Complete the following problems. Be sure to show your work for partial credit.

1. Determine the base of the numbers in each equation for the following operations to be correct:
   (a) $14 / 2 = 5$
   (b) $54 / 4 = 13$
   (c) $24 + 17 = 40$

2. Demonstrate by means of truth tables the validity of the following identities:
   (a) DeMorgan’s theorem for three variables:
      - $\overline{x + y + z} = \overline{x} \cdot \overline{y} \cdot \overline{z}$; and
      - $\overline{x \cdot y \cdot z} = \overline{x} + \overline{y} + \overline{z}$
   (b) The distributive law: $x \cdot y \cdot z = (x + y) \cdot (x + z)$

3. Given the Boolean functions $F_1$ and $F_2$.
   (a) Show that the Boolean function $E = F_1 + F_2$ contains the sum of the minterms of $F_1$ and $F_2$.
   (b) Show that the Boolean function $G = F_1 F_2$ contains only the minterms that are common to $F_1$ and $F_2$.
   Hint: start by identifying a general expression for minterm $i$ (mi) of $F_1$, $F_2$, $E$, and $G$.

4. Given the Boolean function: $F = x \cdot y + \overline{x} \cdot \overline{y} + \overline{y} \cdot z$
   (a) implement it with OR and inverter gates.
   (b) implement it with AND and inverter gates.

5. Show that the dual of the XOR is equal to its complement.

6. Give an example of a truth table requiring between 3 billion and 5 billion rows that can be constructed using fewer than 40 (but at least 1) two-input gates.

7. Convert the following expressions into sum of products and product of sums, and simplify as much as possible:
   (a) $(AB + C) (B + \overline{C} D)$
   (b) $\overline{x} + x (x + \overline{y}) (y + \overline{z})$

8. Simplify the following Boolean equations using Boolean theorems. Check for correctness using a K-map.
   (a) $Y = A \cdot C + \overline{A} \cdot \overline{B} \cdot C$
   (b) $Y = A \cdot B + \overline{A} \cdot B \cdot C + (A + \overline{C})$
   (c) $Y = A \cdot B \cdot C \cdot D + A \cdot B \cdot C + A \cdot B \cdot C \cdot D + A \cdot B \cdot D + \overline{A} \cdot B \cdot C \cdot D + B \cdot C \cdot D + \overline{A}$

9. Simplify the following functions using K-maps, and implement them with two-level NOR gate circuits:
   (a) $F = w \cdot \overline{x} \cdot \overline{z} + \overline{w} \cdot y \cdot \overline{z}$
   (b) $F(w, x, y, z) = \Sigma(5, 6, 9, 10)$