CSEE W3827

Fundamentals of Computer Systems

Homework Assignment 1

Prof. Martha A. Kim

Columbia University

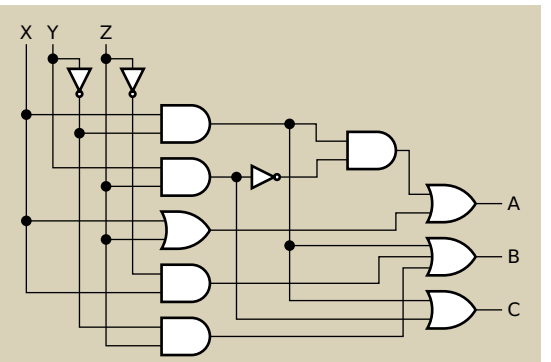
Due September 24, 2015 at 10:10 AM

Write your name **and UNI** on your solutions

Show your work for each problem; we are more interested in how you get the answer than whether you get the right answer.

1. (5 pts.) Prove that the distributive property is true:
   - $X(Y + Z) = XY + XZ$, and
   - $X + (YZ) = (X + Y)(X + Z)$

2. (15 pts.) For each of the following pairs, prove that the circuits are logically equal or unequal.



(a) A and B

(b) B and C

(c) A and C

3. (15 pts.) *Algebraically manipulate* the following arbitrary expression to take the forms specified.

$$\overline{XY + Z} + \overline{Y}Z + XZ$$

(a) Sum-of-products that is not sum-of-minterms

(b) Minimal sum-of-products

(c) Sum-of-minterms

4. (15 pts.) Consider the function M which is true if and only if the input 4-bit number, DCBA, is a multiple of three. The 4 bits, DCBA, encodes values between 0 and 15.

   - Give a truth table for M.
   - Give a minimal, sum-of-products expression for M.
   - Now assume that DCBA encodes only the values between 0 and 9, and inputs 10-15 will never be seen. Give a minimal, sum-of-products expression for this new version of M.

5. (15 pts.) Come up with a way of encoding the numbers 0 through 5 using 3 bits so that adjacent numbers differ in only one bit and the representations of 0 and 5 differ in only one bit as well.

6. (15 pts.) A teacher is forming a team for an inter-school competition:

   - Alice is good at Math, Economics and History.
   - Bob is good at Physics and Maths.
   - Chris is good at English and Physics.
   - David is good at Economics and Physics.

   Draw a schematic for a circuit whose inputs are four wires (one for each student) whose value indicates whether the student is a member of the proposed team. The output is true if and only if a given team is good at all of the subjects (Math, Economics, History, Physics and English).

7. (20 pts.) Which of the following three representations for a playing card requires the fewest gate inputs to check if three cards share a suit (i.e., all three cards are diamonds, or all three are hearts, etc.)? Justify your decision.

   To compute the number of gate inputs, count only AND and OR gates (of any size: 2-input, 3-input, etc.), and sum the total number of inputs across all of the AND and OR gates in the circuit. Inverters are excluded from this count.

   (a) Representing the suit as a 4-bit one-hot number. A one-hot encoding has exactly 1 bit set to 1, i.e., 0001, 0010, 0100, and 1000.

   (b) Representing the suit as a two-bit number: 00, 01, 10, 11.

   (c) Representing the suit with three bits that can be either zero-hot or one-hot: 000, 001, 010, 100.