CSEE 3827: Fundamentals of Computer Systems

Lecture 3

January 28, 2009

Martha Kim
martha@cs.columbia.edu

Agenda

- DeMorgan’s theorem
- Duals
- Standard forms

DeMorgan's Theorem

- Procedure for complementing expressions
- Replace...
  - AND with OR, OR with AND
  - 1 with 0, 0 with 1
  - X with $\overline{X}$, $\overline{X}$ with X

$$\overline{XY} = \overline{X} + \overline{Y}$$

$$\overline{X + Y} = \overline{X}\,\overline{Y}$$

Prove DeMorgan's Theorem

$$\overline{XY} = \overline{X} + \overline{Y}$$

X	Y	$\overline{XY}$
0	0	
0	1	
1	0	
1	1	

X	Y	$\overline{X} + \overline{Y}$
0	0	
0	1	
1	0	
1	1	

Prove DeMorgan's Theorem

$$\overline{XY} = \overline{X} + \overline{Y}$$

X	Y	$\overline{XY}$
0	0	1
0	1	1
1	0	1
1	1	0

X	Y	$\overline{X} + \overline{Y}$
0	0	1
0	1	1
1	0	1
1	1	0

DeMorgan's Practice

$$F = \overline{\overline{A\overline{B}C} + \overline{ACD} + B\overline{C}}$$

DeMorgan's Practice

$$F = \overline{\overline{A\overline{B}C} + \overline{ACD} + B\overline{C}}$$

$$= (A\overline{B}C)(ACD)(\overline{B\overline{C}})$$

$$= (A\overline{B}CD)(\overline{B}+C)$$

$$= A\overline{B}CD + A\overline{B}CD$$

$$= A\overline{B}CD$$

Duals

Duals

- A theorem about theorems
- All boolean expressions have duals
- Any theorem you can prove, you can also prove for its dual
- To form a dual...
  - replace AND with OR, OR with AND
  - replace 1 with 0, 0 with 1

What is the dual of this expression?

$$\overline{X} + \overline{Y} = \overline{XY}$$

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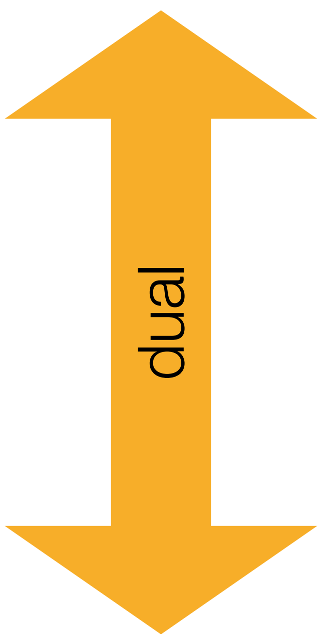
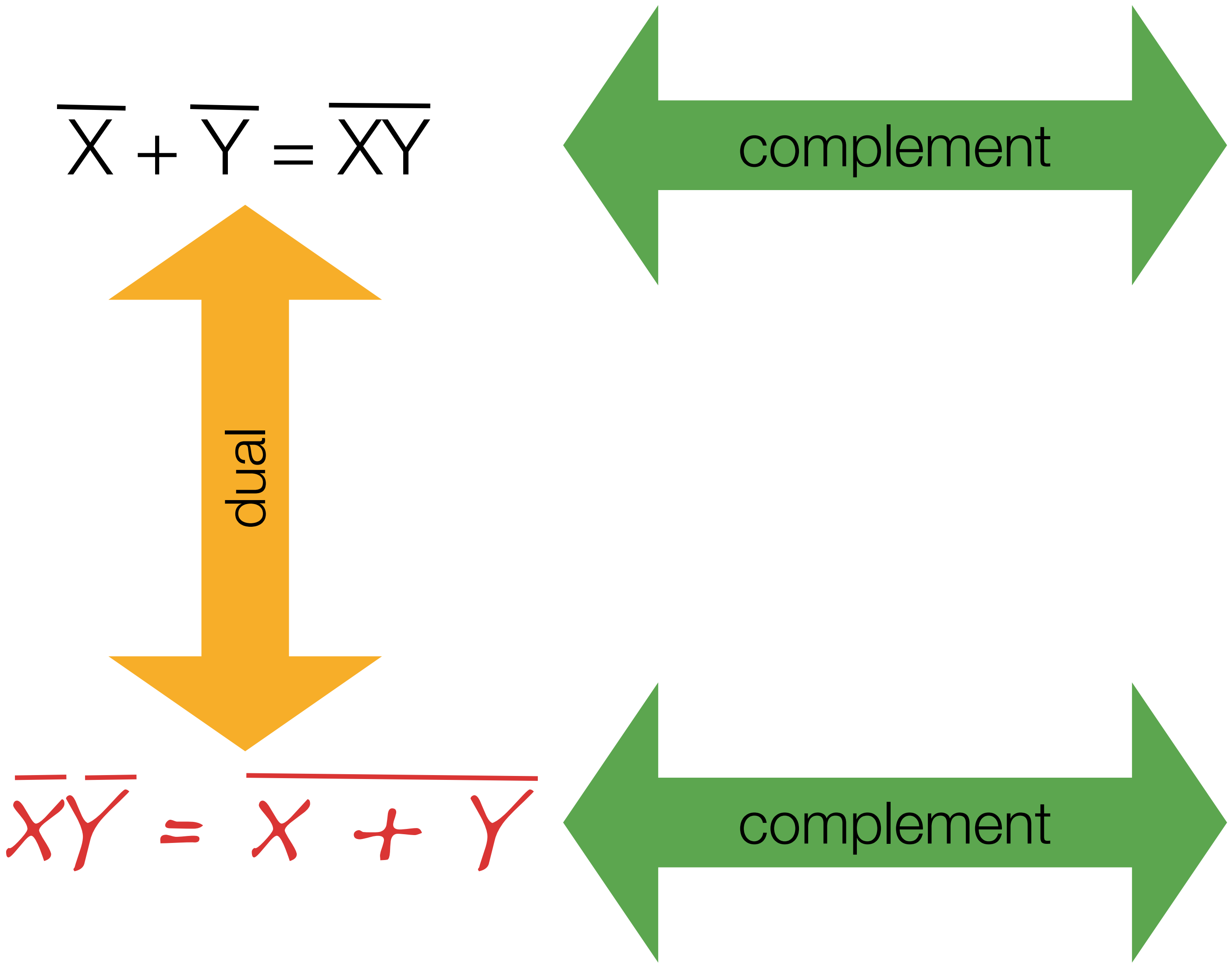
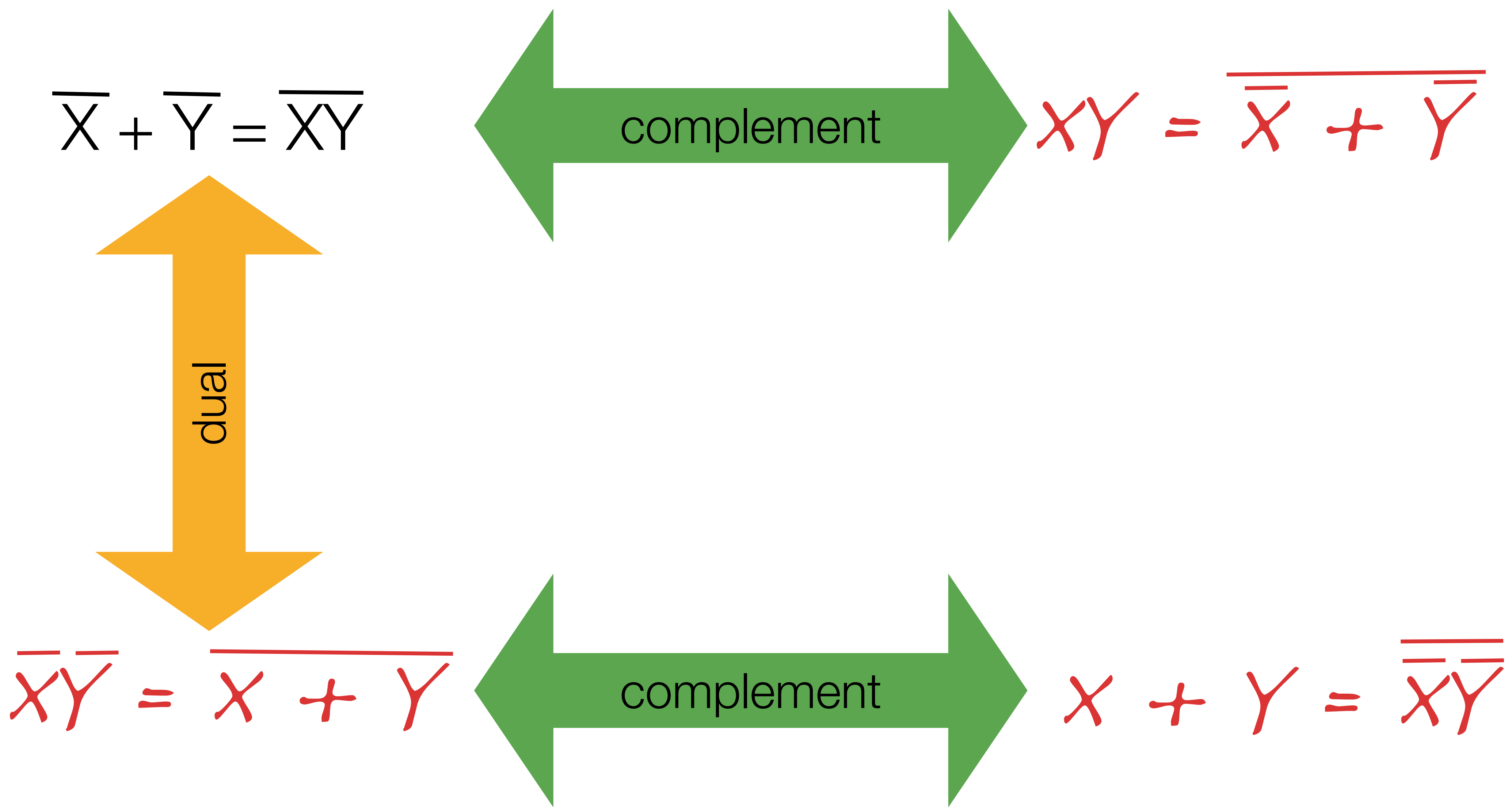


$$\overline{X}\,\overline{Y} = \overline{X + Y}$$

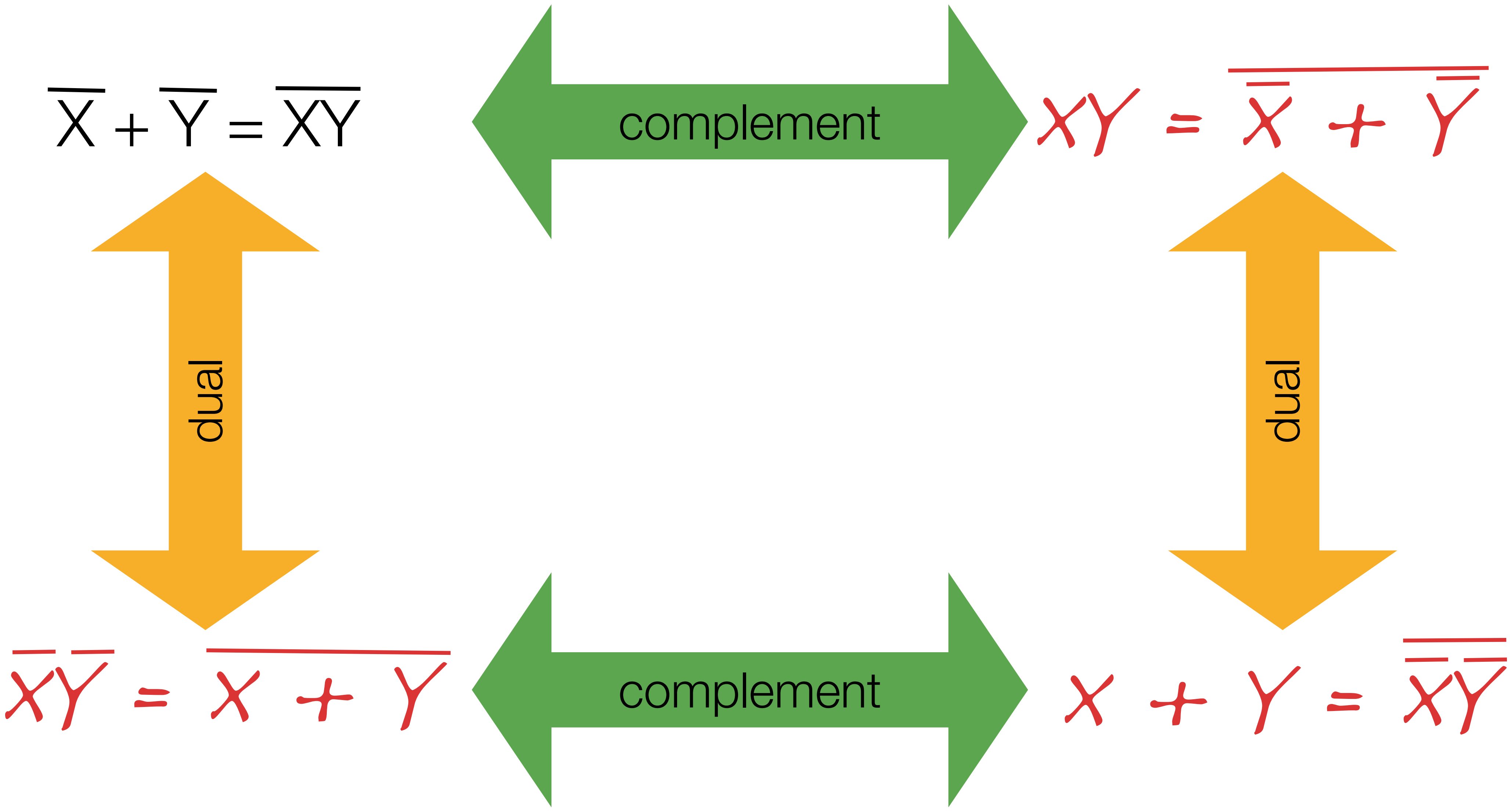
What are the complements of these expressions?



What are the complements of these expressions?

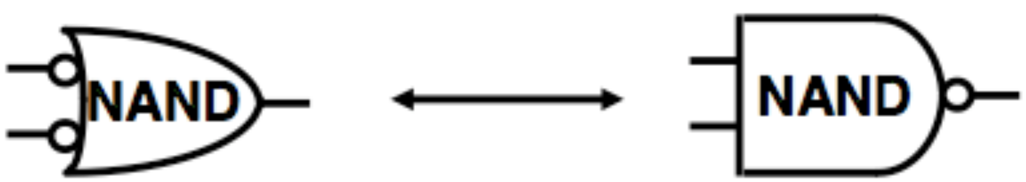


These are also the duals of one another.

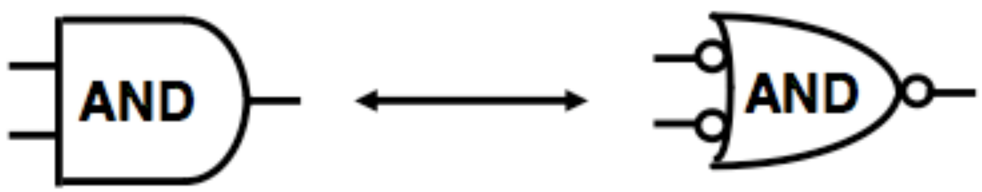


Can be used for gate manipulation.

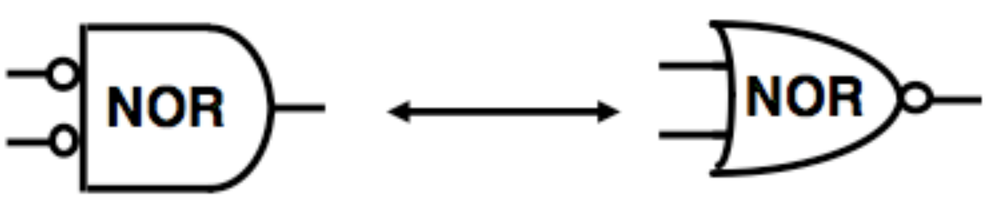
$$\overline{X} + \overline{Y} = \overline{XY}$$



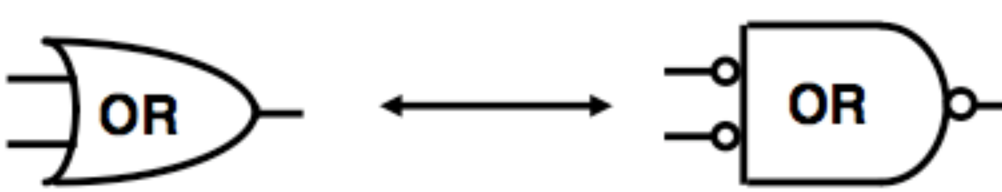
$$XY = \overline{\overline{X} + \overline{Y}}$$



$$\overline{X}\,\overline{Y} = \overline{X + Y}$$



$$X + Y = \overline{\overline{X}\,\overline{Y}}$$



Boolean Algebra: Identities and Theorems

OR	AND	NOT	
$X+0 = X$	$X1 = X$		(identity)
$X+1 = 1$	$X0 = 0$		(null)
$X+X = X$	$XX = X$		(idempotent)
$X+\overline{X} = 1$	$X\overline{X} = 0$		(complementarity)
		$\overline{\overline{X}} = X$	(involution)
$X+Y = Y+X$	$XY = YX$		(commutativity)
$X+(Y+Z) = (X+Y)+Z$	$X(YZ) = (XY)Z$		(associativity)
$X(Y+Z) = XY + XZ$	$X+YZ = (X+Y)(X+Z)$		(distributive)
$\overline{X+Y} = \overline{X}\,\overline{Y}$	$\overline{XY} = \overline{X} + \overline{Y}$		(DeMorgan's theorem)

Standard forms

Standard Forms

- There are many ways to express a boolean expression

$$F = XYZ + XYZ + XZ$$
$$= XY(Z + Z) + XZ$$
$$= XY + XZ$$

- It is useful to have a standard or canonical way
- Derived from truth table
- Generally not the simplest form

Two principle standard forms

- Sum-of-products (SOP)
- Product-of-sums (POS)

Sum-of-products form

- *sometimes also called **disjunctive normal form** (DNF)*
- *sometimes also called a **minterm expansion***

$$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$$

A	B	C	F	$\overline{F}$
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	0	1

$$\overline{F} = \overline{A}BC + AB\overline{C} + ABC$$

Sum-of-products form 2

(variables appear once in each minterm)

A	B	C	F	$\overline{F}$	minterm
0	0	0	1	0	m0 $\overline{A}\,\overline{B}\,\overline{C}$
0	0	1	1	0	m1 $\overline{A}\,\overline{B}C$
0	1	0	1	0	m2 $\overline{A}B\overline{C}$
0	1	1	0	1	m3 $\overline{A}BC$
1	0	0	1	0	m4 $A\overline{B}\,\overline{C}$
1	0	1	1	0	m5 $A\overline{B}C$
1	1	0	0	1	m6 $AB\overline{C}$
1	1	1	0	1	m7 ABC

$$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$$

$$= m0 + m1 + m2 + m4 + m5$$

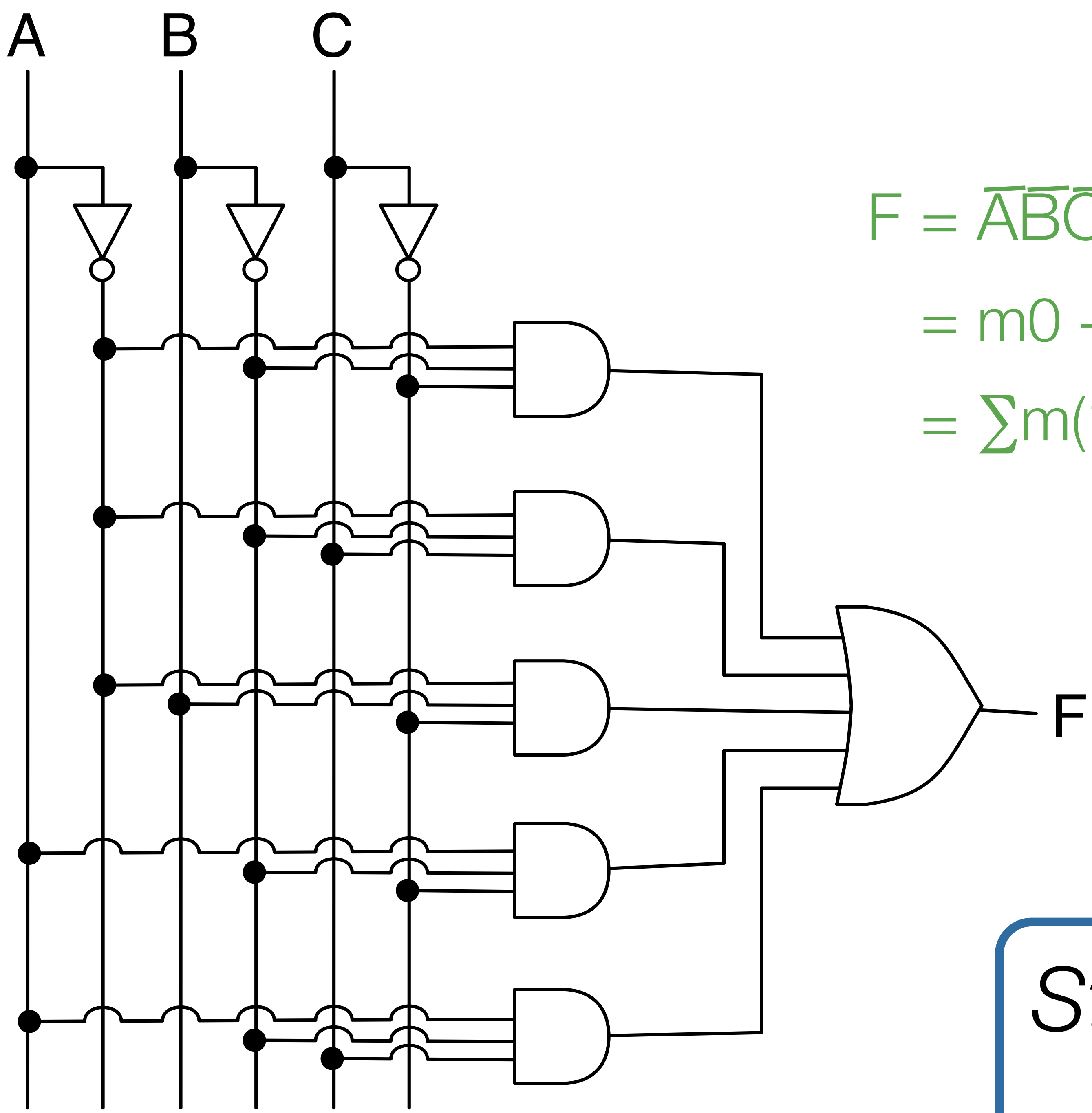
$$= \sum m(1,0,2,4,5)$$

$$\overline{F} = \overline{A}BC + AB\overline{C} + ABC$$

$$= m3 + m6 + m7$$

$$= \sum m(3,6,7)$$

Sum-of-products form 3



$$F = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$$

$$= m0 + m1 + m2 + m4 + m5$$

$$= \sum m(1,0,2,4,5)$$

Standard form is not minimal form!

Two principle standard forms

- Sum-of-products (SOP)
- Product-of-sums (POS)

Product-of-sums form

- *sometimes also called* ***conjunctive normal form*** *(CNF)*
- *sometimes also called a* ***maxterm expansion***

$$F = (A+\overline{B}+\overline{C})\,(\overline{A}+\overline{B}+C)\,(\overline{A}+\overline{B}+\overline{C})$$

A	B	C	F	$\overline{F}$
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	0	1

Product-of-sums form

- *sometimes also called* ***conjunctive normal form*** *(CNF)*
- *sometimes also called a* ***maxterm expansion***

$$F = (A+\overline{B}+\overline{C})\,(\overline{A}+\overline{B}+C)\,(\overline{A}+\overline{B}+\overline{C})$$

A	B	C	F	$\overline{F}$
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	0	1

$$\overline{F} = (A+B+C)\,(A+B+\overline{C})\,(\overline{A}+B+\overline{C})\,(\overline{A}+B+C)\,(\overline{A}+B+\overline{C})$$

Product-of-sums form 2

A	B	C	F	$\overline{F}$	maxterm	
0	0	0	1	0	M0	$A+B+C$
0	0	1	1	0	M1	$A+B+\overline{C}$
0	1	0	1	0	M2	$A+\overline{B}+C$
0	1	1	0	1	M3	$A+\overline{B}+\overline{C}$
1	0	0	1	0	M4	$\overline{A}+B+C$
1	0	1	1	0	M5	$\overline{A}+B+\overline{C}$
1	1	0	0	1	M6	$\overline{A}+\overline{B}+C$
1	1	1	0	1	M7	$\overline{A}+\overline{B}+\overline{C}$

$$F = (A + \overline{B} + \overline{C})\,(\overline{A} + \overline{B} + C)\,(\overline{A} + \overline{B} + \overline{C})$$
$$= (M3)(M6)(M7)$$
$$= \prod M(3,6,7)$$

$$\overline{F} = (A+B+C)\,(A+B+\overline{C})\,(A+\overline{B}+C)\,(\overline{A}+B+C)\,(\overline{A}+B+\overline{C})$$
$$= (M0)(M1)(M2)(M4)(M5)$$
$$= \prod M(0,1,2,4,5)$$

Summary of SOP and POS

	F	$\overline{F}$
Sum of products (SOP)	$\sum m(F = 1)$	$\sum m(F = 0)$
Product of sums (POS)	$\prod M(F = 0)$	$\prod M(F = 1)$

Standard Form Example

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

	F	$\overline{F}$
Sum of products (SOP)		
Product of sums (POS)		

Standard Form Example

A	B	C	F	$\overline{F}$
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

	F	$\overline{F}$
Sum of products (SOP)	$\sum m(1,3,5,6)$	$\sum m(0,2,4,7)$
Product of sums (POS)	$\prod M(0,2,4,7)$	$\prod M(1,3,5,6)$

Converting between canonical forms

	F	$\overline{F}$
Sum of products (SOP)	$\sum m(F = 1)$	$\sum m(F = 0)$
Product of sums (POS)	$\prod M(F = 0)$	$\prod M(F = 1)$

DeMorgans

Next class: systematic minimization