Fundamentals of Computer Systems
Finite State Machines

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Fall 2015
Finite State Machine Components

Current State → Next State

Inputs → Next State

Next State → Current State

CLOCK

Current State → Outputs

Inputs? → Outputs
Moore and Mealy Machines

The Moore Form:

Outputs are a function of *only* the current state.
Moore and Mealy Machines

The Mealy Form:

Outputs may be a function of both the current state and the inputs.

A mnemonic: Moore machines often have more states.
Mealy Machines are the Most General

Another, equivalent way of drawing Mealy Machines

This is exactly the synchronous digital logic paradigm
Moore vs. Mealy FSMs

Alyssa P. Hacker has a snail that crawls down a paper tape with 1’s and 0’s on it. The snail smiles whenever the last four digits it has crawled over are 1101. Design Moore and Mealy FSMs of the snail’s brain.
State Transition Diagrams: Looking for “1101”

Moore Machine: States indicate output
State Transition Diagrams: Looking for “1101”

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State Transition Diagrams: Looking for “1101”

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State Transition Diagrams: Looking for “1101”

Moore Machine: States indicate output

Mealy Machine: Arcs indicate input/output
State Transition Diagrams: Looking for “1101”

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Mealy Machine: Arcs indicate input/output
State Transition Diagrams: Looking for “1101”

Moore Machine: States indicate output

Mealy Machine: Arcs indicate input/output
Moore Machine

<table>
<thead>
<tr>
<th>Next State</th>
<th>Output</th>
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<td><strong>Y</strong></td>
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## Moore Machine

### Next State

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<tr>
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### Output

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### Diagram

![Moore Machine Diagram](image-url)
# Mealy Machine

<table>
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<tr>
<th>S</th>
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<th>S’</th>
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<tbody>
<tr>
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Mealy Machine

<table>
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</table>
More Intuitive Solutions using Shift Registers

Moore Form: Output Depends Only on State

Mealy Form: Output Depends on Input Immediately
FSM Example: A Traffic Light Controller

This controls a traffic light at the intersection of a busy highway and a farm road. Normally, the highway light is green but if a sensor detects a car on the farm road, the highway light turns yellow then red. The farm road light then turns green until there are no cars or after a long timeout. Then, the farm road light turns yellow then red, and the highway light returns to green and cannot be interrupted by a car before a long timer has elapsed.

The inputs to the machine are the car sensor, a short timeout signal, and a long timeout signal. The outputs are a timer start signal and the colors of the highway and farm road lights.

State Transition Diagram for the TLC

Inputs
C = Car sensor
S = Short Timeout
L = Long Timeout

Outputs
T = Timer Reset
H = Highway color
F = Farm road color
State Transition Diagram for the TLC

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State Transition Diagram for the TLC

 Inputs  
 C = Car sensor  
 S = Short Timeout  
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 Outputs  
 T = Timer Reset  
 H = Highway color  
 F = Farm road color

HG
H: G  
F: R  

HY
H: Y  
F: R  

FG
H: R  
F: G

\(\bar{C} + \bar{L}/\bar{T}\)  
\(CL/T\)  
\(\bar{S}/\bar{T}\)  
\(S/T\)
State Transition Diagram for the TLC

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Inputs:
- $\overline{C} + \overline{L}/T$
- $S/T$

Outputs:
- $T$
- $H$
- $F$

Transitions:
- From HG to HG via $H : G$ and $F : R$
- From HG to HY via $C + L/T$
- From HY to HG via $S/T$
- From HY to HY via $C + L/T$
- From HY to FG via $S/T$
- From HG to HY via $S/T$
- From FG to FG via $C\overline{L}/T$
- From FG to FY via $C\overline{L}/T$
- From FY to FY via $S/T$
- From FY to HG via $S/T$
- From HG to FY via $S/T$

Inputs defined:
- C = Car sensor
- S = Short Timeout
- L = Long Timeout

Outputs defined:
- T = Timer Reset
- H = Highway color
- F = Farm road color
State Transition Diagram for the TLC

<table>
<thead>
<tr>
<th>Q</th>
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### State and Output Encoding 1: Binary

<table>
<thead>
<tr>
<th>States</th>
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<tbody>
<tr>
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State and Output Encoding 1: Binary

<table>
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\[
T = \overline{Q_1} \overline{Q_0} CL + \overline{Q_1} Q_0 S + Q_1 \overline{Q_0} (C + L) + Q_1 Q_0 S
\]

\[
D_1 = \overline{Q_1} Q_0 S + Q_1 \overline{Q_0} + Q_1 Q_0 S
\]

\[
D_0 = \overline{Q_1} \overline{Q_0} CL + \overline{Q_1} Q_0 \overline{S} + Q_1 \overline{Q_0} (C + L) + Q_1 Q_0 \overline{S}
\]

\[
H_1 = Q_1
\]

\[
H_0 = \overline{Q_1} Q_0
\]

\[
F_1 = \overline{Q_1}
\]

\[
F_0 = Q_1 Q_0
\]
State and Output Encoding 1: Binary

<table>
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<tr>
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\[
T = \overline{Q}_1 \overline{Q}_0 CL + Q_0 S + Q_1 \overline{Q}_0 (\overline{C} + L)
\]

\[
D_1 = Q_0 S + Q_1 \overline{Q}_0
\]

\[
D_0 = \overline{Q}_1 \overline{Q}_0 CL + Q_0 \overline{S} + Q_1 \overline{Q}_0 (\overline{C} + L)
\]

\[
H_1 = Q_1
\]

\[
H_0 = \overline{Q}_1 Q_0
\]

\[
F_1 = \overline{Q}_1
\]

\[
F_0 = Q_1 Q_0
\]
State and Output Encoding 1: Binary

<table>
<thead>
<tr>
<th>Q</th>
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</table>

\[ T = Q_0 (Q_1 C L + Q_1 (\bar{C} + L)) + Q_0 S \]

\[ D_1 = Q_0 S + Q_1 \bar{Q}_0 \]

\[ D_0 = Q_0 (Q_1 C L + Q_1 (\bar{C} + L)) + Q_0 \bar{S} \]

\[ H_1 = Q_1 \]

\[ H_0 = Q_1 Q_0 \]

\[ F_1 = \bar{Q}_1 \]

\[ F_0 = Q_1 Q_0 \]
State and Output Encoding 1: Binary

\[
T = \overline{Q_0}(\overline{Q_1}CL + Q_1(\overline{C} + L)) + \overline{Q_0}S
\]

\[
D_1 = Q_0S + Q_1\overline{Q_0}
\]

\[
D_0 = \overline{Q_0}(\overline{Q_1}CL + Q_1(\overline{C} + L)) + \overline{Q_0}\overline{S}
\]

\[
H_1 = Q_1
\]

\[
H_0 = \overline{Q_1}Q_0
\]

\[
F_1 = \overline{Q_1}
\]

\[
F_0 = Q_1Q_0
\]
State and Output Encoding 2: One-Hot

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<th>States</th>
<th>Output</th>
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<tr>
<td>HY</td>
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<tr>
<td>FG</td>
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<td>FY</td>
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</table>

<table>
<thead>
<tr>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
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<tr>
<td>Y</td>
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<tr>
<td>R</td>
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State and Output Encoding 2: One-Hot

<table>
<thead>
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<th>S</th>
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<th>T</th>
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## State and Output Encoding 2: One-Hot

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State and Output Encoding 2: One-Hot

\[
\begin{array}{cccc|cc}
Q & C & S & L & T & D \\
0001 & 0 & X & X & 0 & 0001 \\
0001 & X & X & 0 & 0 & 0001 \\
0001 & 1 & X & 1 & 1 & 0010 \\

0010 & X & 0 & X & 0 & 0010 \\
0010 & X & 1 & X & 1 & 0100 \\

0100 & 1 & X & 0 & 0 & 0100 \\
0100 & 0 & X & X & 1 & 1000 \\
0100 & X & X & 1 & 1 & 1000 \\

1000 & X & 0 & X & 0 & 1000 \\
1000 & X & 1 & X & 1 & 0001 \\
\end{array}
\]

\[
T = Q_0CL + Q_1S + Q_2(C + L) + Q_3S \\
D_3 = Q_2(C + L) + Q_3\bar{S} \\
D_2 = Q_1S + Q_2(C + L) \\
D_1 = Q_0CL + Q_1\bar{S} \\
D_0 = Q_0(CL) + Q_3S \\
H_R = Q_2 + Q_3 \\
H_Y = Q_1 \\
H_G = Q_0 \\
F_R = Q_0 + Q_1 \\
F_Y = Q_3 \\
F_G = Q_2
\]

\[
Q & H & F \\
0001 & 001 & 100 \\
0010 & 010 & 100 \\
0100 & 100 & 001 \\
1000 & 100 & 010 \\
\]
State and Output Encoding 2: One-Hot

\[ T = Q_0CL + Q_1S + Q_2(\overline{C} + L) + Q_3S \]
\[ D_3 = Q_2(\overline{C} + L) + Q_3\overline{S} \]
\[ D_2 = Q_1S + Q_2(\overline{C} + L) \]
\[ D_1 = Q_0CL + Q_1\overline{S} \]
\[ D_0 = Q_0(\overline{CL}) + Q_3S \]
\[ H_R = Q_2 + Q_3 \]
\[ H_Y = Q_1 \]
\[ H_G = Q_0 \]
\[ F_R = Q_0 + Q_1 \]
\[ F_Y = Q_3 \]
\[ F_G = Q_2 \]