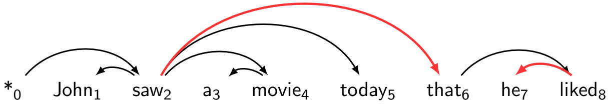

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Penalties

$u(i,j) = 0$ for all i,j

	Iteration 1
$u(8,1)$	-1
$u(4,6)$	-1
$u(2,6)$	1
$u(8,7)$	1

Key

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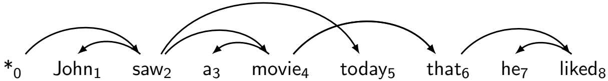

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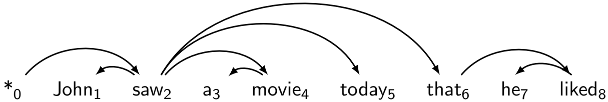

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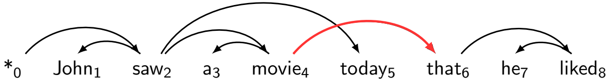

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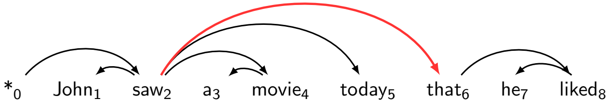

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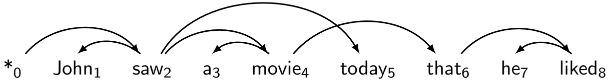

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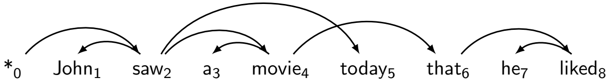
Minimum Spanning Tree



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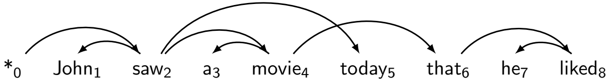

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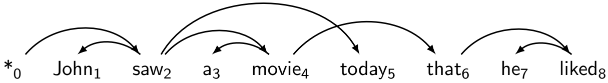

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Converged

$$y^* = \arg\max_{y \in \mathcal{Y}} f(y) + g(y)$$

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Guarantees

Theorem

If at any iteration $y^{(k)} = z^{(k)}$, then $(y^{(k)}, z^{(k)})$ is the global optimum.

In experiments, we find the global optimum on 98% of examples.

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Theorem

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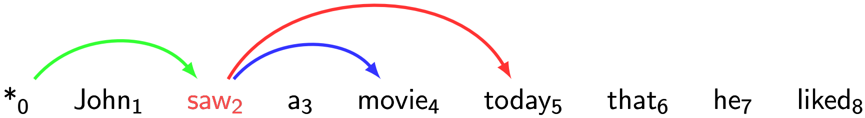
In experiments, we find the global optimum on 98% of examples.

If we do not converge to a match, we can still return an approximate solution (more in the paper).

Extensions

- Grandparent Models



$$f(y) = \ldots + \mathit{score}(\mathit{gp} = *_0, \mathit{head} = \text{saw}_2, \mathit{prev} = \text{movie}_4, \mathit{mod} = \text{today}_5)$$

- Head Automata (Eisner, 2000)

  Generalization of Sibling models

  Allow arbitrary automata as local scoring function.

Roadmap

Algorithm

Experiments

Derivation

Experiments

Properties:

- Exactness
- Parsing Speed
- Parsing Accuracy
- Comparison to Individual Decoding
- Comparison to LP/ILP

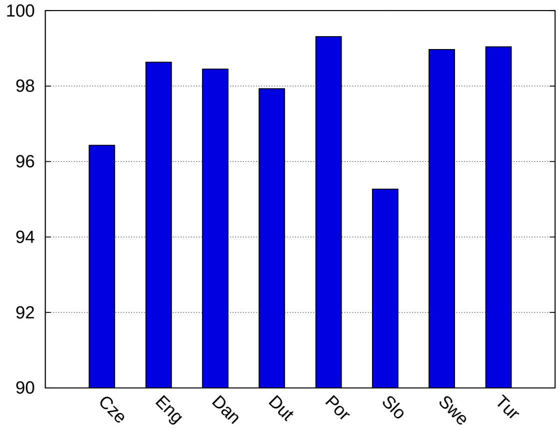
Training:

- Averaged Perceptron (more details in paper)

Experiments on:

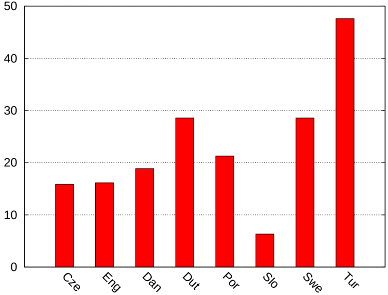
- CoNLL Datasets
- English Penn Treebank
- Czech Dependency Treebank

How often do we exactly solve the problem?

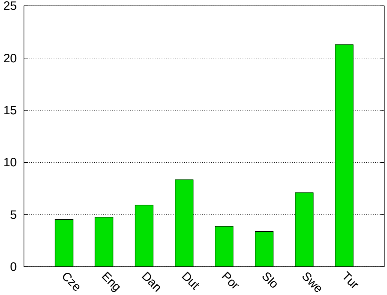


- Percentage of examples where the dual decomposition finds an exact solution.

Parsing Speed



Sibling model



Grandparent model

- Number of sentences parsed per second
- Comparable to dynamic programming for projective parsing

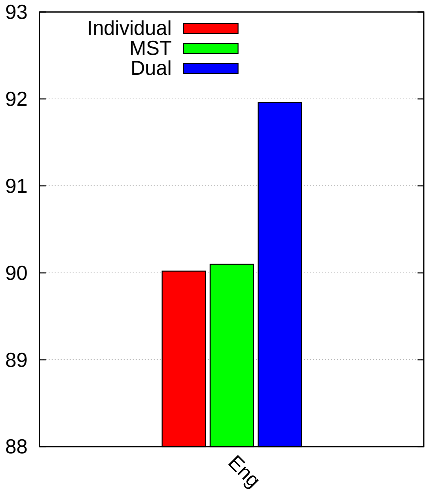
Accuracy

	Arc-Factored	Prev Best	Grandparent
Dan	89.7	91.5	**91.8**
Dut	82.3	85.6	**85.8**
Por	90.7	92.1	**93.0**
Slo	82.4	85.6	**86.2**
Swe	88.9	90.6	**91.4**
Tur	75.7	76.4	**77.6**
Eng	90.1	—	**92.5**
Cze	84.4	—	**87.3**

Prev Best - Best reported results for CoNLL-X data set, includes

- Approximate search (McDonald and Pereira, 2006)
- Loop belief propagation (Smith and Eisner, 2008)
- (Integer) Linear Programming (Martins et al., 2009)

Comparison to Subproblems



F_1 for dependency accuracy

Comparison to LP/ILP

Martins et al.(2009): Proposes two representations of non-projective dependency parsing as a linear programming relaxation as well as an exact ILP.

- LP (1)
- LP (2)
- ILP

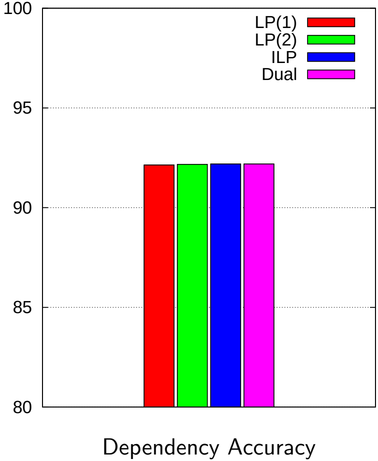
Use an LP/ILP Solver for decoding

We compare:

- Accuracy
- Exactness
- Speed

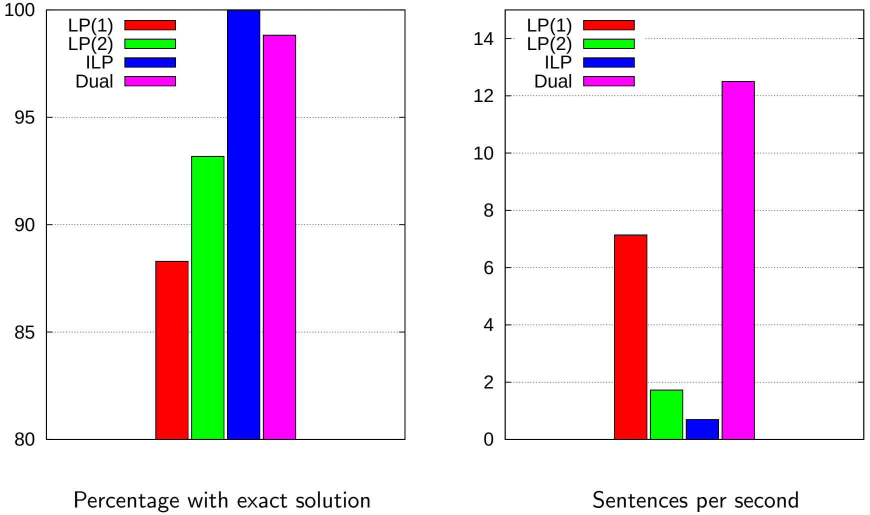
Both LP and dual decomposition methods use the same model, features, and weights w.

Comparison to LP/ILP: Accuracy



- All decoding methods have comparable accuracy

Comparison to LP/ILP: Exactness and Speed



Roadmap

Algorithm

Experiments

Derivation

Deriving the Algorithm

Goal:

$$y^* = \arg\max_{y \in \mathcal{Y}} f(y)$$

Rewrite:

$$\arg\max_{z \in \mathcal{Z}, y \in \mathcal{Y}} f(z) + g(y)$$

$$\text{s.t. } z(i,j) = y(i,j) \text{ for all } i,j$$

Lagrangian: $L(u, y, z) = f(z) + g(y) + \sum_{i,j} u(i,j)\left(z(i,j) - y(i,j)\right)$

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Lagrangian: $L(u,y,z) = f(z) + g(y) + \sum_{i,j} u(i,j)\,(z(i,j) - y(i,j))$

The dual problem is to find $\min_u L(u)$ where

$$L(u) = \max_{y \in \mathcal{Y}, z \in \mathcal{Z}} L(u,y,z) = \max_{z \in \mathcal{Z}} \left(f(z) + \sum_{i,j} u(i,j) z(i,j) \right) + \max_{y \in \mathcal{Y}} \left(g(y) - \sum_{i,j} u(i,j) y(i,j) \right)$$

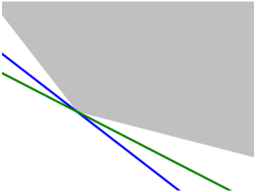
Dual is an upper bound: $L(u) \geq f(z^*) + g(y^*)$ for any u

A Subgradient Algorithm for Minimizing $L(u)$

$$L(u) \;=\; \max_{z\in\mathcal{Z}}\left(f(z)+\sum_{i,j}u(i,j)z(i,j)\right)+\max_{y\in\mathcal{Y}}\left(g(y)-\sum_{i,j}u(i,j)y(i,j)\right)$$

$L(u)$ is convex, but not differentiable. A *subgradient* of $L(u)$ at u is a vector g_u such that for all v,

$$L(v) \geq L(u) + g_u \cdot (v - u)$$



Subgradient methods use updates $u' = u - \alpha g_u$

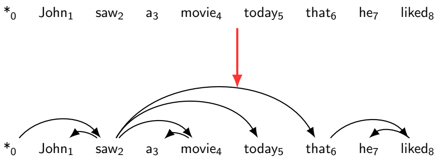
In fact, for our $L(u)$, $g_u(i,j) = z^*(i,j) - y^*(i,j)$

Related Work

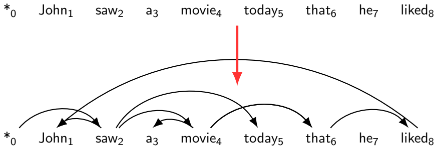
- Methods that use general purpose linear programming or integer linear programming solvers (Martins et al. 2009; Riedel and Clarke 2006; Roth and Yih 2005)
- Dual decomposition/Lagrangian relaxation in combinatorial optimization (Dantzig and Wolfe, 1960; Held and Karp, 1970; Fisher 1981)
- Dual decomposition for inference in MRFs (Komodakis et al., 2007; Wainwright et al., 2005)
- Methods that incorporate combinatorial solvers within loopy belief propagation (Duchi et al. 2007; Smith and Eisner 2008)

Summary

$$y^* = \arg\max_y f(y) \Leftarrow \text{NP-Hard}$$



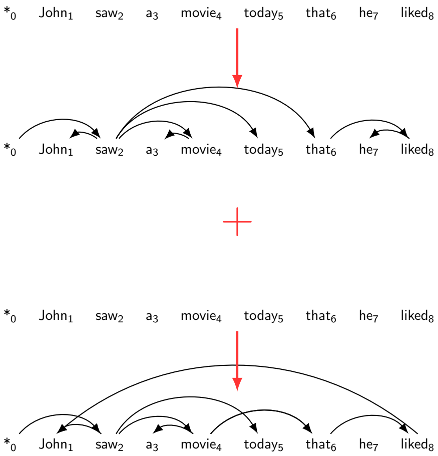
Arc-Factored Model



Sibling Model

Summary

$$y^* = \arg\max_y f(y) \Leftarrow \text{NP-Hard}$$



Arc-Factored Model

Dual Decomposition

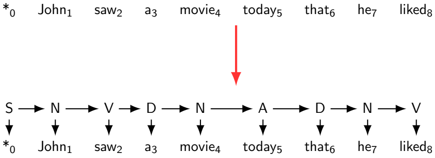
Sibling Model

Other Applications

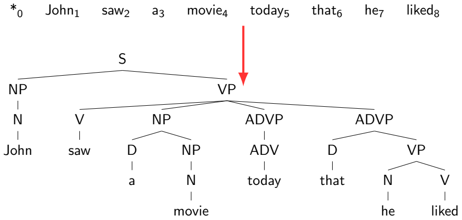
- Dual decomposition can be applied to other decoding problems.
- Rush et al. (2010) focuses on integrated dynamic programming algorithms.
  - Integrated Parsing and Tagging
  - Integrated Constituency and Dependency Parsing

Parsing and Tagging

$$y^* = \arg\max_y f(y) \Leftarrow \text{Slow}$$



HMM Model

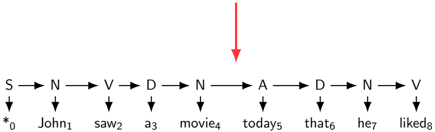


CFG Model

Parsing and Tagging

$$y^* = \arg\max_y f(y) \Leftarrow \text{Slow}$$

$*_0$ John$_1$ saw$_2$ a$_3$ movie$_4$ today$_5$ that$_6$ he$_7$ liked$_8$

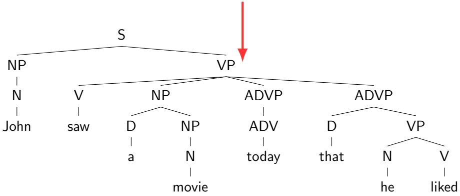


HMM Model

+

Dual Decomposition

$*_0$ John$_1$ saw$_2$ a$_3$ movie$_4$ today$_5$ that$_6$ he$_7$ liked$_8$

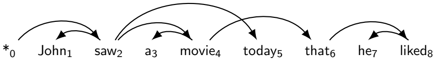


CFG Model

Dependency and Constituency

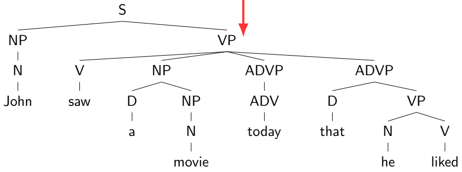
$$y^* = \arg\max_y f(y) \Leftarrow \text{Slow}$$

$*_0$ John$_1$ saw$_2$ a$_3$ movie$_4$ today$_5$ that$_6$ he$_7$ liked$_8$



Dependency Model

$*_0$ John$_1$ saw$_2$ a$_3$ movie$_4$ today$_5$ that$_6$ he$_7$ liked$_8$



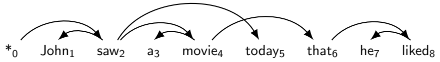
Lexicalized CFG

Dependency and Constituency

$$y^* = \arg\max_y f(y) \Leftarrow \text{Slow}$$

$*_0$ John$_1$ saw$_2$ a$_3$ movie$_4$ today$_5$ that$_6$ he$_7$ liked$_8$

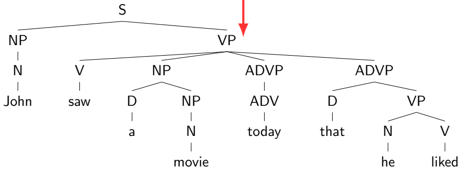
Dependency Model



Dual Decomposition

$*_0$ John$_1$ saw$_2$ a$_3$ movie$_4$ today$_5$ that$_6$ he$_7$ liked$_8$

Lexicalized CFG



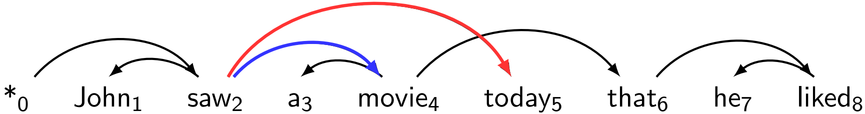
Future Directions

There is much more to explore around dual decomposition in NLP.

- Known Techniques
  - Generalization to more than two models
  - K-best decoding
  - Approximate subgradient
  - Heuristic for branch-and-bound type search
- Possible NLP Applications
  - Machine Translation
  - Speech Recognition
  - "Loopy" Sequence Models
- Open Questions
  - Can we speed up subalgorithms when running repeatedly?
  - What are the trade-offs of different decompositions?
  - Are there better methods for optimizing the dual?

Appendix

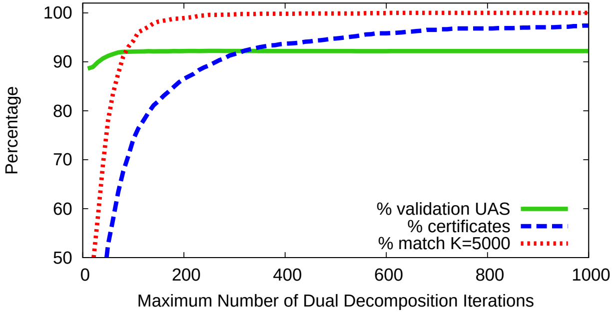
Training the Model



$$f(y) = \ldots + \mathit{score}(\mathrm{saw}_2, \mathrm{movie}_4, \mathrm{today}_5) + \ldots$$

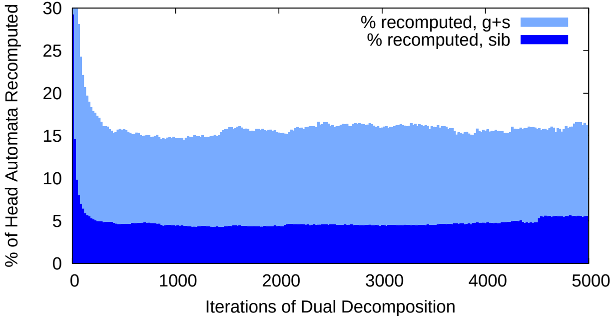
- $\mathit{score}(\mathrm{saw}_2, \mathrm{movie}_4, \mathrm{today}_5) = w \cdot \phi(\mathrm{saw}_2, \mathrm{movie}_4, \mathrm{today}_5)$
- Weight vector w trained using Averaged perceptron.
- (More details in the paper.)

Early Stopping



Early Stopping

Caching



Caching speed