CSEE W3827

Fundamentals of Computer Systems

Homework Assignment 1

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Updated: Due Feb 2, 2016 at 8:40 AM

*Write your name **and UNI** on each page of your solutions.*
Show your work for each problem.
Note your collaborators.

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1. (20 pts.) Add the following numbers *without converting to decimal.*
   (a) 01010_2 and 11001_2
   (b) 713_8 and 405_8
   (c) ABC_{16} and $A78_{16}$
2. (15 pts.) Prove or disprove that the exclusive or operation ($\oplus$) is associative.
3. (20 pts.) Convert the Boolean functions below to minimal product-of-sums and minimal sum-of-products form.
   (a) $x \oplus y \oplus z$
   (b) $z\overline{w} + xy\overline{w} + x\overline{y}z$
4. (20 pts.) Using nothing but 3-input NAND gates (NAND3), give a schematic for f.

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

5. (15 pts.) Use algebraic manipulation (including 2-way DeMorgan's if you wish) to prove the 3-way statement of DeMorgan's below.

$$\overline{x \cdot y \cdot z} = \overline{x} + \overline{y} + \overline{z}$$
$$\overline{x + y + z} = \overline{x} \cdot \overline{y} \cdot \overline{z}$$

6. (30 pts.) Implement $X \oplus Y \oplus Z$ using only the components in the table below.

	Cost (transistors)	Delay (ns)
INV	2	1
AND2	6	2.4
OR2	6	2.4
XOR2	14	6.1

Find the implementation that has

(a) the smallest transistor cost

(b) the smallest delay

(c) the smallest delay-cost product