# CSEE 3827: Fundamentals of Computer Systems

Standard Forms and Simplification with Karnaugh Maps

# Agenda (M&K 2.3-2.5)

- Standard Forms
  - Product-of-Sums (PoS)
  - Sum-of-Products (SoP)
    - converting between
  - Min-terms and Max-terms
- Simplification via Karnaugh Maps (K-maps)
  - 2, 3, and 4 variable
  - Implicants, Prime Implicants, Essential Prime Implicants
  - Using K-maps to reduce
  - PoS form
  - Don’t Care Conditions

# Standard Forms

- There are many ways to express a boolean expression

$$
\begin{aligned}
F &= XYZ + XYZ + XZ \\
&= XY(Z + Z) + XZ \\
&= XY + XZ
\end{aligned}
$$

- It is useful to have a standard or canonical way

- Derived from truth table

- Generally not the simplest form

# Two principle standard forms

- Sum-of-products (SOP)
- Product-of-sums (POS)

# Product and sum terms

- Product term:  logical AND of literals (e.g., $X\overline{Y}Z$)
- Sum term: logical OR of literals (e.g., $A + \overline{B} + C$)

# PoS & SoP

- Sum of products (SoP): OR of ANDs

$$\text{e.g., } F = \overline{Y} + \overline{X}Y\overline{Z} + XY$$

- Product of sums (PoS): AND of ORs

$$\text{e.g., } G = X(\overline{Y} + Z)(X + Y + \overline{Z})$$

# Converting from PoS (or any form) to SoP

*Just multiply through and simplify, e.g.,*

$$G = X(Y + Z)(X + Y + Z)$$

$$= XYX + XYY + XYZ + XZX + XZY + XZZ$$

$$= XY + XY + XYZ + XZ + XZY + XZ$$

$$= XY + XZ$$

# Converting from SoP to PoS

*Complement, multiply through, complement via DeMorgan, e.g.,*

Note: $X' = \overline{X}$

$$F = Y'Z' + XY'Z + XYZ'$$

$$F' = (Y+Z)(X'+Y+Z')(X'+Y'+Z)$$

$$= YZ + X'Y + X'Z \qquad \text{(after lots of simplification)}$$

$$F = (Y'+Z')(X+Y')(X+Z')$$

# Minterms

*e.g., Minterms for 3 variables A,B,C*

| A | B | C | minterm |
|---|---|---|---|
| 0 | 0 | 0 | m0 $\bar{A}\bar{B}\bar{C}$ |
| 0 | 0 | 1 | m1 $\bar{A}\bar{B}C$ |
| 0 | 1 | 0 | m2 $\bar{A}B\bar{C}$ |
| 0 | 1 | 1 | m3 $\bar{A}BC$ |
| 1 | 0 | 0 | m4 $A\bar{B}\bar{C}$ |
| 1 | 0 | 1 | m5 $A\bar{B}C$ |
| 1 | 1 | 0 | m6 $AB\bar{C}$ |
| 1 | 1 | 1 | m7 $ABC$ |

- A product term in which all variables appear once, either complemented or uncomplemented (i.e., an entry in the truth table).
- Each minterm evaluates to 1 for exactly one variable assignment, 0 for all others.
- Denoted by mX where X corresponds to the variable assignment for which mX = 1.

# Minterms to describe a function

*sometimes also called a* ***minterm expansion*** *or* ***disjunctive normal form (DNF)***

This “term” is TRUE when A=0,B=1,C=0

$$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$$

| A | B | C | F | $\overline{F}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 |

$$\overline{F} = \overline{A}BC + AB\overline{C} + ABC$$

# Sum of minterms form

*The logical OR of all minterms for which F = 1.*

| A | B | C | minterm | | F |
|---|---|---|---|---|---|
| 0 | 0 | 0 | m0 | $\overline{A}\,\overline{B}\,\overline{C}$ | 0 |
| 0 | 0 | 1 | m1 | $\overline{A}\,\overline{B}C$ | 1 |
| 0 | 1 | 0 | m2 | $\overline{A}B\overline{C}$ | 1 |
| 0 | 1 | 1 | m3 | $\overline{A}BC$ | 1 |
| 1 | 0 | 0 | m4 | $A\overline{B}\,\overline{C}$ | 0 |
| 1 | 0 | 1 | m5 | $A\overline{B}C$ | 0 |
| 1 | 1 | 0 | m6 | $AB\overline{C}$ | 0 |
| 1 | 1 | 1 | m7 | $ABC$ | 0 |

$$F = \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC$$

$$= m1 + m2 + m3$$

$$= \sum m(1,2,3)$$

# Minterm form cont'd

*(variables appear once in each minterm)*

| A | B | C | F | $\overline{F}$ | minterm |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | m0 $\overline{A}\,\overline{B}\,\overline{C}$ |
| 0 | 0 | 1 | 1 | 0 | m1 $\overline{A}\,\overline{B}C$ |
| 0 | 1 | 0 | 1 | 0 | m2 $\overline{A}B\overline{C}$ |
| 0 | 1 | 1 | 0 | 1 | m3 $\overline{A}BC$ |
| 1 | 0 | 0 | 1 | 0 | m4 $A\overline{B}\,\overline{C}$ |
| 1 | 0 | 1 | 1 | 0 | m5 $A\overline{B}C$ |
| 1 | 1 | 0 | 0 | 1 | m6 $AB\overline{C}$ |
| 1 | 1 | 1 | 0 | 1 | m7 $ABC$ |

$$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$$

$0\ m0 + m1 + m2 + m4 + m5$

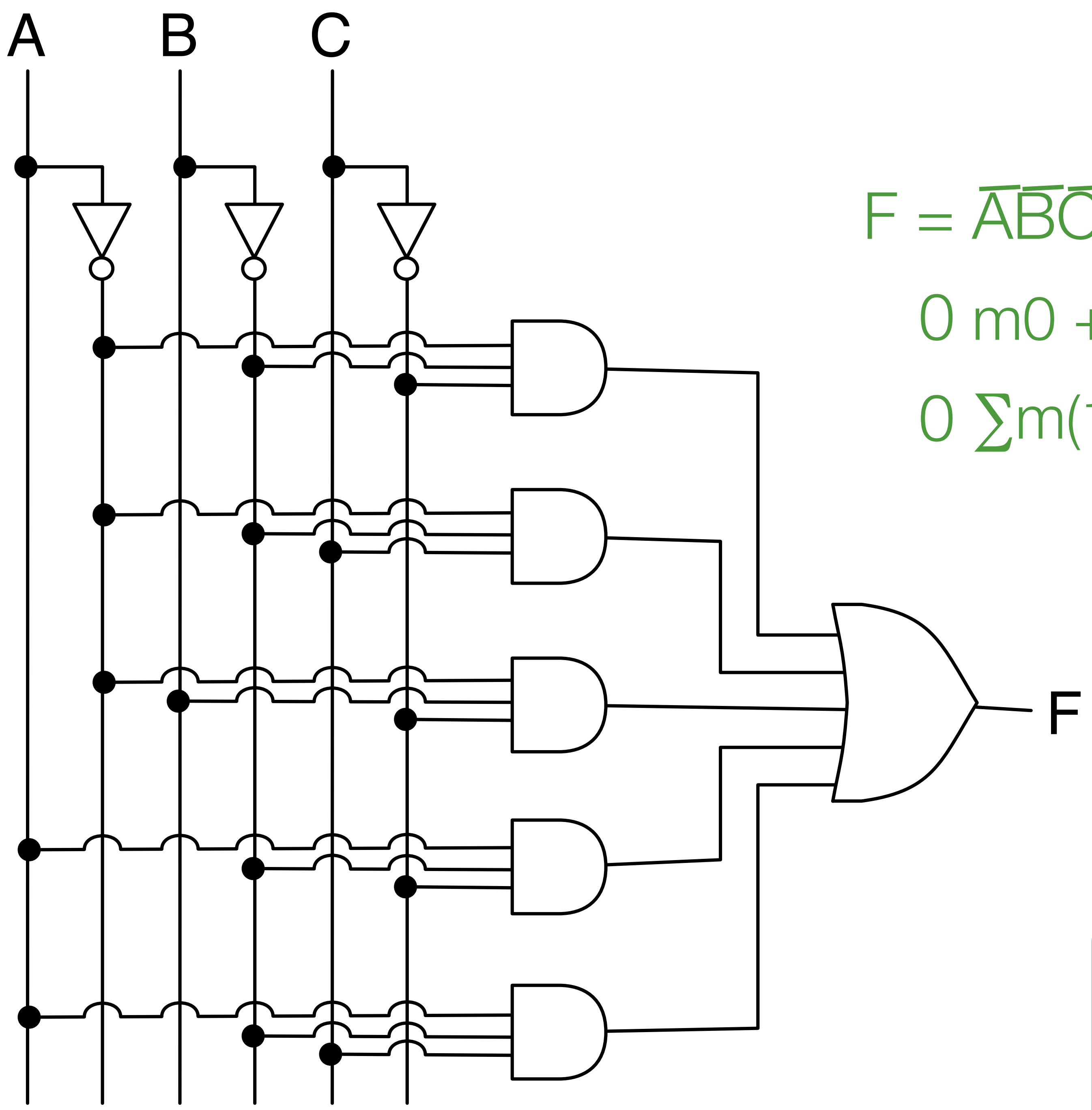
$0\ \sum m(0,1,2,4,5)$

$$\overline{F} = \overline{A}BC + AB\overline{C} + ABC$$

$0\ m3 + m6 + m7$

$0\ \sum m(3,6,7)$

# Minterms as a circuit




$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$

0 m0 + m1 + m2 + m4 + m5

0 $\sum m(1,0,2,4,5)$

*Standard form is not minimal form!*

# Maxterms

| A | B | C | maxterm |
|---|---|---|---|
| 0 | 0 | 0 | M0 $A+B+C$ |
| 0 | 0 | 1 | M1 $A+B+\overline{C}$ |
| 0 | 1 | 0 | M2 $A+\overline{B}+C$ |
| 0 | 1 | 1 | M3 $A+\overline{B}+\overline{C}$ |
| 1 | 0 | 0 | M4 $\overline{A}+B+C$ |
| 1 | 0 | 1 | M5 $\overline{A}+B+\overline{C}$ |
| 1 | 1 | 0 | M6 $\overline{A}+\overline{B}+C$ |
| 1 | 1 | 1 | M7 $\overline{A}+\overline{B}+\overline{C}$ |

- A sum term in which all variables appear once, either complemented or uncomplemented.
- Each maxterm evaluates to 0 for exactly one variable assignment, 1 for all others.
- Denoted by MX where X corresponds to the variable assignment for which MX = 0.

# Maxterm description of a function

*sometimes also called a* ***maxterm expansion*** *or* ***conjunctive normal form (CNF)***

This "term" is FALSE when A=1,B=1,C=0

$$F = (A+\overline{B}+\overline{C})\ (\overline{A}+\overline{B}+C)\ (\overline{A}+\overline{B}+\overline{C})$$

| A | B | C | F | $\overline{F}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 |

Force to 0

# Product of maxterms form

The logical AND of all maxterms for which F = 0.

| A | B | C | maxterm | | F |
|---|---|---|---|---|---|
| 0 | 0 | 0 | M0 | $A+B+C$ | 0 |
| 0 | 0 | 1 | M1 | $A+B+\overline{C}$ | 1 |
| 0 | 1 | 0 | M2 | $A+\overline{B}+C$ | 1 |
| 0 | 1 | 1 | M3 | $A+\overline{B}+\overline{C}$ | 1 |
| 1 | 0 | 0 | M4 | $\overline{A}+B+C$ | 0 |
| 1 | 0 | 1 | M5 | $\overline{A}+B+\overline{C}$ | 0 |
| 1 | 1 | 0 | M6 | $\overline{A}+\overline{B}+C$ | 0 |
| 1 | 1 | 1 | M7 | $\overline{A}+\overline{B}+\overline{C}$ | 0 |

$$F = (A+B+C)\,(\overline{A}+B+C)\,(\overline{A}+B+\overline{C})\,(\overline{A}+\overline{B}+C)\,(\overline{A}+\overline{B}+\overline{C})$$

$$= (M0)\,(M4)\,(M5)\,(M6)\,(M7)$$

$$= \prod M(0,4,5,6,7)$$

# One final example

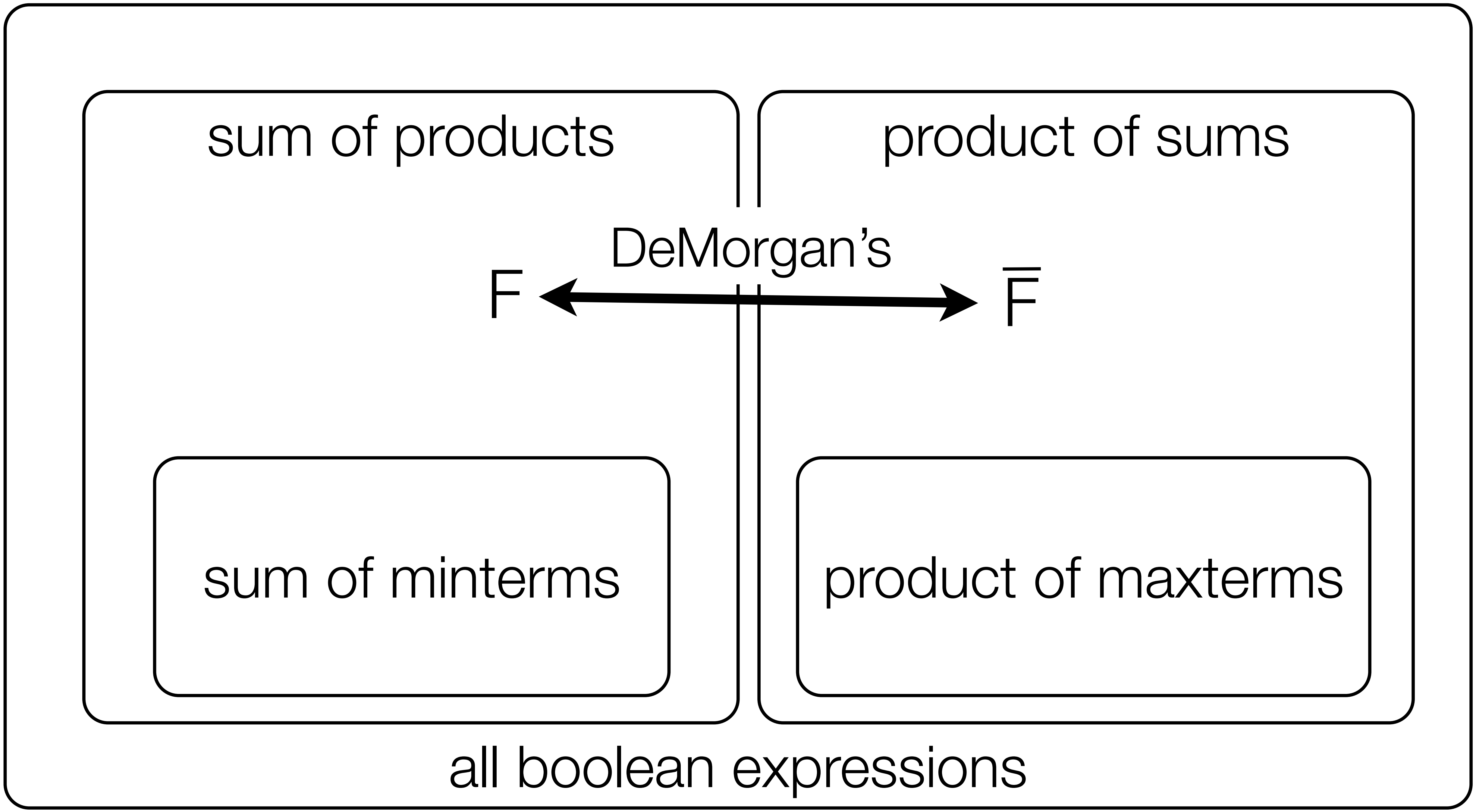
| A | B | C | F | $\overline{F}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |

| | F | $\overline{F}$ |
|---|---|---|
| Minterms (SOP) | | |
| Maxterms (POS) | | |

# Summary of Minterms and Maxterms

| | $F$ | $\overline{F}$ |
|---|---|---|
| Minterms (SOP) | $\sum m(F = 1)$ | $\sum m(F = 0)$ |
| Maxterms (POS) | $\prod M(F = 0)$ | $\prod M(F = 1)$ |

# Relations between standard forms



# Simplification with Karnaugh Maps

# Cost criteria

- Literal cost: the number of literals in an expression

- Gate-input cost: the literal cost + all terms with more than one literal + (optionally) the number of distinct, complemented single literals

Roughly proportional to the number of transistors and wires in an AND/OR/NOT circuits. Does not apply, to more complex gates, for example XOR.

| | Literal cost | Gate-input cost |
|---|---|---|
| $G = \bar{A}\bar{B}\bar{C}\bar{D} + ABCD$ | 8 | 8 + 2 + (4) |
| $G = (\bar{A}+B)(\bar{B}+C)(\bar{C}+D)(\bar{D}+A)$ | 8 | 8 + 5 + (4) |

# Karnaugh maps (a.k.a., K-maps)

- All functions can be expressed with a map
- There is one square in the map for each minterm in a function's truth table

| X | Y | F |
|---|---|---|
| 0 | 0 | m0 |
| 0 | 1 | m1 |
| 1 | 0 | m2 |
| 1 | 1 | m3 |

| X \ Y | 0 | 1 |
|---|---|---|
| 0 | m0<br>$\overline{X}\,\overline{Y}$ | m1<br>$\overline{X}Y$ |
| 1 | m2<br>$X\overline{Y}$ | m3<br>$XY$ |

# Karnaugh maps express functions

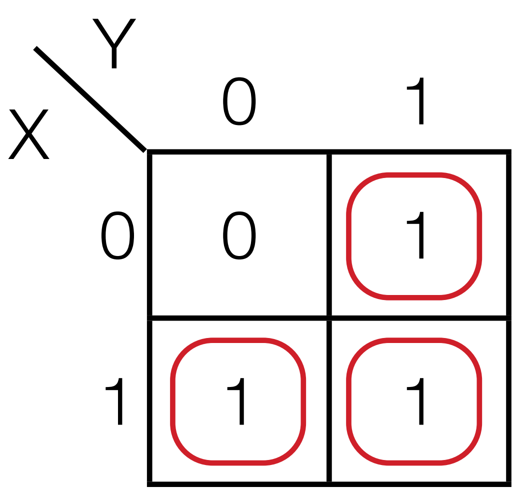
- Fill out table with value of a function

| X | Y | F |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

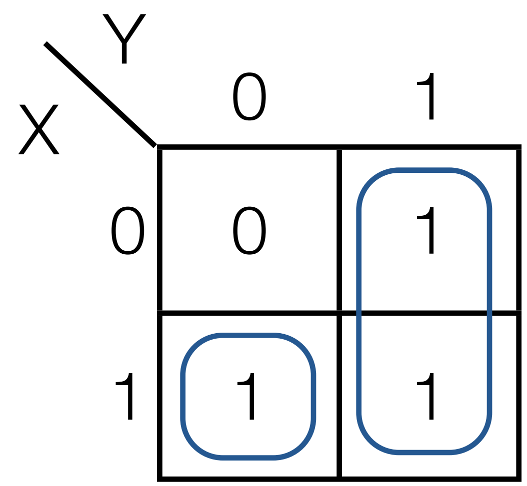
| X \ Y | 0 | 1 |
|---|---|---|
| 0 | | |
| 1 | | |

# Simplification using a k-map

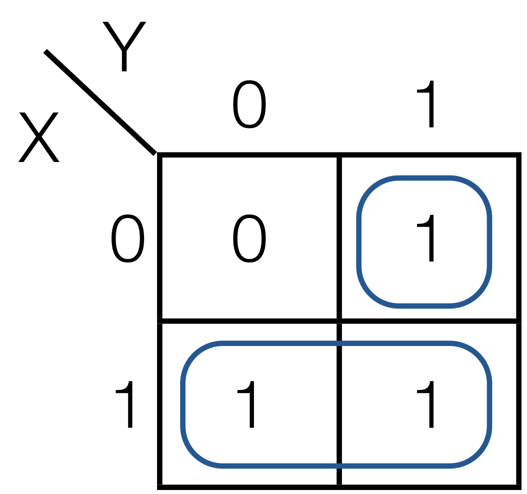
- Whenever two squares share an edge and both are 1, those two terms can be combined to form a single term with one less variable



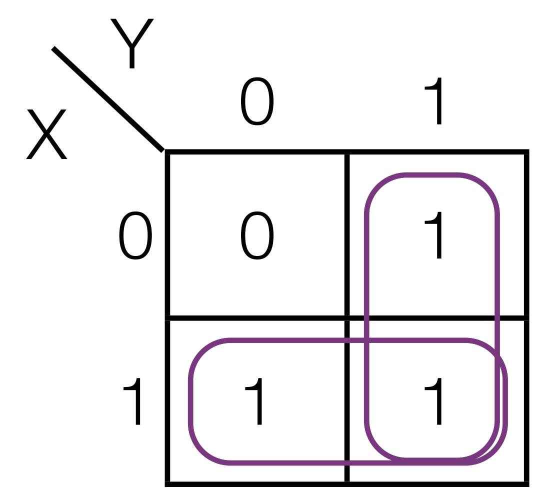

$F = \bar{X}Y + X\bar{Y} + XY$




$F = Y + X\bar{Y}$



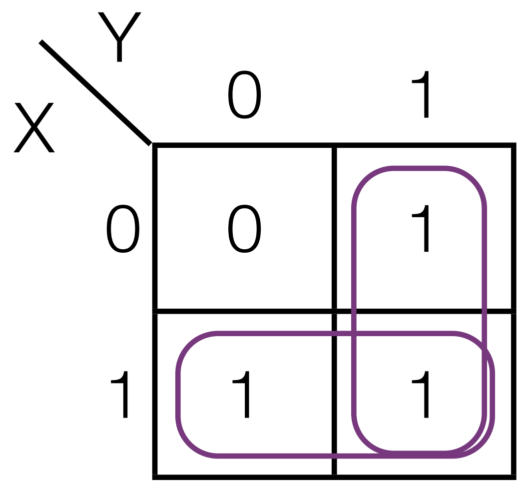

$F = X + \bar{X}Y$




$F = X + Y$

# Simplification using a k-map (2)

- Circle contiguous groups of 1s (circle sizes must be a power of 2)
- There is a correspondence between circles on a k-map and terms in a function expression
- The bigger the circle, the simpler the term
- Add circles (and terms) until all 1s on the k-map are circled




$F = X + Y$

# 3-variable Karnaugh maps

- Use gray ordering on edges with multiple variables
- Gray encoding: order of values such such that only one bit changes at a time
- Two minterms are considered adjacent if they differ in only one variable (this means maps “wrap”)

| X \ YZ | 0 0 | 0 1 | 1 1 | 1 0 |
|---|---|---|---|---|
| 0 | m0 $\bar{X}\bar{Y}\bar{Z}$ | m1 $\bar{X}\bar{Y}Z$ | m3 $\bar{X}YZ$ | m2 $\bar{X}Y\bar{Z}$ |
| 1 | m4 $X\bar{Y}\bar{Z}$ | m5 $X\bar{Y}Z$ | m7 $XYZ$ | m6 $XY\bar{Z}$ |

Y=1

X=1

Z=1

# 4-variable Karnaugh maps

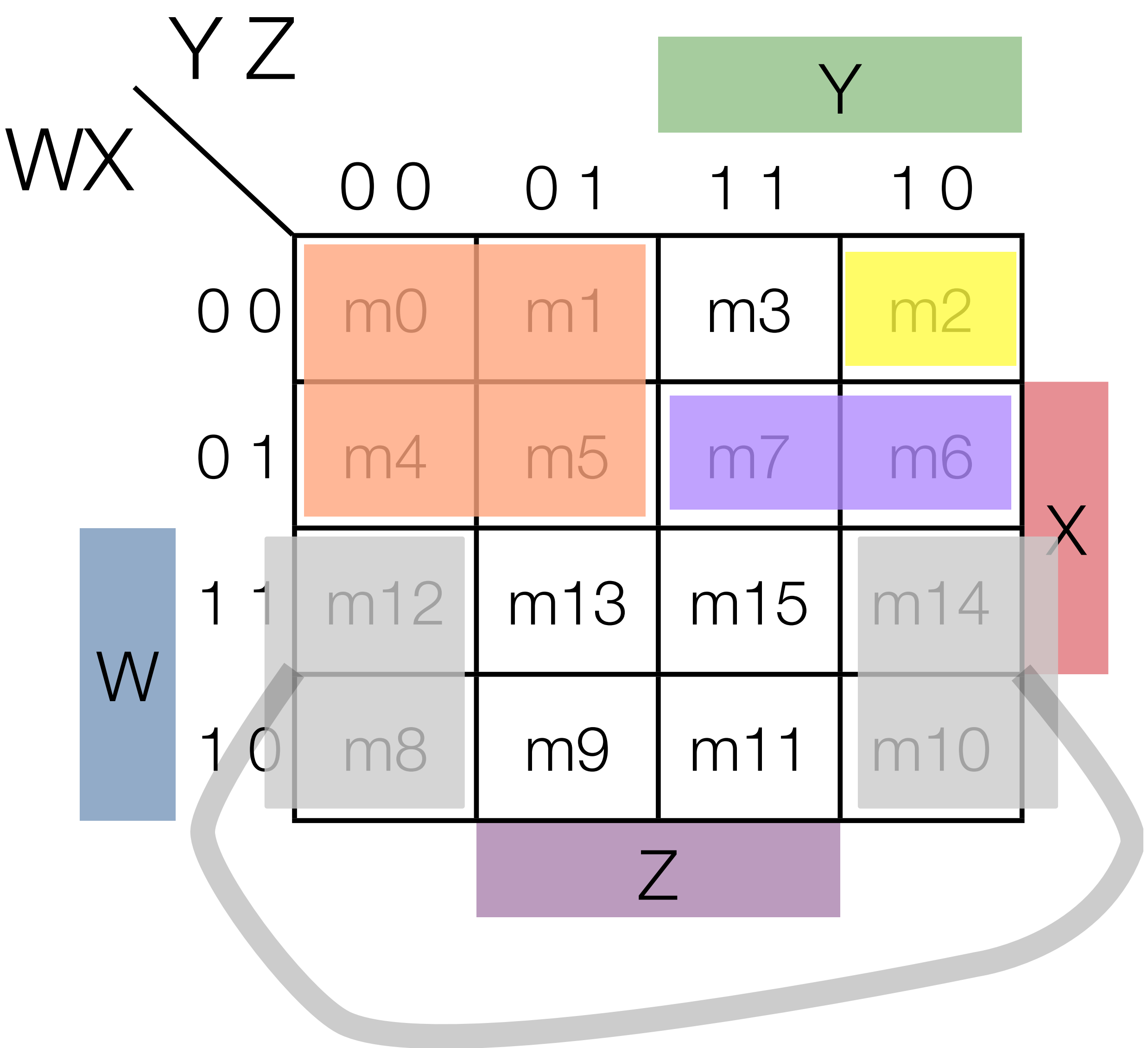
*Extension of 3-variable maps*

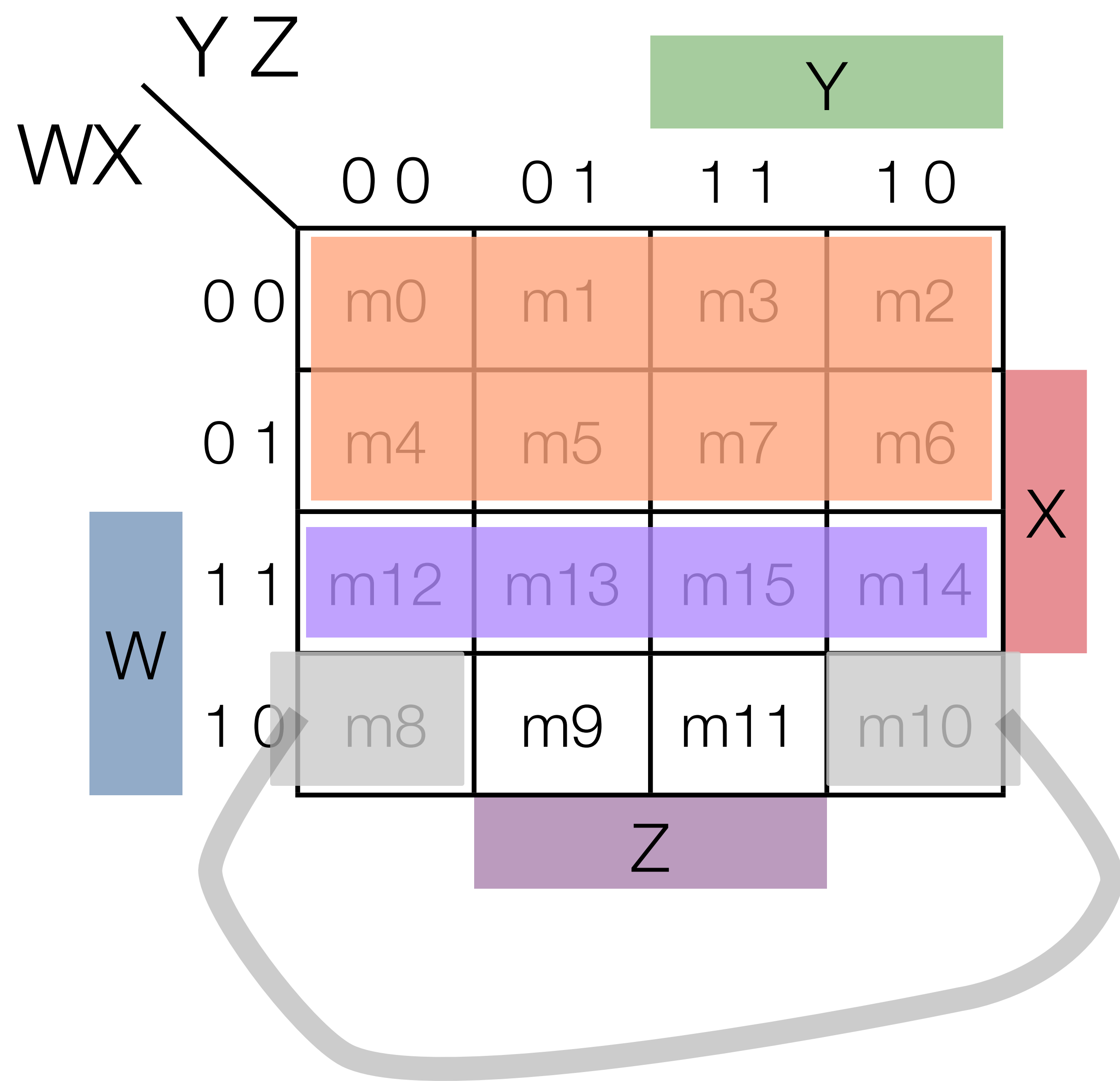
$\overline{W}XYZ$ or $W+\overline{X}+\overline{Y}+\overline{Z}$

| WX \ YZ | 0 0 | 0 1 | 1 1 | 1 0 |
|---|---|---|---|---|
| 0 0 | m0 | m1 | m3 | m2 |
| 0 1 | m4 | m5 | m7 | m6 |
| 1 1 | m12 | m13 | m15 | m14 |
| 1 0 | m8 | m9 | m11 | m10 |

# Implicants

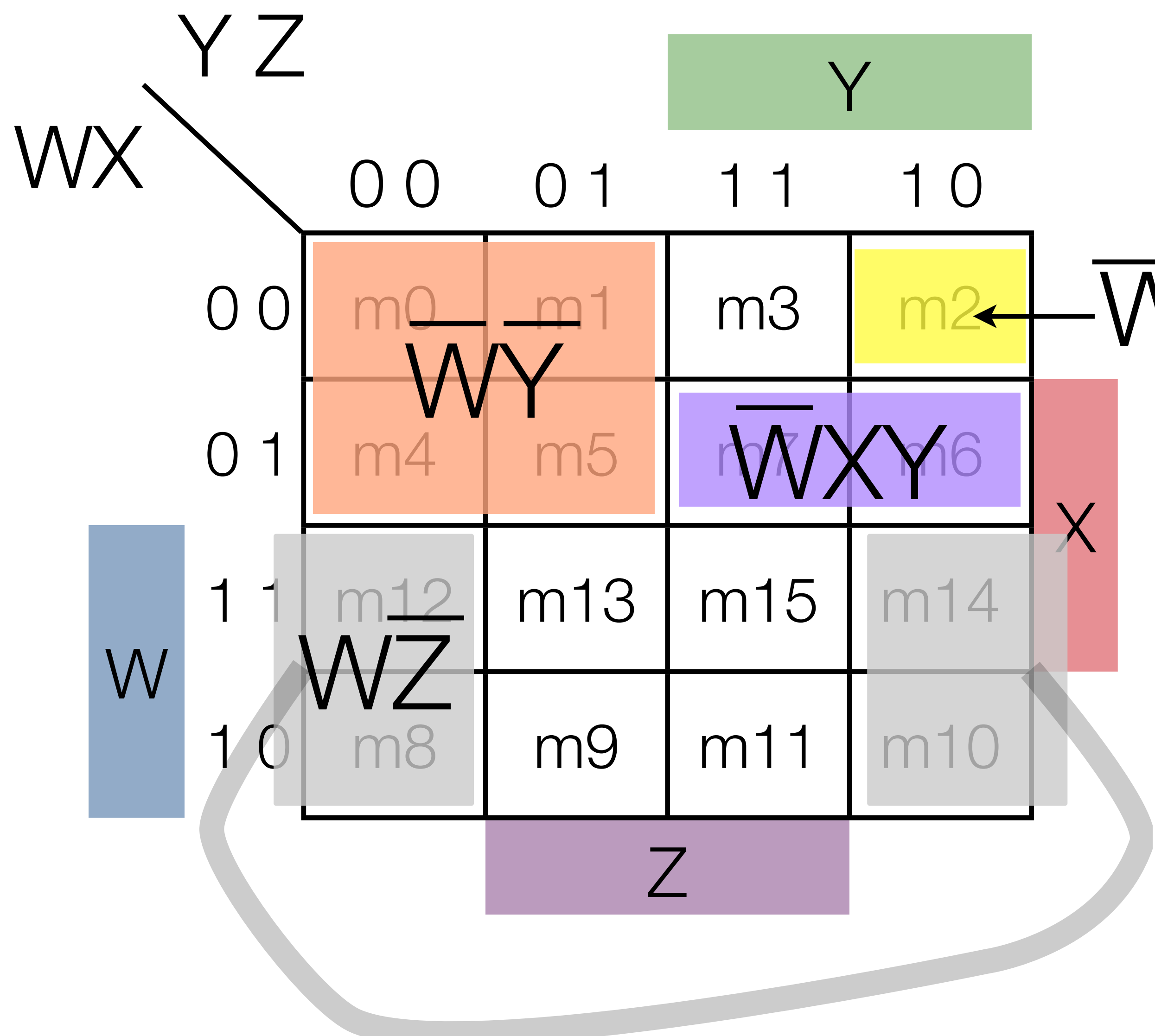
***Implicant**: a product term, which, viewed in a K-Map is a $2^i$ x $2^j$ size "rectangle" (possibly wrapping around) where i=0,1,2, j=0,1,2*

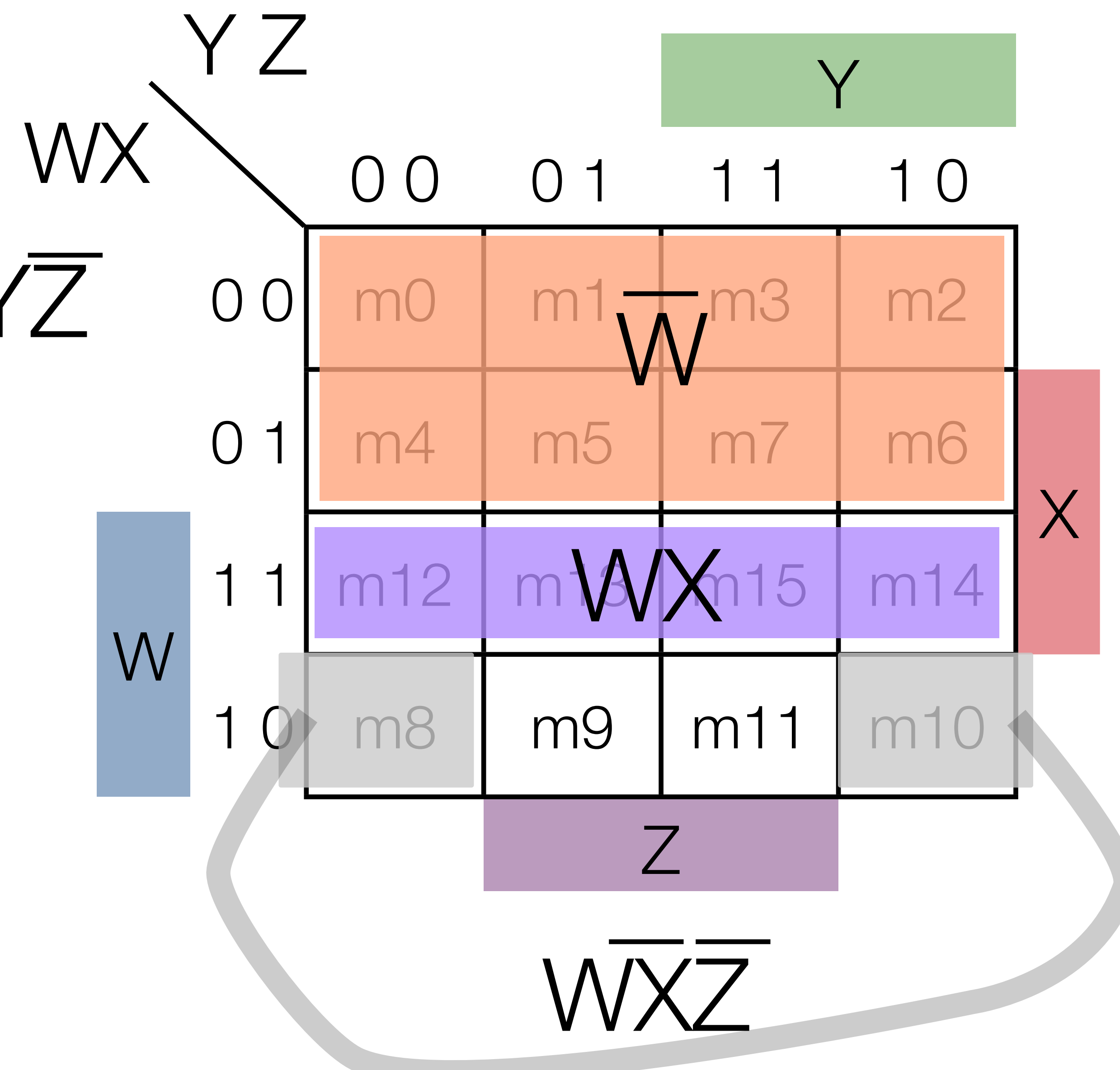


# Implicants

- **Implicant**: a product term, which, viewed in a K-Map is a $2^i \times 2^j$ size "rectangle" (possibly wrapping around) where i=0,1,2, j=0,1,2

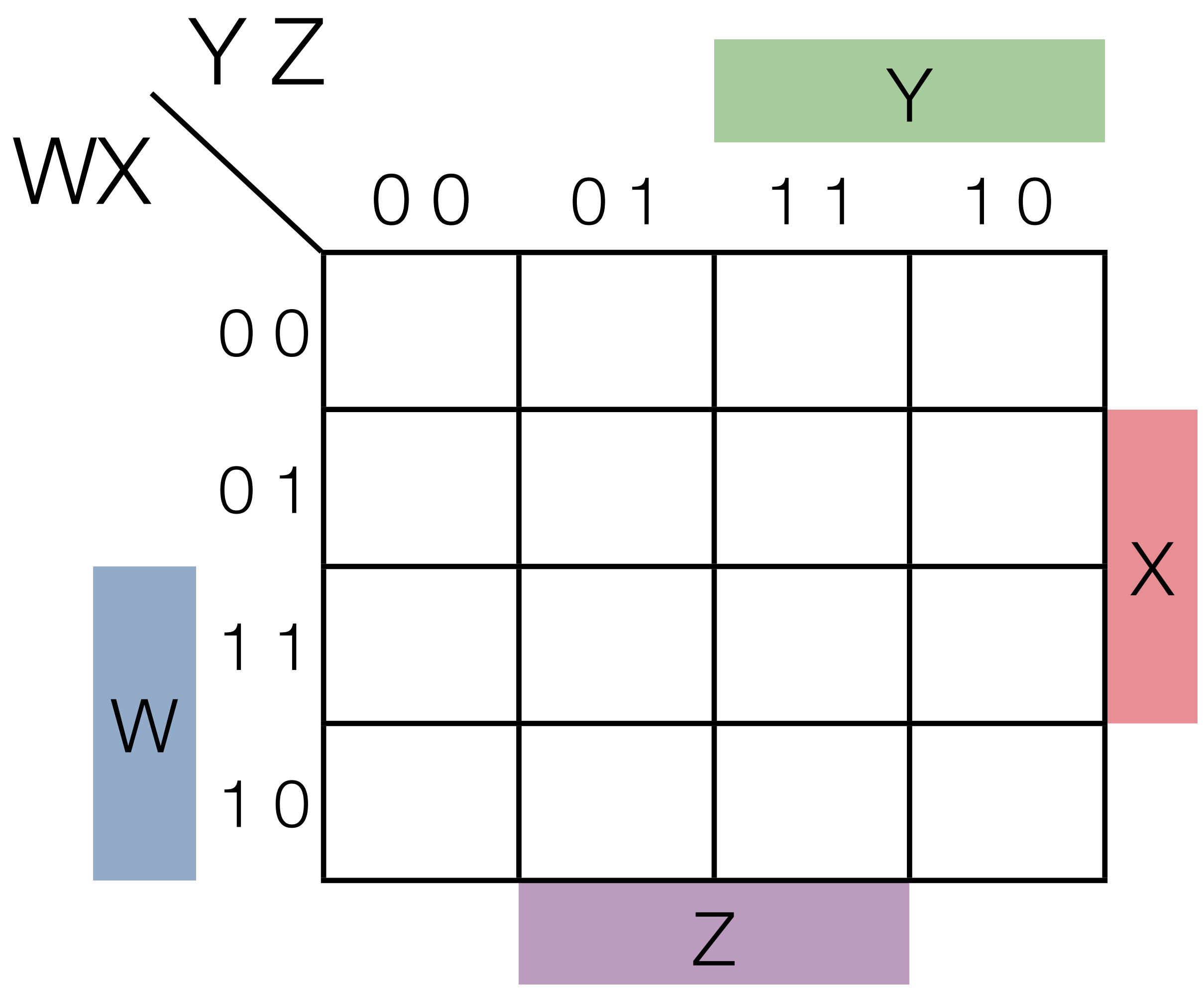



Note: bigger rectangles = fewer literals

# 4-variable Karnaugh map example

| W | X | Y | Z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 |



# Implicant terminology

- **implicant**: a product term, which, viewed in a K-Map is a $2^i$ x $2^j$ size "rectangle" (possibly wrapping around) where i=0,1,2, j=0,1,2
- **prime implicant**: An implicant not contained within another implicant.
- **essential prime implicant**: a **prime implicant** that is the **only prime implicant** to cover some minterm.

# 4-variable Karnaugh maps (3)

- List all all of the prime implicants for this function

- Is any of them an essential prime implicant?

- What is a simplified expression for this function?

| WX \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 1 | 0 |
| 01 | 1 | 1 | 1 | 0 |
| 11 | 0 | 1 | 1 | 1 |
| 10 | 0 | 1 | 0 | 0 |

# Using K-maps to build simplified circuits

- Step 1: Identify all PIs and essential PIs
- Step 2: Include all Essential PIs in the circuit (Why?)
- Step 3: If any 1-valued minterms are uncovered by EPIs, choose PIs that are "big" and do a good job covering

| | | | |
|---|---|---|---|
| 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |

| | | | |
|---|---|---|---|
| 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 |

# Design example : 2-bit multiplier

| $a_1$ | $a_0$ | $b_1$ | $b_0$ | $z_3$ | $z_2$ | $z_1$ | $z_0$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | | | | |
| 0 | 0 | 0 | 1 | | | | |
| 0 | 0 | 1 | 0 | | | | |
| 0 | 0 | 1 | 1 | | | | |
| 0 | 1 | 0 | 0 | | | | |
| 0 | 1 | 0 | 1 | | | | |
| 0 | 1 | 1 | 0 | | | | |
| 0 | 1 | 1 | 1 | | | | |
| 1 | 0 | 0 | 0 | | | | |
| 1 | 0 | 0 | 1 | | | | |
| 1 | 0 | 1 | 0 | | | | |
| 1 | 0 | 1 | 1 | | | | |
| 1 | 1 | 0 | 0 | | | | |
| 1 | 1 | 0 | 1 | | | | |
| 1 | 1 | 1 | 0 | | | | |
| 1 | 1 | 1 | 1 | | | | |

*two 2-bit #'s multiplied together to give a 4-bit solution*

*e.g.,* $a_1a_0 = 10$, $b_1b_0 = 11$, $z_3z_2z_1z_0 = 0110$

# K-Maps: Complements, PoS, don't care conditions

# Finding $\overline{F}$

*Find prime implicants corresponding to the 0s on a k-map*

| WX \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 1 | 0 | 1 |
| 01 | 1 | 1 | 0 | 1 |
| 11 | 1 | 1 | 0 | 0 |
| 10 | 1 | 1 | 0 | 1 |

$$F = \overline{Y} + \overline{X}\,\overline{Z} + \overline{W}\,\overline{Z}$$

| WX \ YZ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 1 | 0 |
| 01 | 0 | 0 | 1 | 0 |
| 11 | 0 | 0 | 1 | 1 |
| 10 | 0 | 0 | 1 | 0 |

$$\overline{F} = YZ + WXY$$

# PoS expressions from a k-map

*Find $\overline{F}$ as SoP and then apply DeMorgan's*

| WX \ YZ | 0 0 | 0 1 | 1 1 | 1 0 |
|---|---|---|---|---|
| 0 0 | 1 | 1 | 0 | 1 |
| 0 1 | 1 | 0 | 0 | 0 |
| 1 1 | 1 | 0 | 0 | 0 |
| 1 0 | 1 | 1 | 0 | 1 |

$$\overline{F} = YZ + XZ + YX$$

DeMorgan's

$$F = (\overline{Y}+\overline{Z})(\overline{Z}+\overline{X})(\overline{Y}+\overline{X})$$

# Don't care conditions

*There are circumstances in which the value of an output doesn't matter*

- For example, in that 2-bit multiplier, what if we are told that a and b will be non-0? We "don't care" what the output looks like for the input cases that should not occur
- Don't care situations are denoted by an "X" in a truth table and in Karnaugh maps.
- Can also be expressed in minterm form:
- During minimization can be treated as either a 1 or a 0

$$z2 = \sum m(10,11,14)$$

$$d2 = \sum m(0,1,2,3,4,8,12)$$

| a1 | a0 | b1 | b0 | z3 | z2 | z1 | z0 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | X | X | X | X |
| 0 | 0 | 0 | 1 | X | X | X | X |
| 0 | 0 | 1 | 0 | X | X | X | X |
| 0 | 0 | 1 | 1 | X | X | X | X |
| 0 | 1 | 0 | 0 | X | X | X | X |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | X | X | X | X |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | X | X | X | X |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |

# 2-bit multiplier non-0 multiplier

| a1 | a0 | b1 | b0 | z3 | z2 | z1 | z0 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | X | X | X | X |
| 0 | 0 | 0 | 1 | X | X | X | X |
| 0 | 0 | 1 | 0 | X | X | X | X |
| 0 | 0 | 1 | 1 | X | X | X | X |
| 0 | 1 | 0 | 0 | X | X | X | X |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | X | X | X | X |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | X | X | X | X |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |

*1's must be covered*
*0's must not be covered*
*X's are optionally covered*

$b_0$ (top), $a_0$ (right), $a_1$ (left), $b_1$ (bottom)

| | | | |
|---|---|---|---|
| X | X | X | X |
| X | 0 | 0 | 0 |
| X | 0 | 1 | 0 |
| X | 0 | 0 | 0 |

$z_3 =$

$b_0$ (top), $a_0$ (right), $a_1$ (left), $b_1$ (bottom)

| | | | |
|---|---|---|---|
| X | X | X | X |
| X | 0 | 0 | 0 |
| X | 0 | 0 | 1 |
| X | 0 | 1 | 1 |

$z_2 =$

# 2-bit multiplier non-0 multiplier (2)

| a1 | a0 | b1 | b0 | z3 | z2 | z1 | z0 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | X | X | X | X |
| 0 | 0 | 0 | 1 | X | X | X | X |
| 0 | 0 | 1 | 0 | X | X | X | X |
| 0 | 0 | 1 | 1 | X | X | X | X |
| 0 | 1 | 0 | 0 | X | X | X | X |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | X | X | X | X |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | X | X | X | X |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |

$b_0$

$a_0$

$a_1$

| | | | |
|---|---|---|---|
| X | X | X | X |
| X | 0 | 1 | 1 |
| X | 1 | 0 | 1 |
| X | 1 | 1 | 0 |

$b_1$

$b_0$

$a_0$

$a_1$

| | | | |
|---|---|---|---|
| X | X | X | X |
| X | 1 | 1 | 0 |
| X | 1 | 1 | 0 |
| X | 0 | 0 | 0 |

$b_1$

$z_1 =$

$z_0 =$

# Final thoughts on Don’t care conditions

*Sometimes “don’t cares” greatly simplify circuitry*

D

| | | | |
|---|---|---|---|
| 1 | X | X | X |
| X | 1 | X | X |
| 0 | 0 | 1 | X |
| 0 | 0 | X | 1 |

B

A

C

$$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}D + ABCD + A\overline{B}C\overline{D} \quad \text{vs.} \quad \overline{A} + C$$